

Publish'd by Thos. Malton April 1st 1774.

t 1482.g.10

A COMPLEAT TREATISE ON PERSPECTIVE, IN THEORY AND PRACTICE;

ON THE TRUE PRINCIPLES OF

DR. BROOK TAYLOR.

MADE CLEAR, IN THEORY, BY

VARIOUS MOVEABLE SCHEMES, AND DIAGRAMS;

AND

REDUCED TO PRACTICE,

IN THE

MOST FAMILIAR AND INTELLIGENT MANNER;

SHEWING

HOW TO DELINEATE ALL KINDS OF REGULAR OBJECTS, BY RULE.

THE THEORY AND PROJECTION OF SHADOWS, BY SUN-SHINE, AND BY CANDLE-LIGHT.

THE EFFECTS OF REFLECTED LIGHT, ON OBJECTS;

THEIR REFLECTED IMAGES, ON THE SURFACE OF WATER,

AND ON POLISHED, PLANE SURFACES,

IN ALL POSITIONS.

THE WHOLE EXPLICITLY TREATED;

AND ILLUSTRATED, IN A GREAT VARIETY OF FAMILIAR EXAMPLES;

IN FOUR BOOKS.

EMBELLISHED WITH

AN ELEGANT FRONTISPIECE, AND FORTY-EIGHT PLATES.

CONTAINING

DIAGRAMS, VIEWS, AND ORIGINAL DESIGNS, IN ARCHITECTURE, &c. NEATLY ENGRAVED.

ALL ORIGINALS;

INVENTED, DELINEATED, AND, GREAT PART, ENGRAVED BY THE AUTHOR,

THOMAS MALTON, the Elder

THE SECOND EDITION, CORRECTED AND IMPROVED; WITH LARGE ADDITIONS.

LONDON:

Printed for the Author; and sold by Meſſrs. Robson, in Bond-ſtreet; Becket, Adelphi, Strand; Taylor, near Great Turn-ſtile, Holborn; Dilly, in the Poultry; and by the Author, No. 56, Poland-ſtreet, Oxford Road, near the Pantheon.

MDCCLXXVIII.

A.D 1778

THE SECOND EDITION, WITH

CONSIDERABLE ADDITIONS,

OF

THE BOOK OF BOOKS;

OR,

A COMPLETE LIBRARY

OF

THEOLOGY AND PRACTICE;

BEING A

COLLECTION

OF

THE CHOICEST TREATISES,

ON

EVERY SUBJECT,

NECESSARY TO BE KNOWN BY

CHRISTIANS.

TO THE

PRESIDENT AND MEMBERS,

OF THE

ROYAL ACADEMY,

For PAINTING, SCULPTURE, and ARCHITECTURE;

INSTITUTED AT LONDON,

By, and under the Auſpices of, his moſt gracious MAJESTY

GEORGE THE THIRD.

Gentlemen,

IF the unremited labour and aſſiduity, with which I have proſecuted the ſtudy of the Science and Art of Perſpective, have thrown any new Light on that moſt neceſſary branch of the Polite Arts, ſo, as to render its Principles clearer and better underſtood, in Theory, more eaſily applicable, in Practice, more generally uſeful and ſubſervient to the Arts, of which, it muſt be allowed to be the foundation, 'tis what I have chiefly aimed at; and preſume, this Work may not be wholly undeſerving your Patronage and Encouragement, although it may not merit your entire Approbation. Such as it is, Gentlemen, I ſubmit to your Candour, and claim your Protection.

The propriety of dedicating ſuch a Work to Gentlemen, who are, undoubtedly, the moſt competent Judges of it, will plead an excuſe for my preſumption; if, on an accurate examination of its Contents, it be found, that I have rendered an apparently intricate Science more familiar, and better adapted to the capacities of young Students, in ſo eſſential a part of their Studies; the neglect of which, amongſt the riſing Artiſts, is much to be lamented. Perſpective ſeems to be looked on as an Appendage, only, which may be diſpenſed with, inſtead of the firſt requiſite; in which, the Student, who would be a Candidate for Fame, ſhould be well grounded.

It

DEDICATION.

It is with reluctance I add, but it is too obvious, that, in many fine Pieces, in reſpect of Deſign, Compoſition, Drawing, or Colouring, there ſeems to be a want of a juſt Idea of PERSPECTIVE.

Pardon me, Gentlemen, I do not mean to depreciate the Merits of ſuch experienced Artiſts, in their ſeveral Performances; for, in my opinion, there only wants a thorough knowledge of Perſpective, to render the preſent Age as famous as any former Æra; which, now, is eſtabliſhed on the moſt permanent and infallible Principles; whereby, the trouble of projecting Objects, perſpectively, is, in the proceſs, greatly abridged and facilitated.

I am,

Gentlemen,

with great reſpect,

Your moſt obedient,

and obliged humble Servant,

THOMAS MALTON.

PREFACE.

IN every Age, and in every Climate, where the Polite Arts are cultivated and encouraged, Emulation, and a deſire of Fame, inſpire the Breaſts of the riſing Generations; the ingenious Mechanic catches the ardent and laudable Flame; Commerce is extended, the People are neceſſarily enriched, and the State becomes potent and formidable. Architecture rears her ſtately Domes and lofty Turrets to the Skies; the ſumptuous Edifices, raiſed by the hand of the able Mechanic, and embelliſhed by the ingenious Sculptor, with breathing Statues and almoſt animated Marble, proclaim abroad the Magnificence of the Founder, and juſtly immortalize the ſkilful Architect; whoſe Works may vie with the moſt celebrated relicks of Antiquity, which ſeem to be of more than human Compoſition. Whilſt Painting, their Siſter Art, aſſiſts and unites in decorating the interior Domes, Cielings, &c. or, in well-choſen Pieces and Subjects, and maſterly Performances, adorns the Stair-caſe, the Gallery, the Drawing-room, and the Cabinet.

Perſpective is allowed by all, who are well acquainted with it, to be the Baſis of all the Polite Arts which have their foundation in Drawing; particularly Painting, or Delineating; for, Colouring does not come within its Rules. The Sculptor and Architect may receive great aſſiſtance from Perſpective. I am ſenſible it is not abſolutely neceſſary to the Art of Deſigning, but it is eſſentially ſo to ſee the effect of the Deſign. An Architect may, doubtleſs, be famous in his Art without Perſpective, but more ſo if he had that accompliſhment; as he would be better able to judge of the effect it would produce, and convey a juſt Idea of it to others, before the Deſign is executed. Drawings, in Perſpective, from particular Stations, where the Building will be moſt conſpicuous, would anſwer in that reſpect, the ſame purpoſe as a Model, and at a much leſs expence.

At a Time when the Arts are in ſuch a Degree of perfection, a Treatiſe on Perſpective may be thought an unneceſſary Publication; ſeeing there are, already extant, many valuable Books on the Subject, containing all the Rules, for practice, which are either neceſſary, or uſeful; and conſequently, unleſs ſome new Principles are propoſed, by which, the trouble of projecting Objects, perſpectively, may be leſſened, it is uſeleſs to increaſe the number. For ſome, with truth, alledge, that, unleſs Perſpective be comprized in a ſmall compaſs, it will never be ſtudied by thoſe, for whoſe uſe it is chiefly intended. On Perſpective, as on other Subjects, there are, indeed, a ſufficient number of Authors; and yet, I may venture to affirm, that, no Subject, whatever, has been worſe handled, in general.

I do not pretend to have found out new Principles, nor do I think, there can or need be any other; thoſe given, by Dr. Brook Taylor, being ſufficient for any purpoſe, whatever; and that, the Principles, on which he has founded his Syſtem, are the moſt ſimple and perfect that can poſſibly be conceived. Notwithſtanding what many, who have not a true Idea of Perſpective, imagine, that there is imperfection in it; that the Rules, preſcribed, do not always produce a true or pleaſing Repreſentation of Nature; I maintain, that all, which can be done by Rule, is performed on the moſt perfect and infallible Principles that can be deviſed; that Perſpective is, abſolutely, at its *ne plus ultra*; where then, it may reaſonably be aſked, is the neceſſity for, or uſe of this Treatiſe?

There are, generally, two Motives, which induce every Perſon to publiſh to the World, his Inventions or Improvements, or Studies of any kind. The firſt is Vanity; or, to give it a milder Appellation, a laudable thirſt of Fame; (I believe, I ſhall not err wide if I affirm, that, Vanity is the firſt, and chief ſpring of all that is great and laudable) the other is the lucrative Emolument, if not always the Conſequence, at leaſt, it may reaſonably be expected an Attendant on Fame; it is ſcarce determinable, whether Vanity or Intereſt ſtimulates moſt to action. To pretend that we are not actuated by either of theſe Motives is the height of folly; for though our Station, in Life, may ſet us above the ſordid views of Gain, yet no Station, whatever, is exempt from Vanity; which, moſt wiſely, for the beſt of purpoſes (the good of the whole) is made a neceſſary ingredient in our Compoſition.

PREFACE.

Now, although I have not the leaſt pretence to the invention of new Principles, yet I am firmly perſuaded, that I have made uſe of thoſe we have to the beſt advantage; that, from the irregular and imperfect Order, as they are given by Dr. Brook Taylor, I have digeſted it into an uſeful and practical Syſtem; not involved in a labyrinth of mathematical Demonſtration, of things which are to little purpoſe in the Art of Delineating; as the voluminous Work of Mr. Hamilton, publiſhed in the Year 1738; which, though a moſt elaborate and valuable Production, has not been of the leaſt uſe, to the Arts it was intended to promote. In the mathematical part, I have not entered further than is really neceſſary, to evince and enforce the Principles, on which the Practice is founded; and, in Practice, I have ſhewn its immediate and abſolute dependance on the Theory. I have by means of an Apparatus, contrived for the purpoſe, united the Theory and Practice ſo together, that he muſt have very little penetration, and a ſhallow Capacity, who does not in a ſhort Time, and with little Study, conceive a clear and comprehenſive Idea of the rationale of Perſpective. I have, every where, made ſuch remarks, and thrown ſuch light on the Subject, as, I flatter myſelf, will make it juſtly deſerve the Title I have given it: my Deſign, throughout, having been to make the ſtudy and practice of Perſpective, at the ſame time, eaſy and entertaining. The Subjects I have choſen to diſplay, and embelliſh it with, are ſuch as are common and familiar; for, to what purpoſe is an intricate and puzling delineation of the Dodecahædron, &c. except, to ſhew that the Author underſtood it? I am of opinion, that the Reader will be much better pleaſed, with the deſcription of ſuch Objects as are frequently before his Eyes, and ſuch as are fit Subjects to introduce into a Picture; or to form a Picture on. I have not been ſatisfied with the common Leſſons for plane Figures, and Solids compoſed of or bounded by Planes, and left the Student to decorate his Buildings with the embelliſhments of Columns, Entablatures, &c. of the ſeveral Orders; but I have, minutely, ſhewn, how to delineate them, ſingly, and to compoſe a regular Building of the ſeveral Materials, deſcribed and treated ſeparately of before.

There are, at this time, but few Gentlemen who are not pleaſed with a fine Picture, which truly repreſents Nature, on Canvas; the Deception being ſo well managed, that, we can almoſt imagine we ſee the real Objects, themſelves, through the aperture of the Frame. For, it is poſſible, in perſpective Delineations, in which there is a true gradation of Light and Shade, and judicious Colouring (on Cielings, &c. which we cannot come near) to deceive the Eye and Judgment, ſo, as to imagine, what we ſee are real, and ſolid Exiſtences.

Although Perſpective is not conſidered as a neceſſary part of the Education of a Gentleman, it muſt be looked on as a genteel and polite Accomplishment, a Qualification for a Prince. For, has not our moſt gracious Monarch cultivated and particularly encouraged it? nay, deſcended to learn its Rules, and to delineate, with his own Hand, pieces of Architecture? Is it poſſible, for a Gentleman to ſee, as he ought, much leſs to judge of a Picture, with true taſte, and with the diſcernment of a Connoiſſeur, without having ſome notion of Perſpective? it expands the Ideas, and makes us ſee Objects as they really appear, to a judicious Obſerver; it makes a Perſon a judge of the diſpoſition, and proportion of the ſeveral parts of a Picture, to each other; the Symetry and Harmony of the whole are perceived; if there be any diſcordance, or unſeemly diſtortion of the Parts, owing to an injudicious choice of the Artiſt, either in the Diſtance or Situation of the Object, or in the Poſition of the Picture, they will be quickly ſeen; eſpecially, when there is introduced regular Architecture, in Buildings of any kind.

The manner, in which Perſpective has hitherto been treated, ſcientifically, is too rigidly and mathematically ſo, to be entertaining, to a Perſon not converſant with the Elements of Euclid; which, though a branch of Univerſity Education, has been treated, in thoſe ſeminaries of Learning, in a manner, which deters numbers from the ſtudy of that moſt uſeful and neceſſary Science, the foundation of all mathematical reſearches. Every Science, and neceſſary Art, ſhould be ſo treated as to allure a Perſon to the purſuit; by making him (in a familiar way)

reliſh

reliſh a ſtudy, for which he has not a natural propenſity. Many Gentlemen imagine, if they neither paint nor draw, that, Perſpective is wholly uſeleſs, and unneceſſary to them; they are greatly miſtaken who think ſo, if they deſire to be accompliſhed, and would beſtow a little time and pains to be ſo.

Of all the mathematical Sciences, the ſtudy of Perſpective is perhaps the moſt entertaining; the pleaſure and ſatisfaction which reſult from it, in delineating, can only be felt, not deſcribed. Aſtronomy (to which Perſpective greatly conduces) is the moſt ſublime of all, and next to it I ſhould rank Perſpective. The entertainment, reſulting from the former, is of a higher kind; we are, as it were, lifted from the Earth, in contemplation of the ſuperior Works of the great Creator, in ſuch wiſe, as to look on all below not worthy of our notice. The advantages to Society, which are deduced from this moſt exalted Science, are many and great; and, being well known, would be impertinent to enumerate. On the other hand, the ſtudy of Perſpective, ſcientifically, is highly entertaining to a rational Mind; to thoſe who would purſue it, I recommend the Works of Mr. Hamilton. I have not choſen to go further into it, than is really uſeful; and yet, to ſome Perſons, I ſhall be thought prolix enough; who want to know it, without the labour of Study; to which, Artiſts, in general, have great antipathy; I mean, to the ſtudy of any thing mathematical.

It is uſual, with moſt writers on Perſpective, to introduce ſo much of Geometry into their Work, as they judge neceſſary for the knowledge of Perſpective. There would be, in my opinion, as much propriety in prefacing every Book, on Literature, with Grammar; leſt the Reader ſhould not be acquainted with Syntax, and the Idiom of the Language. I fairly own it above my ſkill, to draw a line between the neceſſary and unneceſſary, for all is more or leſs ſo. Some Theorems, in Geometry, are only neceſſary for the attaining of others; which, if they cannot be obtained without them, are neceſſary. In ſhort, if the Student be quite unacquainted with Geometry, he is very unfit to ſtudy Perſpective; let him firſt become intimate with Euclid, and then he may ſafely purſue Perſpective; the practical part of one is ſufficient for the other. Wherefore, ſeeing I have compiled a Syſtem of Geometry, to which this Work is ſubſervient, I have wholly omited the Definitions of geometrical Terms, here uſed; becauſe, it is reaſonable to ſuppoſe the Student already acquainted with them; otherwiſe, he muſt firſt ſtudy that Grammar, of the Science of Perſpective.

Perſpective being a branch of the Science of Optics (as it is founded on direct Viſion) it becomes neceſſary to conſider, in the firſt place, the ſtructure of the human Eye, and the nature of Viſion; to conceive (as far as our Intellects can trace) an Idea, how that extraordinary and aſtoniſhing Senſe is performed; which being, in ſome degree, underſtood, we ſhall be well prepared for a clear underſtanding of Perſpective, and enabled to diſtinguiſh between the Repreſentation of an Object on a Plane, and its real Appearance; a circumſtance, which is a great ſtumbling-block to many, who have not rightly conſidered the difference.

I have prefaced this Work with a Section on Light and Colour; and as a more eſſential requiſite, to Perſpective, the ſecond contains a brief deſcription of the Eye, and of Viſion. In the next is contained the foundation of Perſpective; it treats on direct Viſion, compriſed in two Theorems, of univerſal application; which are the very Eſſence of Perſpective. The two remaining Sections, of the firſt Book, are wholly digreſſive; they not being at all neceſſary, to the underſtanding of what is contained in the following Work; the Reader may therefore paſs them over, if he be ſo diſpoſed, without breaking the thread of the Subject. They contain matter of mere Opinion; reſpecting the materiality of Light, Reflection, Refraction, &c.

Book the ſecond contains the whole uſeful Theory of Perſpective, rectilinear and curvilinear; which is ſomewhat copious, by reaſon of the Examples given for illuſtration; and Corollaries, deducible from the Theorems.

PREFACE.

The first Section is a general Introduction, containing many preliminaries, neceſſary to be known, previous to what follows.

The ſecond is alſo introductory, and contains a full explanation of all the various kinds of Projection, ichnographic, orthographic and ſtereographic; with a circumſtantial and comprehenſive Definition of Perſpective, and other introductory matters.

The third is more elementary; it contains full, yet brief Definitions of all the Terms made uſe of in the Theory.

The fourth Section contains the whole Theory of right-lined Perſpective, in fourteen Theorems; from which are deduced ſeveral uſeful and practical Leſſons, in Corollaries, Scholia, &c.

In the firſt ſeven Theorems, is contained all the neceſſary knowledge of Perſpective, relative to Interſections and Vaniſhing Lines, in general; the firſt and moſt eſſential requiſite in Practice.

The eighth Theorem, though ſelf-evident, is intended as a refutation of an abſurd opinion which many entertain, that continued Right Lines, ſeen direct, cannot be repreſented by right Lines; ſeeing that, parallel right Lines always appear to incline towards each other. The ninth teaches all that relates to Planes and Lines parallel to the Picture. The tenth ſhews where the Interſecting and Vaniſhing Points of all other Lines are to be found; and the eleventh determines how to find them. The twelfth and thirteenth contain all that relates to Lines not parallel to the Picture, in the moſt conciſe, yet full and clear manner that I can conceive. Perhaps the Demonſtrations of the laſt may deter thoſe, who are not Geometricians, from examining it with that attention it requires; let ſuch remember, that, in order to practiſe Perſpective, it is not abſolutely neceſſary to be a Geometrician; becauſe, I practiſed it long before I underſtood Geometry; excepting a little practical, and, a ſuperficial knowledge of Lines, in general; which are certainly requiſite; as the whole of practical Perſpective conſiſts in it.

For which end I have ſelected, all that is neceſſary, not only for Perſpective, but, alſo for various mechanical uſes, into one Book (the firſt part of the Royal Road to Geometry) lately publiſhed; perhaps, in the moſt eaſy and intelligent manner, ever yet done; to which I refer the Reader, for the geometrical Conſtruction of all kinds of Figures here uſed; as well as to the Elements, for Demonſtration, in the Theory.

If thoſe Readers, who have neither time nor inclination to become acquainted with the Elements of Geometry, at leaſt acquire ſo much, as that Tract contains; I will vouch for the great advantage he will find to his ſtudy of Perſpective. Indeed it is impoſſible to underſtand or practiſe Perſpective without it; for, being in itſelf, wholly geometrical, the very Language, and Grammar, of Perſpective is Geometry; let them take my word, for once, they will not find their time miſſpent; but, on the contrary, ſo much knowledge, as it teaches, will be purchaſed at a very eaſy rate.

They may, then, boldly venture on the Theory; omiting the Demonſtration where they find they cannot comprehend it; which, in general, they will not find enveloped in myſtery; having treated that part in the moſt familiar manner poſſible, by plain reaſoning only, without other reference than to the Elements, for proof of what is advanced. Let them, at leaſt, by the moſt attentive peruſal of the Theorem, endeavour, to conceive the Premiſes of it; which, in ſome of them, perhaps, may not be perceived, clearly, at the firſt reading; the ſecond will open ſome Ideas, and the third perhaps compleat it; which, if it require a fourth, ſometimes, will not be loſt labour; for, when the Premiſes of a Theorem are clearly underſtood, the proof of it is not difficult, in many caſes. However, whether the proof of what is advanced be perceived or not, it is not eſſential to the Practice of Perſpective; the Reader may give me credit for it, I will be reſponſible for the truth of it; he may be fully ſatisfied that the thing is certainly ſo, and make uſe of that knowledge accordingly.

The Theorem being underſtood, let him, at leaſt, go through one Example in each, as he proceeds; his Ideas will not only be ſtrengthened, but he will, poſſibly, ſee the truth of the Premiſes. Then, the Corollaries, Scholia, &c. are but ſo many eaſy, uſeful, and practical Leſſons, deduced from the Theorems, which will be found of the greateſt uſe in Practice; and, if the Theorem be clearly conceived, the Corollaries will be ſo too.

The fourteenth Theorem has no affinity with, or dependance on the foregoing; it contains the whole Theory of practiſing without Vaniſhing Lines; an ingenious method, but ſeldom practiſed; nor, indeed, is it ſo generally applicable to practice.

The fifth Section contains ſo much of the Theory of curvilinear Perſpective, as is really uſeful in Practice; or neceſſary to be known, by any Artiſt, whatever.

The ſixth is a refutation of ſeveral capital Errors and abſurd Opinions, which many Perſons entertain of Perſpective; and which are, there, clearly and fairly ſtated, and ſhewn to have no real exiſtence.

The third Book is a copious Treatiſe on uſeful practical Perſpective, containing Leſſons and Examples in every neceſſary Caſe, that can occur in Practice; ſhewing alſo its dependance on the Theory, in brief Demonſtrations, where it is neceſſary.

The Method, in which I have treated this uſeful part of Perſpective, is ſuch, as I preſume will ſufficiently recommend it. The Book is divided into twelve Sections, each treating on a diſtinct ſubject from the foregoing; begining with the moſt ſimple and firſt Principles or Elements, and going on, from the moſt ſimple, to the moſt complex Objects, in a regular, progreſſive ſucceſſion.

After the neceſſary Preliminaries and Elements, in the three firſt Sections; the third ſhewing alſo how to find Vaniſhing Lines, and Vaniſhing Points, in general, the firſt requiſite in Practice; in the fourth is contained all the practical elementary Problems in Brook Taylor's Eſſay, which are, of themſelves, a complete Syſtem of practical Perſpective. Theſe Elements, of the following Work, being well underſtood, will be found of great Utility; which has induced me to perfect ſuch as the Author of them had left very imperfect; and ſome, particularly his 21ſt and 22d Figures, are greatly defective; we are nevertheleſs infinitely obliged to him, for giving them to the World, ſuch as they are.

To underſtand this Section clearly, and be enabled to apply the Problems it contains, in all Caſes, practically (which, it muſt be obſerved, is the foundation of the whole) a tolerable ſhare of geometrical knowledge is requiſite; without which, it is impoſſible to turn them to that general uſe, in Practice, in which they are ſo univerſally applicable; inaſmuch that, it may be truly ſaid, he who can comprehend them, perfectly, and apply them in all Caſes, generally, underſtands Perſpective, thoroughly; but, being ſuperficially looked over and not well digeſted, they will be found of little uſe.

This conſideration, perhaps, has induced ſome Perſons to imagine, and affirm, that the whole of Perſpective may be compriſed in a very ſmall Compaſs. But, I muſt make free to tell them, they are under a miſtake; for, notwithſtanding the Principles are few and general, yet, to know how to apply the Rules, in all Caſes that may occur in Practice, cannot be ſo ſoon acquired; if ſo, how comes it, that they are no greater Proficients? Or, can they imagine, that, amongſt the number of able Men, who have wrote on the Subject, none ſhould hit on that much wiſhed-for Expedient? Either they muſt acknowledge that their own faculties are not ſo acute as others (which is paying a ſorry Compliment to their Underſtandings) or they muſt ſuppoſe that there is more in it than they at firſt imagined. 'Tis true, the whole of Practical Perſpective is compriſed in this and the foregoing Sections; yet, how few would be able to apply thoſe Problems, though clearly underſtood, in delineating a regular piece of Architecture, is but too obvious to enlarge on; without previous, practical Leſſons, diverſified variouſly, to familiarize them. For which reaſon, it is neceſſary to ſhew their application, in various Examples, which I have not been

ſparing of. I have not, as others have done, ſtudied the moſt eaſy poſitions of Objects; but I have endeavoured to give the moſt pleaſing, pictureſque, and natural Repreſentations, and render the moſt difficult Poſitions eaſy.

Some Perſons know a great deal (in their own opinion) becauſe they know, perhaps, that all Right Lines, which are parallel, have the ſame Vaniſhing Point; who yet, know not how to fix and aſcertain any one, except the Center of the Picture, and that often very abſurdly (indeed they ſeldom, if ever, uſe any other) nor how to proportion a Line perſpectively; and further, to ſhew their conſummate knowledge in it, if you ſhew them a piece of Perſpective, in which there is not one Line tending to the Center, or Point of View, they will call that Point on the Picture, in which the principal Lines converge, the Point of View; ſo that, there may, by this criterion, be ſeveral Points of View in the ſame Picture; a circumſtance too palpably abſurd to need a refutation.

If this be to know Perſpective, it may indeed be very ſoon acquired; it may be taught to a Child. But, when they have drawn a Line to its proper Vaniſhing Point, how that indefinite Line is to be proportioned, in order to repreſent, certain finite parts of the Original, they are wholly ignorant, though the chief requiſite in Practice, and the principal buſineſs of the fourth Section of the third Book.

However, not to diſcourage them, from attempting to acquire ſo neceſſary a part of their Studies, they may be aſſured that a fixed reſolution and perſeverance, will ſoon ſurmount every apparent difficulty. Some are alarmed, and even frighted, at the ſight of ſo many Lines in a Diagram, for a Leſſon in Perſpective, imagining they can never comprehend them all. Certainly, they muſt, to a perſon wholly ignorant of the uſe and meaning of them all, appear a general confuſion; but when they have ſet about it, with a reſolution to know, and have well digeſted the Definitions of them, they will, at one glance, comprehend the uſe of ſeveral; and the reſt, by analizing them regularly, will ſoon become familiar to the Eye, and to the Underſtanding.

This, I believe, has induced ſeveral Perſons, who have wrote on Perſpective, to deal very unfairly in it, in not giving all the neceſſary Lines in their Diagrams. I have, in mine, given all, and more than are neceſſary to be drawn, at once; for let it be obſerved, that the Radials of Lines, for determining Vaniſhing Points, ſeldom need be drawn; and ſeveral of the operative Lines, or Viſual Rays, which are only uſed for cuting others, need not be drawn, at all; a Ruler being applied to the two Points (the Eye and the original Point) the indefinite Line may be cut, without drawing the whole Line, which could not anſwer, in a Diagram, ſo well as drawing the whole, in order to ſhew the direction of the Line; beſides, in Practice, one part is effected and the Lines rubbed out before others are drawn, which cannot be done in a Diagram, all muſt remain together.

Others, again, there are, who have almoſt determined to make an effort, only want to know ſo much of the Practice as to be able to project a plain Building, &c. and ſeem quite afraid of attempting the Theory; becauſe there is occaſion for Geometry, which, to many, is a terrible Bugbear; ſo that, rather than exert themſelves, they do without it, though the firſt eſſential, and chief requiſite perhaps of their Profeſſion. They want to know Perſpective, but are afraid to venture on the Theory. They deceive themſelves if they imagine it an unneceſſary Appendage; can it be unneceſſary to know how, and why, before we attempt any thing, practically? is not one, the means of effecting the other? I do aver, that the ſhorteſt way to acquire Perſpective is to underſtand it firſt, in Theory; the Practice of it will readily follow.

How different are the diſpoſitions of mankind. After I had (from the Jeſuits) acquired (as many would ſuppoſe) a proficiency in Practical Perſpective, I was far from being ſatisfied with all that could be obtained from that Author; and although I was not a Geometrician, at that time, and he not giving any Theory, I ſet about contriving means to have conviction of the truth of his Rules; by which means, I

fell inſenſibly into the method uſed by Vignola and Sirigatti; than which, nothing can be more convictive; and I was accordingly ſatisfied, that the Rules, given, might be depended on.

But, having acquired ſuch a proficiency in Geometry, as to enable me to comprehend Brook Taylor's Principles, how trifling, how limited appeared all I had learned of it before; I was even aſhamed of my own productions, ſome of which had, undeſervedly, been admired: I found, indeed, I ſcarce knew what was meant by Perſpective, till then. The greater progreſs I made in Geometry, the clearer, and more comprehenſive Idea I had of his Principles; which, though founded on common obſervation of Objects (ſupported by Geometry) are beyond every thing ever thought on, by any who had wrote on the Subject; and yet, it is ſo ſimple, when underſtood, that 'tis ſtrange it ſhould not have been brought to light ſooner.

Having, in the fourth Section of the third Book, diſplayed the whole Elements of Practical Perſpective, the fifth begins the application of them to real uſe; the whole of which, conſiſts in finding the repreſentation of a Line or Point, in any Plane, any how ſituated: for, Points are the extremes of Lines, and Lines of Surfaces; and, Solids, or Objects, are bounded by their Surfaces. A Plane, however ſituated, it is manifeſt, is the ſame, in all its properties; its ſituation, in reſpect to the Horizon, is, therefore, of no conſideration, in Perſpective; and, to delineate Plane Figures is all that can be done by Rule, with abſolute certainty; the apparent Contours of ſome round Objects are ſomewhat difficult to determine, yet they may be done ſufficiently correct. In this Section, the Conſtruction of all kinds of plane Figures, perſpectively, in Planes any how ſituated, is compriſed; and, in the ſixth is ſhewn how to conſtruct Solids, of Planes only; by which, any Plane Building may be projected.

The ſeventh Section teaches, fully, how to repreſent Mouldings in general, and to break the ſame at the Angles of a Building, &c. internal or external, right-angled or otherwiſe, with other neceſſary, and decorative, parts of Architecture, &c. as Triglyphs, Conſoles, Modilions, &c. alſo, how to form a Pediment.

The eighth is wholly adapted to circular and round Objects of all kinds; as Arches, Columns, Tuſcan and Doric Baſe and Capital, round Steps, Wheels, Vaſes, and circular Mouldings; including alſo the Ionic and Corinthian Capital.

Being now furniſhed with all the materials of a Building, the ninth Section ſhews how to compound them, and to form a Building; from the moſt plain and ſimple to the moſt elegant and rich, decorated with the various Orders; alſo detached Buildings, Views, &c.

The tenth is for internal Subjects, as Rooms, inſide views of Churches, Arcades, Staircaſes, and Cieling-pieces, repreſenting Domes, and Cupolas; which, though ſomewhat particular, is founded on the ſame invariable Principles.

The eleventh is adapted for houſhold Furniture; as Tables, Chairs, Cabinets, Bookcaſes, Beds, &c. alſo, for Machines, Coaches, &c.

The twelfth treats of inclined Pictures and Planes, in general, containing Leſſons in the moſt difficult caſes, from known data; in which, the excellency of the new Principles is exemplified.

Book the fourth treats on the Perſpective of Shadows, which is indeed a copious Subject, and much more might be ſaid of it; but, as I am ſenſible, that very few take the neceſſary pains to project their Shadows by Rule (general Effects being all that is ſtudied, or regarded) it would therefore be to little purpoſe to give Rules, which will ſeldom, if ever, be followed. Nor is it at all neceſſary, or even practical, to project every Shadow, mathematically; it would be attended with great loſs of time, and perhaps, in ſome Caſes, produce a bad effect, though truly projected. In this branch of Perſpective, licence may therefore be taken, juſtifiably, provided they do not run into groſs and palpable abſurdities; ſuch as projecting the Shadows of Objects, already immerſed in Shade; or, as I have ſeen in the works of eminent Maſters, Shadows caſt both ways, in the ſame Picture, to the right, and to the left.

I have

I have alſo ſeen, the Shadow of a curve Line repreſented by a Right Line, when the Luminary was not nearly in the Plane of the Curve; which, with others, leſs obvious, ought carefully to be avoided, and guarded againſt.

Nevertheleſs, there are certain general Rules may be given, which ought ſtrictly to be adhered to; and never, on any account, departed from. I have, therefore, in this Work, given ſo much as I conceive neceſſary to be known; for, in order to be converſant in Shadows, it is abſolutely neceſſary to be acquainted with the invariable Law of Nature, in the projection of Shadows; although it may not always be neceſſary to follow her dictates, implicitly.

The effect of reflected Light on Objects, from other Bodies, in vicinity with them, is likewiſe treated of in this Book; alſo the effect of Diſtance, uſually underſtood by the term Keeping, properly, aerial Perſpective; both which, contribute greatly to the perfection of a Picture; inſomuch that, without due regard being had to both, a Picture, ever ſo well deſigned and delineated, or coloured, will be but a flat and ſpiritleſs performance.

And laſtly, the reflected Images of Objects on the ſurface of ſtill Water, or on poliſhed Mirrours, in any Poſition, are, in the laſt Section of this Book, treated on.

Having, now, given a brief account of the Work, I ſhall juſt make one more obſervation, in reſpect of the notions of ſome Artiſts; who being always accuſtomed to ſketch by ſight, only, and having not the leaſt conception of projecting Objects by any Rule, they vainly imagine that Perſpective will teach them how to repreſent Objects, exactly as they appear to the Eye, without knowing any thing of their Proportions or Situations, in reſpect of each other, or of the Picture, which is impoſſible; ſo that, when they find there is a neceſſity for taking and applying their real Meaſures, or Proportions, being ſo foreign to what they conceived of it, they imagine ſo many obſtacles to lie in the way, that, they ſoon deſiſt from the attempt; chooſing rather, to remain in ignorance, and proceed with uncertainty, than take the neceſſary pains to acquire, what, being acquired, they would deem invaluable.

He, therefore, who can hit on ſuch an Expedient, as to teach them how to delineate Objects truly, by Rule, without applying any Rule at all, will be likely to meet with great encouragement from thoſe indolent Artiſts. I acknowledge my incapacity for the taſk; and therefore, I ſhall abide by the Rules, as they are given in the following Work.

A TRUE

CASE,

BETWEEN THE AUTHOR,

OF A LATE PUBLICATION,

His PRINTERS, and PAPER MERCHANTS.

TRUTH WILL EVER PREVAIL.

OF all the calamitous accidents, which, of late years, have happened in this Metropolis, none has attracted the attention of the Public, more than the Fire, at Meſſrs. Cox and Bigg's, Printers, in the Savoy, on the evening of March the 2d, 1776; on account of the many valuable literary Productions which were deſtroyed there; ſeveral of which were completed, and ready for Publication. Amongſt the reſt, was the Compleat Treatiſe on Perſpective, in folio, publiſhing by Subſcription, by Thomas Malton, and intended to be publiſhed in March. The following is a true Caſe between that Author and Meſſrs. Cox and Bigg; alſo, between him and Meſſrs. Wright and Gill, Paper-Merchants, in Abchurch-Lane, Lombard-ſtreet.

Thoſe Gentlemen were alſo ſufferers, by that Fire, to the amount of 140 Pounds, for Paper, credited Meſſrs. Cox and Bigg. Who being lately removed from the Strand, into the Savoy; and having taken in another Partner, * they had been making Conveniences, for carrying on a more extenſive Buſineſs. In conſequence of which, they had neglected to inſure, till all was completed; their former Policy being forfeited, or of little value, now. They likewiſe intended to inſure in another Office (the Union) every thing (if I am truely informed) was either ready to be executed or ordered to be ready. Perhaps the Money might not be ready; their neglect of ſo important and neceſſary an affair may, on that ſcore, admit of ſome vindication, in their favour; nevertheleſs, it was an unpardonable negligence, having ſo much of others Property on the Premiſſes, and in their poſſeſſion.

Although the Author of the Treatiſe on Perſpective was, by no means the greateſt ſufferer, by that accident; reſpecting the quantity or value of what he loſt; yet, all circumſtances conſidered, his Caſe was ſingularly hard; and, the treatment, he afterwards experienced, from them, and others, fellow-ſufferers,

* A Mr. Nettleton; who, if I am not misinformed, was but nominally ſo, in truſt for Mr. Wells, Mercer, on Ludgate-hill. Be that as it may, it is certain, Mr. Wells was the perſon who advanced the Money for Nettleton; and, there is not the leaſt doubt, but that he was to be a ſharer in the profits of the Buſineſs, as a joint Partner.

cruel and inhuman; ſuch as no Perſon, who had not taken ſome pains to diſſolve every ſocial tie, nay, to diveſt himſelf of humanity, could have determined to carry into execution, at the time; it ſeems as if they were determined to cruſh him, if poſſible, by their premeditated, and, in ſome meaſure, their united efforts. As the diſintereſted Public are often miſled, in forming a judgement of ſuch matters, as they are no way concerned in, I was induced to lay the real and true ſtate of the Caſe before them, in the moſt candid and impartial manner.

This Treatiſe on Perſpective was the produce of many years ſtudy, the reſult of my leiſure hours, when I followed the profeſſion of a Cabinet-maker, with little ſucceſs; and which I quited ſeven Years ago, encumbered with as many Children, ſince increaſed one. Let any one picture to himſelf, the ſituation of a Perſon, who had quited a Buſineſs, not worth following, burthened with ſuch a Family; having no other means to provide for their ſupport, than the produce of his ſtudies, advanced almoſt to the meridian of Life.

I began printing the Work about three years ago; but, as all ſuch Works are uſually longer in hand than is conſiſtent with the Author's convenience what between the Engravers, Printers, Paper-merchants, and the encumbrance of eight Children, I found myſelf, when in the middle of my work, not in the moſt eligible ſituation; the encouragement, at firſt, not being adequate to ſuch an arduous and expenſive Undertaking. Notwithſtanding, I kept joging on, with reſolution and perſeverance, though but ſlowly; for Engravers are a hungry ſort of Gentlemen, who will not dig without eating, and well too; when, or before, they have done their Work, they immediately want their Money, and frequently much more than they deſerve; many of them being as mere Mechanics as any whatever.

Being thus circumſtanced, it is no wonder the Work went ſo ſlowly on; the Engravers work'd for me rather than be idle, expecting to be paid ſome time; the Printers had a very convenient jobb to ſet their Men on, when they had nothing elſe to do; and thus, we went gently on for ſome time, each laying the blame on the other. As time and perſeverance will ſurmount great obſtacles, the Royal Road to Geometry was firſt completed and publiſhed, in December 1774, which was well received by the Public. When both were finiſhed, I deſired to have them ſent Home, altogether, to ſave the trouble and expence in ſending often; but, as there are reaſons for every thing, they alledged ſome plauſible excuſe, to let them abide in their Warehouſe, and ſend for them as I wanted; which, judging the true reaſon to be ſecurity for Payment, I ſupinely acquieſced in.

When they moved from the Strand into the Savoy, they moved my Work, without giving me the leaſt intimation of it, although I had requeſted them to ſend it Home long before; and ſurely, as it was to be moved, it was as eaſy to have ſent it home to me, as to carry it farther from me. Not long after (for reaſons beſt known to them) they refuſed to deliver any more to my Orders, repeated ſundry times, to my great hindrance in delivering the Work to the Subſcribers, and conſequently to my loſs (as it happened) in not delivering ſo many as I might have done, or in making them ready for ſale; near a hundred Subſcribers wanted Books when the Fire happened. At the ſame time, they required Security for payment of the remainder of the expence in Printing, Paper, &c. which they had ſupplied. *Query*, If a perſon retains my Property, againſt my Will, as ſecurity for payment, is he not accountable for it, if the Property be deſtroyed? At length, by continual duning, obtaining Notes or accepting Drafts, they were over-paid, before the Fire. *Query*, 2*d*. Whether a perſon has a lawful right to be paid for Work done, which was never delivered,

although

although requeſted? and whether, the Property being deſtroyed, the Money paid ſhould not be refunded, in proportion to the Property loſt? There requires, I preſume, no great ſagacity, or deep knowledge in Law, or, at leaſt, in Equity, to determine.

A few weeks before the Fire, Mr. Bigg, as uſual, came to me, and deſired me to give him a Note of hand, or two, for the balance of their Account, as they were in great want of Money, that day; having in his hand a ſlip of paper, on which he had taken (as he ſaid) the ſum total from the Book; I think, about twenty ſeven Pounds. Now, as I never yet had a regular account of the whole, but only for the book of Geometry completed, and part of the Perſpective; and as there had been ſome diſpute in reſpect of the Money paid, which is perfectly clear by the Receipts given; by which, and the accounts I had, I was fully ſatisfied that very little was due to them; and conſequently, I objected to the ſettling an Account I had never ſeen; for which, every one who knows any thing of the World, muſt have condemned me for a Fool. But, he being very urgent and importunate, I agreed, merely to ſerve their preſent emergency, as I thought, to give Notes for twenty pounds fifteen ſhillings, provided they kept back another of ten pounds, due the week after, for two months longer; for which, I gave him another, then, to raiſe money, which was ſent me to pay the firſt. Mr. Wells, at that time, being concerned with them, thoſe Notes were depoſited in his hands; and the firſt was paid, after the Fire, as it was properly due before; the other, I was fully determined not to pay, unleſs compelled by Law; as they were ſcarce legally obtained, and for which I never received value in any ſhape; but, at the time I gave them, I was then going on with an Appendix to the book of Perſpective, which induced me to it; as I concluded, that before the Notes became due, I ſhould be as much in their Accounts; nor, did I doubt their probity, when we ſhould come to ſettle them.

Being thus ſituated with the Printers, I ſhall take my leave of them for a while, and introduce to the Readers acquaintance, two very worthy Gentlemen, Meſſrs. Wright and Gill; with whom I had contracted for Paper, to the amount of 85l. of which I paid them 32l. with intereſt, and gave two Notes for 28l. each, bearing Intereſt. One of theſe Notes was paid, with Intereſt, and fifteen Guineas of the other; ſo that there remained, due to them, twelve pounds and five ſhillings when the Fire happened.

It is much eaſier to conceive than deſcribe an Author's ſituation; who, having gone through the fatigue and expence of ſuch a laborious Work, and completed it, ready for publication, expecting to reap the harveſt of his labours; when, lo! a fatal Bar to his hopes, deprives him, in one Hour, of that reſcource he had ſo long and ſo ardently panted after. No redreſs, no remedy but Patience; the labour, and expence of printing it muſt be repeated, before he ſits down to enjoy the fruits of his paſt labours, in quiet; ungrateful taſk; Patience, ſurely, never had a more ample ſubject, for exerting her dominion and influence over the Mind.

Such was my ſituation, on the third day of March, 1776. It is not eaſy to paint the Proſpect which at firſt preſented itſelf; the Harveſt of my labours deſtroyed, when juſt ready for the Sickle. But, calling Reaſon to my aid, I formed the plan of my future proceeding; and comforted myſelf with the reflection, that the Engravers, Printers, and Paper Merchants were ſo near paid; there remaining, to the laſt, the trifling ſum of 12l. five ſhillings only, as above; which, conſidering that they were very eminent, that is, very rich Men, and great traders, I never imagined they would be urgent for ſuch a trifle, to them, at ſuch a time, being reputed to be worth 40 or 50 thouſand Pounds, each. What

pitiful

pitiful Ideas muſt thoſe have, who never felt the ſublime pleaſure of heaping up Money. There is not, perhaps, a pleaſure the World affords that can be compared with it; ſo exquiſitely ſublime, that all inferior pleaſures are wholly abſorbed in it. Thoſe gentlemen having loſt 140 Pounds, by the aforeſaid Fire, it was a terrible blow to their repoſe; but they, with others of the ſame caſt, took care to ſecure, to themſelves, the fragments of all that was left on the Eſtate, in Debts, &c. wholly excluding all ſuch claimants as myſelf, from having any ſhare in the ſpoil. Conſequently, having loſt my preſent dependance, it was judged a proper time to require payment, of my Debt to them, having ſtood long in their Books; which ſingle circumſtance is, to a lover of Money, a ſufficient plea for levying it by diſtreſs; the ſolid argument, that the culprit is not in a ſituation to pay, with convenience to himſelf, is an argument that has no weight; the moſt ſolid and convincing is, you muſt and ſhall pay. Sound, counting-houſe, reaſoning; ſuch as, I make no doubt, every retailer or manufacturer of Goods, for which he has obtained credit of ſome hungry, rapacious, over-bearing, wholeſale Dealer, has experienced one time or other.

In conſequence of the above reaſoning, I received a Letter from Wright and Gill, requeſting immediate payment, as they were really in want of Money. I doubt not the truth of their wanting Money, for that they will to the end of their days. Such penurious Money-rakers make no diſtinction between the Wants of others, who want the neceſſaries of Life, and theirs; which is, merely to accumulate, and heap money on money. This Letter was duly anſwered, requeſting them to add a little more to their common ſtock of Patience, on account of the misfortune which had happened, in which we were common ſufferers; and that I would wait on them in a few Days. Accordingly I attended their Levee, when I ſaw both Wright and Gill: the following is the ſubſtance of what paſſed between us, on that Interview.

Well Mr. Malton (ſays Mr. Wright) I hope you are come to ſettle that account, now; it has ſtood a long time in our Books, and it is very diſagreeable to us, to keep Accounts ſo long open. Yes Sir, ſays I, if you pleaſe we will ſettle it, but it is not convenient to pay you; as you know the loſs I have ſuſtained, you cannot imagine that I am in a ſituation to pay money yet, having loſt the means I had of obtaining it. Why, we are informed that your loſs is trifling, a mere ſhuffling pretence to delay payment. Sir, I ſcorn the appellation of a Shuffler; * my loſs Sir, is greater than yours, which I can juſtify; but, let that be as it may, I have loſt near half the Paper I had from you, with the additional expence of printing, annexed to it. Well, but I knew it had ſtood long, and ought to have been paid before the Fire happened; that he knew I had got a valuable Work, and that I might pay them if I would. What ſound, logical reaſoning; how futile muſt ſuch arguments appear, to an impartial and diſintereſted perſon. That it ſhould have been paid long ago was, now, but little to the purpoſe; it was not paid; and they knew I had loſt that, with which I hoped to have paid them and others, ere now; and the very identical Property I had of them, inſtead of making Money of it; which, had I done, they would be juſtifiable in the ſtep they took to obtain it. All the arguments I could make uſe of availed little or nothing; they had loſt too much already, by Cox and Bigg, who, he (Mr. Wright) ſaid, deſerved to be hanged; and, I make no doubt, they were his real Sentiments. Good God! is it poſſible any human Being could ſo far diſgrace human Nature? as to ſay a perſon deſerved to be hanged; for a piece of negligence, in which, they had not loſt,

* An opprobious term, much uſed by wholeſale Dealers; when an honeſt induſtrious Manufacturer, cannot make his Payments in due time.

on

on the whole, above a hundred pounds (who were reputed worth a hundred thouſand) the other had loſt their Whole, diveſted of all they had, in the World. And I am alſo of opinion, that if, in order to pay them the paltry ſum of 12 or 13 pounds, I had ſold my Copy-right for half, or a third part of its value, they would have highly applauded me, for my Honeſty. I ſhould look on ſuch a perſon in the ſame light, as one who had rob'd me of fifty pounds value, when he knew he could not make ten ſhillings, to himſelf, by his villainy. No, had they never been paid, I know the value of the Work; I have grappled with every difficulty to preſerve it my own Property, entire, and it muſt be ſome more dire Calamity, to me, that ſhall wreſt it from me.

When all I could ſay availed nothing, I propoſed, that, if they would deduct the Intereſt I had paid (as ſurely, in ſuch a caſe, they could not be hurt if they got the whole Debt, without Intereſt) I would endeavour to pay them in a certain, ſhort time, mentioned. I muſt allow them to think and judge for themſelves, they ſaid; and generouſly propoſed, if I paid them in the time, to charge no Intereſt from that time; but they wondered how I could expect what I propoſed, after ſo long credit. I then propoſed to give them a Book for the Intereſt, and put them down as Subſcribers; ofwhich, to ſell it, they might make two pounds, at leaſt. No, the Book was uſeleſs to them. Wretches, who had ſubſcribed ten Guineas, juſt before, to make a ſound in print, or for ſome other ſinifter view, yet refuſe to encourage a Work, for which they had furniſhed Paper, to the amount of 85 pounds; and the Subſcription to be paid by the intereſt of the money; and, after I had loſt, by fire, near half the Paper I had of them. What pity, that any, who either write or print, ſhould contribute to enrich ſuch ſelfiſh mercenaries, by conſuming their Paper. I then told them, that they were the beſt judges what they had to do, and muſt purſue ſuch meaſures as appeared to them the moſt elegible; I could not pay them yet. They, then, aſſured me they would not wait much longer; in which they ſpoke the truth; as all ſuch like religiouſly obſerve their Word, in paying or in being paid; who frequently make their boaſt, that they are always punctual in their payments; by way of reproach, to thoſe who are not in a ſituation to pay them. As if there was any merit in being punctual, to thoſe who have it always in their power; or a reproach to others, who have not; provided they do not unneceſſarily conſume and embezzle their getings, and are induſtrious to get.

Being, as I thought, in perfect ſecurity, on their account, notwithſtanding what had paſſed; thinking, as being Men, they could not, and as Tradeſmen (for their credit) they durſt not put their menaces into execution. But I reckoned without my Hoſt; for, on Saturday the 8th of June, a gentleman Officer (one Mr. Armſtrong, of Carey-ſtreet, Lincoln's-Inn) waited on me, from them, with a ſcrap of Parchment, and ſlyly told me I was his Priſoner. On whoſe account, Sir, ſays I? let me ſee your Warrant. I read its contents, deſired him to walk up ſtairs with me, and I would ſettle the Account; having, fortunately, received as much that morning, which I had then in my pocket. What the conſequence would have been, had I not been ſo lucky, I will not ſay; but, as it was, it ſupplied me with courage to ſupport it, with becoming fortitude.

Thoſe who never had the honour to be acquainted with ſuch like Gentlemen, executors of the Law, cannot have the leaſt idea of the delicacy, the humanity, the tender feelings of ſome of thoſe Harpies in office; who are dead to every ſenſe of feeling, where their Intereſt is concerned. My Wife (ever anticipating woe) ſuſpecting his buſineſs, followed cloſe in the rear; when there followed, perhaps, the moſt entertaining Scene that can be imagined; of which, I will endeavour to give a true Picture.

There were, in my Study, three of my Children, Boys, who were juſt come home, from a School in Yorkſhire, the oldeſt juſt turned of twelve years. Hav-

ing been rather a heavy Tax on me, for three years and a half, of thirty Guineas a year, I had now got them Home; not merely to ſave that expence, * or ſome part, but rather to inſtruct them myſelf, now; having, as I thought, finiſhed the work I had in hand. As ſoon as my Wife entered the room, as if ſhe followed a Spectre; I hope my dear (ſays ſhe) you need not go with the Gentleman? No, my dear, I don't intend it, ſays I; indeed but you muſt, ſays he; Sir, I ſhall not go out of the Houſe (replied I) I ſhall pay the Demand you have on me. On which, he ſtarted up, and collared me; and puting on one of his fierceſt magiſterial looks, Sir, ſays he, do you know who I am? Do you know, that I repreſent the Sheriff of Middleſex? He has a worthy Repreſentative, ſays I; but if you repreſent the Devil † I care not; I can pay your demand on me, and therefore, will not go with you. Immediately he calls up his Hell-hound; who, ere this, had gained entrance, and, like a Ruffian, attempted to drag me down ſtairs. The whole Houſe was alarmed; my Wife, ſtriving to ſcreen me from their violence, was handled ſomewhat roughly; the Boys ſet up their ſhrileſt Pipes, to a fine old tune; up runs a Daughter and the Maid; a Lady, on the firſt floor, was frighted almoſt to death; in ſhort, it was the moſt diverting ſcene that can be imagined; what pity, the Authors of it were not preſent, to enjoy it, and glut themſelves with their juſt Vengeance §. Reſiſtance being vain, I enquired where his habitation was; being informed, in Carey-ſtreet, a Coach was called, and away we drove, taking a Son with me, who happened to come in. He had to trudge into Abchurch-Lane, to the Attorney (Mr. Conſtable) to get a Bill of debt and coſts. If he had happened to be gone to his Country houſe, I perhaps, have taken up my reſidence in Carey-ſtreet, till Monday; it being a cuſtom with thoſe Gentlemen, to ſeize their prey on a Saturday, in order to detain them the next Day, if matters cannot be ſettled ſoon. The Bill was making out, when a meſſenger was ſent, to order twenty ſhillings to be tacked to it, for Intereſt; which, with the expences of one pound ſixteen ſhillings, brought it up to fifteen pounds, and one ſhilling. That buſineſs being done, the Office ſearched, and the Money paid, I regained my Liberty, and returned home, to comfort my diſconſolate Family.

I ſhould be glad (for the ſake of thoſe, whoſe misfortunes may ſubject them to the inſults of ſuch Wretches) that ſome humane Perſon, of the Law, would, in public print, let ſuch know, how far thoſe Gentlemen have authority to tyrannize over thoſe, who are already too much oppreſſed; and how they may be puniſhed, for over acting their part. What! is an unfortunate Man, in credit, to be treated, by ſuch Reptiles, like a Cutpurſe, or a Felon? If I was under a miſtake in point of Law, he ſhould have ſet me right, in a proper manner. But why ſhould a Perſon be obliged to go with them (the idea of which is terrible to a Perſon in Credit) when he can anſwer the demand againſt him? Is it to ſqueze the laſt Shilling from him, by all means poſſible? A Shilling or two for a Coach; a Shilling for being lock'd up; a Bottle or two of Wine; and what they can extort for ſearching the Office, (though they have no right to demand any thing), 'tis their own Buſineſs; and at their peril they detain a Perſon, if there be nothing more againſt him. It would be a real Bleſſing to the unfortunate, if every Sheriff, entering on his Office, would make it his firſt buſineſs to enquire, ſtrictly, how thoſe, to whom they delegate ſuch Power, have acted, in the execution of their Office; and either ſuſpend for awhile, or expel them entirely, if they be found, as too many might be found, to deſerve it.

* Let none be ſurprized at the mention of ſaving. Having three, at once, it was agreed for ten Guineas each, otherwiſe it is twelve Pounds a Year; out of which, I will maintain it, they clear five or ſix, for teaching; then where three are ſo many in a Family, with œconomy, there may be a ſaving.

† *Query*, Whether it was not a more juſt, and ſtriking Repreſentation?

§ To give them a better Idea, they may ſee a faint reſemblance, delineated in a Print, in the firſt Number of a Publication, five or ſix Years ago, called the Gentleman's Muſeum.

paltry

I ſhall, now, take my leave of thoſe generous and true, thoſe fair and upright Traders; who, by means of their grea power to monopolize a moſt neceſſary Article, for many miles tropolis; who could have the barbarity to arreſt an induſtrious paltry Sum of twelve Pounds five Shillings, the remains of near after having loſt near half the Paper he had of them (as related above he paid them Intereſt for their Credit; amounting, on the whe Pounds, twelve Shillings, and ten Pence.

Would it not have redounded more to their Reputation, to have ſen and addreſſed me thus? Well, Mr. Malton, we are ſorry that you are ſufferer with us, in that unfortunate Accident; which, to us, is triflin cauſe we are able to bear it. But for you, who having juſt completed ſuch truly laborious Work; burthened, as you are, with ſo large a Family, to be thus fruſtrated in reaping the emolument of your indefatigable Labour and Induſtry, we are really concerned. Having ſeen and experienced your Probity, we freely offer to credit you Paper, for reprinting the Work; which, we are glad to hear, merits the approbation of the Public; as we ſhall be happy in being inſtrumental to the rewarding of ſo much Merit. There was a ſmall Balance left unpaid; had not our loſs been already conſiderable, we would ſtrike it out of our Books; we cannot ſuppoſe you are in a ſituation now to pay; but if you approve of it, we will add it to the next Account; you may, hereafter, be in a better ſituation; we will endeavour to contribute towards it; we will each of us be a Subſcriber to your Work: as for the Intereſt you have paid, it ſhall be remited, 'tis enough that we get the Principal now. I muſt obſerve, here, that when Mr. Wright ſaid, he knew I had got a valuable Work, I aſked him, what better I was for it, at preſent? would they let me have Paper, for reprinting it? The anſwer was, yes, if I would give them ſecurity for it. You aſked for none before, ſaid I, nor will I attempt to give you any now; ſurely, now the merits of the Work is eſtabliſhed, I am not in a worſe ſituation than when I began it. But I deſire not your credit, I can have it elſewhere, and be ſerved at a lower price.

Having thus diſcharged a duty to the Public, in reſpect of ſuch Blood ſuckers; I have only further to add, that I am fully perſuaded, they were inſtrumental to the following affair. Being fortunate, in geting their Money, I muſt, in courſe have it in my power to pay the other, alſo; but if I had, I was far from having the inclination, ſeeing I never had value for the Money, delivered to me; but, on the contrary, detained from me, by their Caprice, till it was deſtroyed. With what confidence could they think of compelling me to pay, for Work done, when they were conſcious of being the ſole cauſe of my loſing it? I ſhall find means (doubt not) to do myſelf juſtice.

On Wedneſday the 26th of June, I received a Letter from an Attorney on Mr. Wells's account; giving me to know, that Mr. Wells had put into their hands, the two Notes, for twenty Pounds fifteen Shillings, with order to arreſt me, if they were not paid the next Day, or on Friday. I waited on Mr. Wells the next Day, and let him know how I was ſituated; and that, if I muſt pay the Notes, they muſt allow me Time; but that, Cox and Bigg were the Perſons properly accountable for them. I knew not, then, that he was a party concerned, otherwiſe, than having advanced, as he ſaid, a thouſand Pounds, which he had loſt; he deſerved to loſe it, for his negligence, in not ſeeing them inſured; by which, others ſuffered ſo much. But, as ſuch, generally, are better acquainted with the affairs of thoſe who are indebted to them, than themſelves, he knew I could pay them, then, if I would. Perhaps I could; but I had twenty more important, and neceſſary demands for all the Money I could get.

I wrote

. Day, and ſtated the Caſe in the moſt candid and in-
the true ſituation of my affairs open to him; with the
family, as I had to provide for; and that, in three Weeks
ake my utmoſt endeavours to pay one of the Notes. But, what
ordinary, although I had requeſted it, over and over, of Mr.
had promiſed I ſhould, yet, I could never be indulged with an
that there was againſt me; to have, at leaſt, the ſatisfaction that I
much indebted to them.

tanding all I had ſaid or wrote to Mr. Wells, I was arreſted in a
after, on Saturday (as uſual) the 13th of July. As I never imagined
y durſt, or that any Perſon, who was not a Jew or a Barbarian, could
e proceeded to compulſion, in ſuch an affair, I was determined not to pay
till I had advice concerning it; and, having the long Vacation before me,
conſequently gave Bail. But, finding I had no Plea to ſupport a Trial, as I
could not give direct and poſitive proof, that he was a Partner in the Buſineſs;
nor, indeed, did I know it then; and, being at a diſtance from Home,
I ſent Orders for paying it, before the Term commenced. I muſt, however,
do Mr. Wells this juſtice; he took no more than the Principal; I paid no ex-
pences, nor did he charge Intereſt on the Notes; like the other hungry, rapaci-
ous Wretches, Wright and Gill, who charged to the laſt Shilling of Intereſt.

Thus the affair ſtands, at preſent, between me and the Printers. I have
waited till now, in hopes they might be in a ſituation, ere this, to reprint the
Work; and, by that means, make me ſome reparation, without inconvenience
to themſelves. But, I am never the nearer; the Partnerſhip is diſſolved, and
therefore, I have but little to expect on that ſcore; nor could I longer delay
putting the Work to Preſs. The Public may now be aſſured, that it will be
regularly publiſhed in Numbers, or Parcels, every Month.

I cannot ſuppoſe that Meſſrs. Cox and Bigg are inſenſible to the peculiar
hardſhip of my Caſe, as they are conſcious that I ſuffered, entirely, through
their caprice; but I think, had they repreſented the Caſe properly, and as it
really was, they might have alleviated it; and, at leaſt, have acquitted me of
thoſe Notes, I had, imprudently, but with good intention to them, laſt given;
eſpecially, as they were not gainers by the payment. I have only, now, to wait
with Patience, to ſee what will be the reſult of the Brief now impending,
and collecting from Houſe to Houſe, throughout every Pariſh in Eng-
land, &c. obtained through the Policy and Intereſt of Mr. Wells; who,
although he would diſdain to have his Name appear in it, will not, I preſume
refuſe to accept the ſhare of it, which may be allotted to Mr. Nettleton; for
that, the World may be aſſured, is the ſole motive that induced him to intereſt
himſelf ſo much in it.

I hope for my own, as well as for their Emolument, that the Public will be
liberal; they never had a more intereſting occaſion. Although they have en-
deavoured to make my loſs appear trivial (as it certainly is, compared, in bulk,
with ſome other ſufferers) I do here affirm, that I loſt 350 Books of Geometry,
entirely, and near two hundred (out of five) of the Perſpective, which entirely
fruſtrated all my hopes of Emolument, from the publication of that Work for
the preſent.

BOOK I.

OPTICS or VISION.

SECTION I.

Of LIGHT and COLOUR.

TO treat on any Science, properly, it is neceſſary to begin with the Elements of it, or firſt Principles; which is the reaſon, as I ſuppoſe, that ſome writers, on Perſpective, have prefaced their Works with a chapter on Optics and Viſion. Some of whom, not daring to advance any thing of their own, on the ſubject, have favoured the World with extracts from Sir Iſaac Newton's and Smith's Optics; which, indeed, is very little to the purpoſe, as the much greater part is no way ſubſervient to Perſpective; conſidering it not merely optical, but purely a mathematical Science, ſupported on the firm Baſis of Geometry.

The ingenious Mr. Hamilton, in his Stereography (a work which does great credit to the Author) to which I acknowledge myſelf indebted, has given ſome judicious obſervations on the ſubject of Viſion; and ſome, which are rather exceptionable. But, the Theory of Colours (if that can be called a Theory, which is ſo little known, and nothing in it demonſtrable) he, very wiſely, declines enlarging on; as not being ſubſervient to Perſpective, nor indeed to Painting: I mean the Theory of the Priſm, which, Sir Iſaac Newton and others have ſo copiouſly expatiated on.

I would by no means have it thought that I intend, or have the leaſt deſire, to derogate from the character of ſo great a Man, whoſe Name I highly revere; but I am perſuaded, that the purſuits of the greateſt Men are, ſometimes, in themſelves, trivial and merely amuſing; when the character of a great Man is eſtabliſhed, it gives a ſanction of conſequence to his moſt inſignificant amuſements; for, why may not the moſt profound reaſoners relax from their intenſe ſtudies, and amuſe themſelves, ſometimes, with trifling matters? by which means, things of great utility have been brought to light; as muſt have been the caſe in reſpect of the Priſm: nor do I find that he has perfected what he had in view, but left his obſervations for others to improve on, having (in his own words) not inclination to take it up again, and purſue it further, as he had intended.

Had the Theory of Colours, as deduced from the Priſm, been amongſt the firſt and chief of that great Man's purſuits, I am much in doubt, if the reputation he has acquired had ever been eſtabliſhed, at leaſt on that Baſis; things of infinitely more importance to the Community, fixed his credit (moſt deſervedly) on the higheſt pinnacle of Fame; for, what uſeful and neceſſary knowledge has been communicated to mankind, by this acquiſition to the Science of Optics? which (with ſuch, apparently wondrous, ſagacity and penetration) he has explored and given to the World.

To define, with any degree of preciſion and perſpicuity, what Light is, is not poſſible; ſeeing we cannot comprehend, and enter into an accurate diſquiſition of Fire; which is the only cauſe we can conceive of Light. From the different and obſtinate refrangibility of the different Colours produced by a Priſm, when applied to the Sun's Beams, it is concluded, that Light is, in its nature, heterogeneous; that it is compoſed of Particles, of different qualities, which produce very different effects on Objects. I cannot, from any thing I have yet ſeen or read on the Subject, give my aſſent to that opinion; for, I believe, that what we call Light (the vivid glow occaſioned by any luminous body) is perfectly homogenial; and that it is not compoſed of Particles; which implies that it is material, a Body; for, how elſe can it be a compoſition of Particles? which, by means of the Priſm, are ſuppoſed to be diſſevered, and ſeparated from each other.

But, what reaſon can be aſſigned, that a Priſm ſhould have this moſt extraordinary power of ſeparating them; owing to its form only *, not the matter of which it is compoſed: But it is not owing to either, as it is manifeſt by Experiment. Allowing the rays of Light to be thus ſifted or ſeparated, what uſe is made of it, or what further knowledge is deduced from it?

'Tis obvious, that the ſame Object, having its Surfaces differently diſpoſed, or ſituated, exhibits different Colours, according as its Surfaces are ſituated to the Light; that is, to that quarter, from which any thing luminous cauſes an illumination of them; and thoſe parts, which are moſt directly oppoſed to it, are moſt intenſe in Colour; whilſt the oppoſite Faces (the Light being obſtructed by the Object) are almoſt deprived of Colour; which they would be entirely, but for other Objects in vicinity with and oppoſed to them: which other Objects being ſtrongly illumined, are ſaid to reflect Light to them. Hence, a concluſion is drawn, that Objects have, in themſelves, no inherent Colour; but that, all the ſenſe we have of different Colours, in the ſame or in different Objects, is entirely owing to their different qualities and modifications (or, rather, to the conſtruction of their Surfaces, ſimply) by which, they are fitted and diſpoſed to reflect different coloured Rays, ſome more copiouſly than others. But, it does not neceſſarily follow, that they have no inherent Colour; for although Objects, when oppoſed to the Sun (that is, when nothing opake intervenes) exhibit very different, and more brilliant colours than otherwiſe; yet, if its Rays are obſtructed, inſtantaneouſly (the Light around them being then ſuppoſed ſtagnated), they ſtill exhibit Colour, and of the ſame kind, or hue, though leſs vivid, and greatly inferior in the degree of it.

The chief Argument advanced in favour, and the only reaſon, of weight, aſſigned, in ſupport of their Hypotheſis, is, that Objects exhibit no Colour, when they are entirely deprived of Light; which argument cannot be denied; becauſe, no Object, in ſuch Caſe, is perceptible by means of Viſion; without Light our Optics are of no uſe at all. It is the ſame if we cloſe our Eyes, when Objects are fully illumined; but, certainly, the Colour, as well as the Object, remains, whether, by ſhuting our Eyes, we perceive it or not. I look on this as a ſpecious, and ſubtle, not to call it a ſophiſtical way of reaſoning; ſince, without Light, there can be no Viſion; and conſequently, the ſenſe of Colour, more than of the Figure, Magnitude, or Situation of Objects, cannot be communicated. But, if Light be conſidered as a Medium exiſting of itſelf, without the Luminary, how can there be a privation of it, if it fills the whole ſurrounding ſpace, as Air does? which, ſeeing that, Air is alſo conſidered as a material Body, is ſomewhat repugnant to the eſtabliſhed and univerſally received Maxim, that, two Bodies cannot fill or occupy the ſame Space, at the ſame Time. And, if Light be a Medium, of what uſe is it, without the Luminary? or, how can there be a perpetual emiſſion, from the Luminary, the whole Space being, every where, filled with Light? and alſo, if it be material, what becomes of it? To ſuppoſe that it is abſorbed, by the Earth and other Planets, would be ridiculous in the higheſt degree; becauſe, the whole Surfaces of all the Planets bear no ſenſible proportion to the immenſe Space between

* A Priſm, as defined by Euclid (Def. 13. 11.) is a Solid, contained by Planes, of which, two oppoſite are equal, ſimilar, and parallel; all the reſt are Parallelograms; ſo that, every Parallelopiped is a Priſm, by which, being right-angled, no effect of Colour is produced. Now, as it depends, ſolely, on the inclination of one Plane to another, an acute angled Pyramid (in which all the Planes are Triangles) or fruſtum of a Pyramid, is the ſame, in reſpect of Colour, as a Priſm.

them

them, in which there is no obſtruction to the progreſſion of Light, flowing from the Luminary in all directions.

That Light ſhould exiſt in darkneſs (notwithſtanding the Scriptures ſay, that Light was created before the Sun) is not only a direct contradiction in terms, but to reaſon and common ſenſe, and the nature of things. Alſo, that Objects exhibit no Colour, when deprived of Light, is certain; becauſe they are no longer viſible; but, ſince every other quality remains, it is moſt probable (Colour being perceptible by ſight only) were they objects of Viſion, in total Darkneſs, Colour would remain likewiſe: For, although Colour cannot be perceived without Light, yet it is not the efficient cauſe of different Colours, in Objects compoſed of different materials.

In treating on Colours, the learned Boyle gives us a wonderful account of a blind Man's diſtinguiſhing Colours by the touch; which can only be by the aſperity or roughneſs of the ſurface of the coloured Body, and which is conſtrued as favouring their Opinion. It may be poſſible, that a difference in Colours, artificially laid on the ſurfaces of Bodies, may be felt; or, that the qualities of Dies or Stains may alſo affect Bodies ſo, as to make them ſenſible to the touch. But, will any perſon venture to ſay, that if Bodies, ſtained or otherwiſe coloured, were poliſhed or made equally ſmooth, the difference, in Colour, could be perceived by feeling; or, that the natural Colours of Wood, Metals, or Stones, nearly of the ſame qualities, except Colour, when equally poliſhed, could be diſtinguiſhed by that mode of Perception? I affirm they could not; nor can I give any credit to ſuch aſſertions. Then, ſince that, Bodies, equally hard and equally poliſhed, exhibit different Colours, I ſee no reaſon for ſuppoſing that the cauſe of their differing in Colour is owing to the different texture or conſtruction of their Surfaces; or, that it is poſſible to diſtinguiſh them by their aſperity; for, Bodies of equal hardneſs, and being equally poliſhed, muſt have Surfaces alike, except in Figure and Colour.

Light, however propagated, has, I preſume, the ſame properties and influence; and, conſequently, acts the ſame on Bodies. I would aſk then, what is the reaſon, why Blue and Green are ſcarce diſtinguiſhable by the light of Fire? If the action and reaction of Light, on the ſurfaces of Bodies, be uniform, by a certain law, according as they are fitted and diſpoſed to reflect any particular ſpecies of colour-making Rays, whence ariſes the difference in this caſe? I ſhould be glad to hear or read an ingenious diſquiſition of that point; as, I do not doubt, there are Men of ſagacity and penetration ſufficient to ſet it in the cleareſt light, and make it as evident, to the underſtanding, as any common Caſe in any Science whatever.

To return to the Priſm. It is certainly true (and it is a curious and entertaining Experiment) that, through an acute angled Priſm, we perceive Objects very differently coloured from what they really are by Nature; alſo, by applying the Priſm, properly, to the Sun's Beams, there is exhibited a curious and moſt extraordinary Phænomenon, *viz.* a diverſity of Colours, the moſt lively and beautiful that can be conceived; and, the more uncoloured the Surface is, on which they fall, the more vivid they appear.

Now it is certain, that here is a perception of Colour, on the ſurfaces of Bodies, which is not inherent; and ſo is there when the Sun's Beams, only, fall on them: yet, we cannot from thence conclude, that they have no inherent Colour. The Colours which the Priſm exhibits are Red, Orange, Yellow, Green, Blue, Indico, and Violet or Purple. But why is there, here, made a diſtinction of ſeven Colours, when, in reality, there are not above three or four ſimple Colours in Nature? (unleſs Black and White may be called Colours) *viz.* Red, Yellow, Blue *; and Green; which may alſo be compounded of Blue and Yellow. Indico is Blue, Orange and Purple are Compounds, or mixed Colours. Now, if any Philoſopher, or Artiſt whatever, can, from theſe three or four ſimple and brilliant Colours, produce all the variety which we ſee in Nature; or, did the Priſm exhibit di-

* It is ſaid in Emerſon's Optics (Cor. 3. Prop. 9.). There are as many ſimple Colours as there are degrees of Refrangibility; and therefore an infinite number. And, hence (in Cor. 5.) it is plain, if the Sun's Light conſiſted but of one ſort of Rays, there would be but one Colour in the World. In one place, he ſays, Objects have no Colour naturally; and in another, they become coloured, by reflecting the Light of their own Colour, more plentifully than others.

ſtinctly, all the ſimple Colours which are in Nature, only, without mixture, I ſhould then be better diſpoſed to give credit to their Theory: or, when I can have conviction, that a mixture of all theſe Colours, together, in any ratio, will produce a perfect White (as Snow) I may then be a perfect Proſelyte; till then, I am perſuaded that I muſt diſſent from their Opinions.

I ſhall, in the next Place, enquire, how a Priſm, particularly, has the wonderful property of ſeparating the different coloured Rays of Light. The Priſm is a body of Glaſs, or it may be of Cryſtal, or other pellucid, uncoloured Stone; is the property then in Glaſs, or Stone? No; in the Figure that is made of it, only; amazing! that the diſpoſal of the ſame Matter into different Forms ſhould produce ſuch very different Effects. Glaſs, diſpoſed into a portion of a convex Sphere, will make Objects, ſeen through it, appear magnified; and, being oppoſed to the Sun's Beams, will collect real Fire; aſtoniſhing indeed! If the Surfaces be concave, Objects, ſeen through them, appear leſs than they really are; in the forms of Priſms it has various effects; but we do not ſee the effect of exhibiting Colours through a right-angled Priſm; I mean, a four-ſided one, whoſe oppoſite faces are parallel, and the Angles right ones; neither has it that effect through the right Angle, of a right-angled triangular Priſm, but only through the acute Angles. The cauſe, then, of exhibiting Colour is not in the Matter itſelf, or Figure, nor in the Surfaces, ſimply, but in the inclination of the Surfaces to each other; for, being parallel, or at right angles, or nearly ſo, they do not produce that effect. We may as well aſk, why convex Surfaces collect Fire (but, that is well known) rather than plane or other parallel Surfaces, as, why a Priſm, whoſe Planes are inclined to each other, ſhould have the property of exhibiting the different Colours I have mentioned, rather than one that is right-angled; and I am perſuaded, that, with all the ſagacity and penetration Man is endowed with, he will never be able to account, truly, for either.

Yet, I do not condemn all enquiries into the cauſes of the various effects which we perceive in Nature, but think the purſuit rational, and truly commendable, when it is founded on certain Data, and real Hypotheſis; and the reſult of our reſearches productive of real utility, deducible from it, as in ſome other branches of Optics. It is aſſerted, by ſome, that the perfection of Teleſcopes is owing to the Theory of the priſmatic Colours. As I am not converſant in the mechanical conſtruction of Lenſes and their application to Teleſcopes, I cannot affirm that it has not been ſubſervient thereto; but, in my opinion, it has not the leaſt apparent tendency to benefit Mankind accruing from it. Certain I am, that it will never be of uſe to a Painter, to compound his Colours and form a Theory thereof; by means of which, he may ſooner, and with certainty, arrive at perfection, in his Art: and I muſt needs ſay, that the attempt made by Brook Taylor, in the Appendix to his ſecond part, does not ſhew the Doctor's judgment, in that, to be of a piece with the reſt of the Work.

The Sun's Beams paſſing through Glaſs, whoſe Surfaces are neither parallel nor perpendicular to each other, or nearly ſo, exhibit the various Colours, ſpoken of above, in ſome degree, according to the inclination of the oppoſite Surfaces, and purity or clearneſs of the Glaſs; through the thick part, near the knob, in the middle of a table of Crown Glaſs, I have often obſerved the Phænomenon, in a Window of thoſe Squares: but, it is alſo obſervable, that the effect ceaſes when the Sun ceaſes to ſhine on them. It is alſo certain, that Objects appear coloured otherwiſe than what Nature aſſigned them, when we look through Mediums denſer than Air; whoſe Surfaces, through which we look, are inclined to each other in certain Angles; and conſequently, there will be the effect of Colour produced, in ſome degree, which is not natural to Objects, in looking through Object-Glaſſes, or Lenſes, with which Teleſcopes are conſtructed; ſeeing that, their ſurfaces are inclined, at the edges. The buſineſs, therefore, of an Optician, in this reſpect, is, ſo to contrive and diſpoſe his Glaſſes, as to diveſt the Object of the borrowed Colours, which do not belong to them, naturally; and I could almoſt affirm, that the perfection, ſpoken of, has been found out by repeated trials; not from any certain eſtabliſhed Law, deduced from the Theory of the priſmatic

Colours: for, if a certain Theory had been eſtabliſhed, what hindered the immediate perfection of them, as ſoon as the cauſe of the imperfection was known, and certain Laws eſtabliſhed, whereby the unnatural effect of Colour might be removed from the Field of View?

From what has been obſerved, reſpecting the effect of Colours produced by the Priſm (unleſs it can be proved that the particles of Air, through which, Light muſt neceſſarily paſs, are Priſms) to infer, that Nature has given no inherent Colour to Objects, is bold and aſſuming; it is alſo groundleſs, ſeeing that the Colour, which is natural to each, remains when the Sun does not ſhine on them, though it differs greatly in the degree of it. We may with as much reaſon conclude, that Objects are much larger than we perceive them to be with the naked Eye, becauſe they appear ſo, when view'd through Glaſs whoſe ſurfaces are convex; and, we may as well enquire how Vegetables, as Flowers, Fruit, &c. ſpring out of the Earth, and are adorned with all that beautiful variety of Colours which we ſee in Nature; by what means they are continually varying in their Colours, from their firſt formation, in embryo, to maturity; as why, the Priſm exhibits the various Colours of the Rainbow, to which they are, in a great meaſure, ſimilar.

Colours produced by alkaline Salts, &c. and mixtures of different Fluids, with all ſuch like chymical operations, I look on, equally, as entertaining Experiments; but no way productive of any advantage, accruing from it, to the Theory of Colours neceſſary for a Painter to know, in order to reduce it to real uſe, in practice.

I ſhall not, therefore, treſpaſs any longer on the Reader's Time, as it is entirely foreign to the purpoſe of Perſpective; but confine my further obſervations to that part of Optics relative to Viſion, only. In reſpect of direct Viſion, it is certainly of uſe and ſubſervient to Perſpective; and eſſentially neceſſary to give a clear Idea of the Principles on which it exiſts, both in Theory and Practice.

Colour, whether inherent or incident to Bodies, is not material towards a perſpective Deſcription or Delineation; 'tis the Figure of the Object, only, it contemplates.

All ſolid Bodies, whatever, become objects of Perception from their Figure or Colour. Figure is more particularly inſeparable from our Ideas of Matter, than Colour; it being impoſſible, in the nature of things, to have any notion of Extention, abſtracted from the Idea of Figure, which limits the Space Bodies occupy or fill. It is the ſame with or without Light, and may be perceived by the ſenſe of Feeling only. From the known external Figure, and ſituation of an Object, may be delineated, by mathematical rules, a true perſpective Repreſentation, on a Plane; which, by the help of Light and Shade, will raiſe a perfect Idea in the Mind, of the figure of the Object, and the different forms and poſitions of its Surfaces, in reſpect of each other, without the aſſiſtance of Colour; but, to exhibit a true and natural Picture of Objects, and create a perfect Idea in every circumſtance, Colour is abſolutely neceſſary; and is the laſt degree of perfection, with which the omniſcient Creator has embelliſhed Nature, that can be given to a Picture.

Light, conſidered as a Medium, by which Viſion is convey'd to the Eye, is of too refined a nature for my ſpeculations. But, as the great Author of Nature has given a portion of Reaſon to every human Being, we have certainly a right to make uſe of that Reaſon; and, if we will exert it, properly, it is poſſible, nay certain, that one Perſon may penetrate as deeply into the myſteries of Nature as another, though not bleſs'd with quite ſo much Learning.

We are told by ſeveral great and profound Philoſophers (to be a Mathematician is not neceſſary) that we perceive Objects, only by means of Rays of Light, reflected from every point in their Surfaces to the Eye; which enter there, and form an Image, or Picture of the Objects perceived, on the Retina, or fine Membrane which ſurrounds all the back part of the Eye, internally. The Retina is ſaid to proceed directly from the Optic Nerve, which dilates or ſpreads itſelf as beforementioned; and, the impreſſion being made thereon, we are further told, is conveyed, by the Optic Nerve, to the Brain, or ſeat of Perception.

But, why do they ſtop here? I expected, and ſhould be glad to be conveyed into the inmoſt receſſes of the Brain, and be ſhewn or told, how the Image of the Object, on the Retina, is there perceived; for I muſt own it is aſtoniſhing! that, from what is perceived or felt within, we ſhould have a true Idea of the Figure, Colour, and Magnitude, Situation and Diſtance of Objects, which are external, or ſituate without the Eye.

Now, after all this parade, and pompous diſplay of great ſagacity and deep penetration, what does the Sum total amount to? Why, that the Object is perceived, i. e. the Mind is ſenſible of the exiſtence of ſuch Objects as it perceives; and that, the Viſion of them is conveyed in right or direct Lines from the Object to the Eye, or from that wonderous Organ to the Object; but how, or in what manner, remains as much a myſtery as before. So that, after all which has been ſaid on the Subject, and, allowing the Image, formed on the Retina (inverted or otherwiſe) as perfect as they pleaſe, what nearer are we? where is the Perception of that Image or Picture of the Object? How the Mind, or Soul, perceives the Image on the Retina, any more than the real Object, we are juſt as much at a loſs to account for as ever.

In vain, therefore, does Man torment himſelf, in endeavouring to explore the hidden myſteries of Nature, which are for ever hid from our reſearches. Let us then purſue what is within our reach, and not an airy Phantom, which will ever elude the purſuit. In mathematical Sciences we have ſome certain Data, whereon to frame Hypotheſes; and although we muſt not expect perfection, except in Theory, yet, we can diſcover ſo much of Truth as gives us ſufficient encouragement to purſue it, and makes ample amends for the imperfection of human nature.

I do not intend, nor ſhall I attempt to explain the nature of Viſion, or enquire into the true cauſe of it; being fully convinced of the inſufficiency of our reaſoning faculties for ſuch a diſquiſition. It is ſufficient, for the ſcience of Perſpective, to know, or even to ſuppoſe that it is conveyed to the Eye, the ſeat of Sight, in Right Lines. But, whether it be by means of Rays * of Light, reflected from all parts of Objects to the Eye (as is the general Opinion) or whether the Allwiſe former of the Eye has given a power to that truly wonderful and amazing Organ, to convey to the Mind, by Viſion, the perception of Objects by other means, I do not mean to make the ſubject of my enquiry. Yet, I muſt acknowledge, that I have ſtrong objections againſt the general Opinion, as it is now received and almoſt univerſally aſſented to; *viz.* that the perception we have of external Objects, from Viſion, is by means of Rays of Light, reflected from all parts of their Surfaces to the Eye; and that thoſe Rays are material, or compoſed of Matter.

But, as it is of no conſequence, in Perſpective, by what means Viſion is performed, ſo it be conveyed in Right Lines, which may paſs for an Axiom; ſeeing that, the refraction of the Rays (if there be any) in the Air which ſurrounds us, is ſo very little, in Objects which are at an immenſe diſtance beyond the whole Atmoſphere, it cannot be ſenſible at any diſtance we can perceive Objects, within it, ſuppoſing the refraction uniform or regularly curved, between the Object and the Eye. But, I rather ſuppoſe the refraction of the Rays to be at the common Surface of two Mediums, only; at entering any other Medium, denſer or rarer, than that through which they firſt paſs, and thence proceed again in Right Lines; conſequently, there is no Refraction at all, within the Medium; either in Air or Water, I ſuppoſe Viſion to be convey'd in Right Lines, if the Object and the Eye are both in the ſame Medium.

At the ſame time, I would not have it thought, that I ſuppoſe any Rays to paſs, by means of Reflection, from opake Bodies; and in luminous ones, the Light, which proceeds from them, filling a concave Sphere, as far as they can be ſeen, I do not ſuppoſe to proceed in Rays, nor in Planes (as a modern Author has ſuppoſed); the Hypotheſis is abſurd; ſeeing that, the whole Space is illumined in every part, it is impoſſible, that from every Point in the ſurface of the luminous Body, Rays can proceed to an immenſe diſtance, filling all the Space between.

* I make uſe of this Term, though I have not the leaſt conception of a Line, or thread of Light, called a Ray; much leſs of a Pencil, or bundle of Rays.

Suppoſe

Suppoſe every Point in the ſurface of the Sun to emit a Ray of Light, proceeding in Right Lines from the Center; it is evident, that every Ray, at the diſtance of the ſemi-diameter, will fill a Space four-fold of its firſt dimenſions, *viz.* in a duplicate ratio of the Diſtance; at the diſtance of the whole Diameter, it muſt fill a Space nine times as large; i. e. it increaſes in proportion to the Squares of the Diſtance; what, then, will be the magnitude of a ſingle Ray of Light, proceeding from the Sun to Jupiter or Saturn, before it reaches thoſe Planets*? or how can the whole of the Space it occupies be filled (as it certainly is) with Light, which proceeded from a ſingle Point, in the Luminary only? Can one individual Ray be ſhattered into thouſands, into millions, lying cloſe to each other? what direction do they proceed in, from the Luminary? not in the direction of a Right Line from the Center, I preſume.

I know it may and will be alledged that it expands as it proceeds, ſtill filling the whole Space. I preſume then, it cannot proceed in Rays, or Right Lines from the Center of the Luminary, as it is imagined. Air may be rarified and expanded to a very great degree; alſo, Water and other Fluids are expanded, as in ſteam, by Fire, or, in Miſts and Vapours, by Exhalation, till it floats on Air. But can one drop, or the minuteſt particle of Water be expanded infinitely? as a Ray of Light, emited from one ſingle Point in the Luminary, muſt, in this Caſe, neceſſarily be; elſe, how could we ſee thoſe Stars, whoſe diſtances are, to all ſenſe, infinite?

Theſe things, which may amuſe and paſs for orthodox with the generality of mankind, cannot be ſatisfactory to a Perſon who dare think for himſelf, and is furniſhed with the means to do ſo. Therefore, ſince all which can be ſaid about it is, at beſt, but an ingenious conjecture, and ſince it is in no wiſe ſubſervient to Perſpective, I ſhall at preſent take my leave of the Subject, to purſue that part or branch of Optics, which is eſſentially ſo, reſpecting direct Viſion; and after that, in a ſeparate Chapter, or Section, I ſhall give my objections to the received Opinion of the cauſe of Viſion.

* I find by calculation (from the Tables in Harris's Uſe of the Globes, which I ſuppoſe are as much to be depended on as any other) that the diſtance of the Earth, from the Sun's Center, is 212 ſemi-diameters of the Sun, nearly. Wherefore, if, inſtead of a Point, we ſuppoſe a certain quantity of Light to flow from one ſquare Inch in the ſurface of the Sun; each ſide of that ſquare Inch will, at that diſtance, be increaſed to 212 inches, or 17 feet 8 inches; the ſquare of which is 44944 ſquare inches. Conſequently, a quadrangular Pyramid, whoſe Baſe is 17 feet 8 inches ſquare, and its altitude, the diſtance of the Sun from the Earth, is filled with Light, from one ſquare Inch, only; and conſequently, each ſingle Ray, proceeding from every Point in its Surface, will, at that diſtance, be multiplied, or magnified 44944 times.

At the diſtance of Saturn (by the ſame Tables) a ſquare Inch, and conſequently a ſingle Ray, will be encreaſed or multiplied 4,149,369 times; its diſtance being 2037 ſemi-diameters of the Sun, nearly; conſequently, a quadrangular Pyramid, whoſe Baſe is 169 feet 9 inches, on each ſide, and its altitude 777 millions of Miles, is filled with Light, flowing from every ſquare Inch on the ſurface of the Sun; an amazing circumſtance indeed! if it is ſuppoſed, or conſidered to be a material Body. But, how far Light may proceed, from the Sun, beyond the realms of Saturn, is not yet, I preſume, determined.

SECTION II.

Of the ſtructure of the EYE, its parts deſcribed, and a ſhort INTRODUCTION to the nature of VISION.

THE conſtruction of an Eye, that moſt wonderful organ of Sight, by means of which, we have intelligence of the Objects which ſurround us, is various; yet has nearly the ſame Parts, in all the animal Creation. The Parts are few, and apparently ſimple, but aſtoniſhing in their mechaniſm; as, indeed, are all the animal Functions; but, what is there in Nature which is not ſo? The Figure of an entire human Eye, within its Socket is globular, of ſomewhat more than an Inch in Diameter; the Furniture of which are as follow.

Plate I. Fig. I. Let HIKL be a vertical Section through the center of the Eye, which, in this Figure, is conſiderably larger than Life; in order, that the ſeveral parts may be deſcribed with greater accuracy.

The external part, from H to I, is called the Cornea, or Horny Coat. It is perfectly tranſparent, and is ſomewhat more convex than the part which is within the Socket, by means of a fine, clear, aqueous or watry Humour, between the Cornea and the Iris (the coloured Circle within). Immediately behind the Iris, (g h i k,) through which there is in Aperture, in the middle, to let in Light (or Viſion by means of Light) is a whitiſh Subſtance (M N) like a ſtrong Jelly, or cold Glue, of a moderate conſiſtence. It is as clear and pellucid as Cryſtal; from which, it is called the Cryſtalline Humour. It is in every reſpect of the ſame nature and uſe as a double convex, microſcopic Lens; and is ſaid to be more convex on one Side (the inward) than the other. Its uſe is ſuppoſed to contract the Rays of Light, by which Viſion is convey'd, to a Focus, beyond this Humour, in the Center of the Eye, at E; fromwhence, they become diverging, and fall on the bottom or back part of the Eye, at *a* c *b*.

The Cryſtalline Humour (MN) is ſaid, by thoſe who are acquainted with and accuſtomed to diſſections of the Eye, to be furniſhed with muſcular Fibres; by means of which, it is made either more or leſs convex; or, by contracting them, is brought forward or otherwiſe as occaſion requires; in order, to render the Images, or Pictures of Objects perfect, in the bottom of the Eye, as the Objects are nearer to or further from the Eye; all whch ſeems conſonant to Reaſon.

The Aperture or Pupil (h i) is alſo, by ſome ſuch means, expanded or contracted, as there is occaſion for more or leſs Light to enter. I have obſerved, in the Eye of a Child, in a Room juſt light enough to diſcover it, the Pupil enlarged to three tenths of an Inch in diameter; and immediately, on bringing into a full Light, it was contracted to one tenth, or leſs; amazing Structure! and this is performed involuntarily, whether we will or no. There can ſcarce be any Perſon who has not made the experiment; on going into a darkiſh Room, he can ſcarce, at firſt, diſtinguiſh any thing; when preſently, he will, after the Aperture in the Iris is open'd to a proper dimenſion, diſcern Objects diſtinctly; and again, on going into a full Light, he cannot bear to look on any illumined Object, till the Pupil is contracted; it is even painful to the Eye, at firſt, going out of a dark Room into a ſtrong Light, eſpecially if the Sun ſhines out.

All the back part of the Eye, within, is lined with a fine Membrane of a moſt curious and delicate texture; called the Retina, from its reſemblance of net-work. The large cavity, V V, which is bounded by the Retina, backward, and the Cryſtalline Humour, before (which is contained within it) is filled with a glutinous Fluid like the white of an Egg; being perfectly pellucid and uncoloured, it is called the vitreous or glaſſy Humour; in which the Rays converge, at E, the center of the Eye; and, croſſing each other, in that point of contraction, they fall diverging on the Retina, and form the Images of Objects thereon; the ſenſation, of which, is ſuppoſed to be communicated, by the Optick Nerve (which is connected with the Retina) to the Senſorium, in the Brain.

As the manner in which the Rays of Light are refracted, in paſſing through the various humours of the Eye, is entirely conjectural; ſeeing that, through a ſmall pin-hole, applied cloſe to the Eye, at q, there may be perfect Viſion, of an Object of any magnitude; in which Caſe, the whole Syſtem of Rays fall on the Comea in one Point; how, then, are they refracted? As, this Eſſay is not intended as a Treatiſe on Optics, I ſhall not inſiſt on any thing; but, ſhall only ſuppoſe the Viſual Rays (when the Eye is naked) to converge and croſs each other in the center of the Eye, at E (without conſidering their Refractions) from whence, I ſuppoſe, they diverge in equal Angles, falling on the Retina, at *a* c *b*, in the back part of the Eye.

DEFINITIONS.

1. The AXE of the EYE is a Right Line paſſing through the Center of the Eye and the Center of the Aperture or Pupil. As cEC.

The Point c, where the Axe cuts the Retina, is the place where Viſion is perfect; which point is varied as the Eye is moved, or its Axe directed to different Objects, or to different parts of an Object.

2. VISUAL RAY. If AB be ſuppoſed an Object of Sight; the Right Lines AE, BE &c. from all parts of the Object to the Eye, or to its Center (E) under which, or by the means of which, the Object is ſeen, or ſuppoſed to be ſeen, are called Viſual Rays.

3. OPTIC ANGLE. By Optic Angle may be underſtood, either a plane or ſolid Angle.

If the Object AB be conſidered as having no dimenſions but length, only; i. e. if it be a Right Line, ſimply; the two Viſual Rays EA, EB, from the Eye to each extreme, form a Plane Angle, AEB; which is the Optic Angle, under which, the Line AB is ſeen. And, AED or DEB is the Optic Angle of the Segment AD or DB.

4. CONE, or PYRAMID of RAYS. If the Object of Sight be either globular, or circular; as ABG; the Viſual Rays, EA, EB, &c. from the Eye to every part of the extremes of its Surface, being towards the Eye, form a Figure reſembling a Cone, which is called the OPTIC CONE. Fig. 2.

But, when the Object is either a right lined Solid, or Plane Figure, as CD; the Viſual Rays EF, EC, EH, &c. form a ſolid Angle, compoſed of ſeveral plane Angles, FEC, CEH, &c. which, being of a pyramidal form, is called a PYRAMID of Rays.

The Viſual Rays croſſing each other, in E, and, from thence, become diverging to the Retina, each oppoſite plane Angle being equal, † there is formed a ſimilar Pyramid, cEd, which is called the oppoſite Pyramid; and E is their common Vertex. † 2. 1. El.

The Figure, ab or cd, which is the Baſe of the oppoſite Cone or Pyramid of Rays, is the Image or Picture of the Object, AB or CD, on the Retina.

The correſponding Characters ſhew in what manner the Image is inverted.

Let AB be ſuppoſed an Object, direct before the Eye, that is, perpendicular to its Axe (EC). It is ſuppoſed to be ſeen, by means of the Viſual Rays EA, EB, EC, &c. from every part or point in the Object, towards the Eye, whether it be Line, Surface, or Solid; which, from the point of contraction (E), proceed diverging, to the back part of the Eye, where they fall on the Retina, and form the Picture, *a c b.* of the Object AB.

It may be alledged, that it is impoſſible for the Points A and B to be ſeen, at the ſame time; ſeeing, the Rays, from thoſe Points, do not enter the Pupil. I grant they cannot; nor F and G, diſtinctly; nor indeed, little more than a Point, without moving the Eye; we have but a confuſed intimation of the ſurrounding Objects, at a little diſtance from that Point, to which the Axe of the Eye is directed; of which, every Perſon's own Experience muſt ſoon convince him. But, the quick tranſition of the Eye, or the direction of its Axe, from one part of an Object to another, is ſo little attended to, that, many imagine the field of View is much larger than it really is. As for the refraction of the Rays, through the various Humours of the Eye, I ſhall leave for the ſpeculation of thoſe, who are bleſs'd with a more fertile imagination than I have any pretenſion to.

Plate I. Fig. 1. The Image of the extreme point A, by means of the Viſual Ray A E, falls on the Retina at *a*, and the other extreme, B, is repreſented at *b*; and C E, the Axe of the Eye, conveys the Image of the point C to c, in the center of the Retina. From which it is evident, that the Picture of an Object, formed on the Retina, is neceſſarily inverted.

Nor is there any thing ſurprizing in this, as ſome may imagine. For, ſuppoſe A the top, and B, the bottom of the Object. It may be imagined, from the inverted poſition of the Image, *a c b*, that the Object muſt neceſſarilly appear upſide down. But, if we conſider, that the ſenſation of the Point A, being perceived or felt at *a*, is by the direction of the Viſual Ray A E, which determines its place in the Object; the Mind, by long experience, having acquired a habit of determining that part of an Obejct, perceived at *a*, to be above, and that at *b*, below; by the ſame reaſoning it is manifeſt, that the points of Object on the right hand are pictured on the left, and thoſe to the left, on the right; and which, is no matter of ſurprize at all; no more than, that, by the ſame experience, we have acquired an Idea of the diſtance of any Object perceived.

If a Perſon, who was born blind, could, when grown to maturity, be made to ſee, he would have no Idea at all of Diſtance, or the ſituations of Objects; which way was up, or which down; and would as ſoon attempt to lay hold of the moſt diſtant Object, which he perceived by the ſenſe of ſeeing, only, as others which were near him; it being impoſſible to diſtinguiſh, merely by Sight, whether the Point perceived at c, on the Retina, be at *C* or C, or at any diſtance beyond *C*, in the direction of E C. But, by Experience and a familiarity with Objects, according to their known magnitudes or diſtinctneſs of parts; or, according to the length of Ground, which we imagine to lie between us and the Objects, we judge them to be at ſuch or ſuch a Diſtance. Whereas, when we look at the Heavens, one Star appears as far off as an other; nor can we form the leaſt Idea of their diſtance from ſight, ſimply; for, a Star may as well be ten thouſand millions of Miles off as ten Miles, its Magnitude being in Proportion to its Diſtance. (See Fig. 7. A, B and C.)

The Art of Seeing is acquired, regularly and progreſſively, as all other Arts or Knowledge whatever. Do not we ſee a Child attempt to catch the Moon or other ſtriking Object, though at an immenſe Diſtance? but, growing up in a familiarity with them, by common and frequent experience, they become ſo intimately connected with our Ideas, that we form a judgement of their Magnitudes and Diſtances, inſtantaneouſly, at firſt ſight; and alſo of their Situations, in reſpect of themſelves and of each other.

It may, to ſome Perſons, appear ſtrange to call Seeing an Art; but that it is ſo is certain, although Nature has furniſhed us with that moſt wonderful Organ of Sight, the Eye; which, being perfect, is not poſſible to be open (with ſufficient Light for the purpoſe) but we immediately ſee what is before the Eye. A Child ſees as ſoon as it is born; but it ſees, without judgement, or diſcerning what it ſees. So would a Perſon born blind, and continued ſo, till he was mature in Judgment, in every other kind of Perception. Let us ſuppoſe ſuch a Perſon, immediately made to ſee. What Ideas he would have of what he merely ſaw, at firſt opening his Eyes 'tis impoſſible to ſay; but, of this I am well aſſured, that he could not diſtinguiſh the moſt familiar Objects, without Experience; as Figure and Colour are things of which he had formed no conception.

There are not many more oppoſite, regular Objects, than a Cube and a Sphere, or Globe; nor any, more eaſily diſtinguiſhed, by feeling; yet, even theſe could not, with certainty be known, merely by ſeeing them, which was one and which the other; becauſe, Experience had not taught, in what manner ſuch Objects affect the Eye. In reſpect of Colour, 'tis impoſſible he could have any Ideas, by which he could diſtinguiſh them; they would be, to him, as if never exiſting before. Of Diſtance, he could have no Conception; for being wholly unacquainted with the real Magnitudes of Objects and the effects which Diſtance has on them, their relative and apparent Magnitudes could not be diſcerned, ſo as to know that one was farther off than another.

This

This may appear ſtrange, to thoſe who have not conſidered it with attention; but I maintain it to be a truth, founded on ſound reaſon. For, as he knows nothing of the difference between real Magnitudes, and the apparent Magnitudes of Objects, according to their ſeveral Diſtances; he could never imagine that a Man, or a Horſe, which he ſaw at a Diſtance, was of the ſame kind as thoſe he ſaw near him; till Experience had ſhewn, that the difference in apparent Magnitudes is, merely, the effect of Diſtance. So, of any other Object, as a Houſe, for inſtance, into which, as he finds he can freely enter, being near, he could never imagine that to be a Houſe, which by means of its Diſtance, appeared not ſo big as his Head. Conſequently, ſince he could not imagine that a Man, or a Houſe, at a Diſtance, was a Man or a Houſe, of the ſame Magnitude as others which were near him, he could form no Idea of Diſtance, by ſight; beſides, the difference in Effect and Colour, occaſioned thereby, are things entirely new; of which, he had not, till now, any Ideas.

Now, the judgement we form of Diſtance, &c. being acquired early, and grown up with us, is ſo little attended to, that at firſt thought of theſe things, we ſuppoſe it to be merely from Sight; than which, no Senſe is more deluſive and uncertian. The deceptions of Viſion are many and frequent; as, when we look at Objects through refracting Mediums, i. e. through any tranſparent Medium whatever, ſolid or fluid; according to the different denſity of the Medium, or to its Figure, &c. we frequently imagine an Object to be nearer or farther off, larger or ſmaller than it really is; or to be where it really is not; nay, it is poſſible, by Light and Shade, judiciouſly diſpoſed, and the aſſiſtance of Perſpective, to deceive both the Eye and Judgment.

Of all the Senſes, that of Sight is by much the nobleſt; it is, nevertheleſs, the moſt deceptive. He who can moſt deceive the Eye in the art of Delineating and Colouring is eſteemed the greateſt Genius. It is poſſible, to deſcribe the repreſentations of many Objects ſo very accurately on a plane Surface, that (as I have already obſerved) with the aſſiſtance of Colour, and a juſt diſtribution of Light and Shade, they would, in the true Point of view, deceive a judicious obſerver, and appoar to be real Objects. As a proof of the deception, in Viſion, in reſpect of Magnitude, ſimply, I ſhall give a moſt ſtriking inſtance; of which, though it has fallen within the cognizance of Thouſands, few, or none, that I have converſed with on the ſubject, have formed a juſt Idea, or right judgment. I have aſked ſeveral Perſons, what were their Ideas of the apparent magnitude of the Moon? moſt have concluded, that it appears, when full, as large as the crown of their Hat; ſome, as a common Plate, or ſmall Diſh. I ſhould be glad to know, by what ſtandard ſuch a judgment could be formed? for, unleſs ſome Diſtance, of the Object to be compared with, be determined, no compariſon can be made; as, it may appear as large as the Dome of St. Paul's; but not, I am perſuaded, at any diſtance we can ſee that Object, by the Light of that Luminary *. Now if we ſuppoſe the Plate laid on a Table, before us; or the Hat in one Hand, when we make the compariſon, 'tis, perhaps, as random a ſuppoſition as can be conceived; and, without ſome determined Diſtance, it is vague and undeterminate. I know it will ſurprize many; but I do affirm, that, at the greateſt diſtance we can hold any thing to compare with the Moon, it does not appear larger than a Silver Penny.

Now, if Seeing be not an Art, to be acquired by Experience, how comes it, that ſome Perſons ſee with more judgment than others? Since as I am perſuaded, that, all human Eyes (being perfect) ſee alike by Nature; and conſequently, all would judge alike, of what they ſaw; but, it is certain we do not; of which, ſtriking inſtances might be given. A Picture, or an Object, it is manifeſt, muſt ſtrike every Eye alike, in the ſame Point of view; how is it then, that the moſt judicious and accurate repreſentation, of a well-known Object, does not communicate, to every beholder, the ſame Idea of the Original?

* There was a Moon-Light Piece (by a capital Artiſt) in the Strand Exhibition, laſt, in which (if I remember well) the Moon appeared near that Object, and almoſt of equal Magnitude.

It would ſcarce gain credit, to affirm, that, on aſking a Perſon, who was admiring a large, and well-executed, Print of St. Paul's Cathedral (one who had lived, ſome Years, near that well-known Object) what it repreſented? When, after a ſtrict attention to it, I was anſwered, with an Interrogation, is it Weſtminſter Bridge? 'Tis true, it was a Female; but, ſhe had Eyes, and could diſtinguiſh between a Cow and a Horſe, at firſt ſight, as well as any other Perſon.

It is, to me, almoſt impoſſible to account for this; as I have known many Children, of thre or four Years old, ſpeak with certainty of very paltry Repreſentations. I think it can ariſe, only, from their inattention to the Objects they ſee; as it is impoſſible, that a Child can have had more Experience than a Perſon grown to maturity. Nevertheleſs, it is certain, that ſome have much more Judgment of what they ſee, with leſs Experience; or, at leaſt, they acquire it much leſs time.

That Viſion is conveyed in Right Lines to the Eye, I preſume no perſon will attempt to diſpute (I would be underſtood to mean, of ſuch Objects as are ſituate on or near the Earth; and when no other Medium, but Air, is between the Object and the Eye). I ſhall, therefore, give it as a general Axiom; which, nevertheleſs, may be illuſtrated or ocularly demonſtrated, by ſtraining a fine ſilken thread, to a Right Line, from any Point, as *F*, of an Object (*DF*) to the Eye.

Then, if any other Object, as QR, be interpoſed, as ſoon as it touches the Thread E*F*, they will appear to be in contact; and if it be ſo interpoſed, as to hide the Point *F*, the Right Line or Thread, which may repreſent a Viſual Ray, will be refracted, or broken into two Right Lines ER, R*F* forming an Angle, ER*F*.

Hence it is evident, that the Points F and *F*; alſo, the Point C or *C* being in the ſame Right Line with *C*2, will perfectly coincide with each other, to the Eye, at E.

SECTION III.

Containing a brief THEORY of DIRECT VISION.

THEOREM I.

OBJECTS appear to have that Proportion to each other, reſpectively, as the Angles under which they are ſeen.

Let *a* c *b* repreſent a ſection of the Retina, a portion of a concave Sphere; and, let A B be an object direct before the Eye; divided, in any Ratio, in D and F.

Fig. 1. DEM. It is evident that the Object A B, by means of the Viſual Rays E A, E B, is ſeen under the Angle AEB; and the ſeveral Parts of A B, as A F, F D, and D B, are ſeen under the ſeveral Angles A E F, &c.

But, the oppoſite Angles *a*E f, f E d and d E *b*, are, reſpectively equal to the Angles A E F, F E D, and D E B — — — 2. 1. El.

And, the Images *a* f, f d and d *b*, on the Retina, of the Originals, A F, F D, and D B, are the meaſures of thoſe Angles, reſpectively (Th. P. Ang.*) the Retina being ſuppoſed a portion of a Sphere; each part is, therefore, equally diſtant from the Center.

But, the Ark *a* f is to the Angle *a* E f, as the Ark f d is to the Angle f E d, and as d *b* to d E *b*. — — — — — 19. 6. El.

* (Th. P. Ang.) refers to the Theory of Plane Angles, in my Treatiſe of Geometry.

Therefore, the Images or Pictures on the Retina, and conſequently, the apparent magnitudes of Objects are in the ſame ratio, or proportion, as the Angles they ſubtend at the Eye. Q. E D.

Nor is there occaſion to have recourſe to the Images on the Retina; but, allowing the Viſual Rays E A, E B, &c. to be Right Lines, and an ark of a Circle, of any radius, being drawn, cuting thoſe Rays; the portions of the Ark, l m n o, intercepted between the Viſual Rays, as l m, m n, &c. meaſure the ſeveral Angles A E F, F E D, &c. reſpectively; and conſequently, the parts A F, F D, &c. of the Object A B, will appear to the Eye, at E, in proportion to the Arks l m, m n, &c. which are equally diſtant from the Eye.

All remote Objects appear equally diſtant from the Eye; as the Planets, Stars, &c. appear as if they were all in the ſame concave Sphere; of which every Eye is the Center. So likewiſe, when we ſtand in the line of direction of two, three, or more Objects, they appear to touch each other; nor could we judge, merely from ſight, which was fartheſt off, or which the largeſt.

For, ſuppoſe the Viſual Rays E F, E D, E B, produced to *F*, *D* and *B*; any other Objects, as *D F* and *B D*, being in the ſame direction with F D, and D B, and touching the ſame Viſual Rays (E F, E B) will coincide with F D and D B; the leſs will hide the greater from Sight, and appear to be of equal magnitude.

2. I am ſomewhat ſurprized at what Mr. Hamilton has advanced, in Art. 6, Sect. 1. where he ſuppoſes, that Objects do not appear in proportion to the Angles they ſubtend, but in proportion to the Tangent of half the Angle; and imagines the Retina to be, nearly, a Plane, in the center of the ſeat of Viſion.

How Mr. Hamilton could conceive and endeavour to account for ſuch a ſtrange and unwarrantable Opinion I cannot imagine. If, as he has truly aſſerted, the ſpace on the Retina, ſo far as there is diſtinct Viſion, does not exceed an Angle of one or two Degrees, on every ſide of the Axe; which is as much as to ſay, it does not exceed an Angle of two or four degrees; and I am fully convinced, that it does not exceed one Degree; what difference can it make, whether that ſmall portion of the Retina be a Plane or a portion of a Sphere? he certainly could not ſuppoſe a large portion of the Retina to aſſume that Form.

Now, if the Retina does aſſume the form of a Plane, of the ſpace *a b*, then it is certain that the Images, *a f*, *f d*, and *d b*, on that Plane, (of which, the Right Line *a b* may be ſuppoſed a Section) are not in proportion to the Angles A E F, F E D, and D E B, equal *a E f*, *f E d*, and *d E b*, reſpectively.

Dem. For, the Tangent c f or c d is not in proportion to c a or c b as the Angle c E f is to c E a; becauſe, c f is equal f a (A F being equal F C) therefore, the Angle c E f is not equal f E a, or c E a double c E f. — — 3. 6. El.

But, *a b* is parallel to a b; therefore, *a f*, *f c* and *c b* are in the ſame Ratio, as a f, f c and c b. — — — — — Cor. to 6. 6. El.

Thus it is clear, if the Retina does take the form of a Plane, that Objects do not appear in proportion to the Angles they ſubtend; but, as it is the only inſtance in which I ever knew it conteſted, ſo, I am perſuaded, it will never be univerſally adopted, on that Hypotheſis. However, as it is merely conjectural, it ſeems to me more rational, to ſuppoſe the Retina to retain its original ſpherical Form, which the all-wiſe Creator has given it, than, that it ſhould, continually, at every motion of the Eye, be changing to that of a Plane.

Wherefore, on the preſumption that the internal Eye is truely ſpherical, it is very reaſonable to ſuppoſe, that, Objects appear, exactly, in proportion to the Angles under which they are ſeen; and not in proportion to the Tangent of half the Angle, as Mr. Hamilton aſſerts, and has endeavoured to demonſtrate, in 7 and 8 of the ſame Section: which cannot be as himſelf has ſtated it, but on the ſuppoſition of the Retina aſſuming the form of a Plane, which is abſurd to ſuppoſe. Or

Plate I. Fig. 1. granting it really ſo, the portion of the Retina, which the compaſs of an Angle of one or two Degrees, at moſt, takes up, is ſo very ſmall, that it is impoſſible to diſtinguiſh it, in a Sphere of an inch in Diameter, from a Plane. As the 360th part of the circumference of a Circle, of a ſmall radius, is not diſtinguiſhable from a Right Line; conſequently, there could be no difference in the apparent magnitude of an Object, in either Caſe, which is ſeen under ſo ſmall an Angle. As, the ſmall portion at d, on the Retina, ſubtending an Angle, at E, of about five Degrees, is not diſtinguiſhable from a Right Line.

3. It is alſo evident, if the Viſual Rays are refracted in paſſing through the aqueous Humour, before they enter the Pupil (unleſs that Refraction be rectified again at the Cryſtalline Humour or elſewhere, and, by that means, proceed in the ſame direction to the point of convergency, in the center of the Eye) that Objects cannot appear exactly in proportion to the Angles they ſubtend. Nor have I any conception how a Viſual Ray, AE, falling remote on the Cornea, can be refracted ſo as to paſs in the ſame direction through the Aperture, h i, to the point of convergency, at E, and from thence to the Retina.

For, ſuppoſe a Viſual Ray, A *e*, from any Point A (ſeen very oblique) falls perpendicularly on the Cornea, at *e*; if it paſs at all through the Pupil, h i, whilſt the Axe is directed to C, it muſt paſs in the direction *eg*; where it falls on the Cryſtalline, at *g*, and where, it is poſſible it may again be refracted, in the direction *g h*, to the Retina; making an equal Angle with the Axe, (E C) as the original Ray, A e; in which Caſe, the Image of the Point A is ſeen at *h*, in its proper poſition.

But, it is manifeſt, that, whilſt the Axe remains in the direction E C, the point A cannot be ſeen, at all, nor even the point F, diſtinctly; the intimation we have of Objects, at a leſs diſtance from C, is ſo very confuſed, that, without moving the Eye, we ſcarce perceive their exiſtence; of which, every Perſon's own experience will ſoon convince him. Unleſs the Axe be directed to any Point (as E) no perfect Image of that Point is formed in the Mind, or on the Retina. Conſequently, the Rays go direct through the various humours of the Eye; as when we look through a Pinhole; for, how can it be otherwiſe? ſeeing, it is manifeſt they muſt all paſs through the ſame Point; (q is then their common point of convergency) the Image formed on the Retina is, in every reſpect, the ſame (except in Hue, for want of ſufficient Light) and of the ſame Magnitude as it appears to the naked Eye. Which ſingle circumſtance explodes, in my opinion, the notion of Refraction at the Cornea and through the various Humours, in the paſſage of the Rays to the Retina. For it is manifeſt, that, if the Point of convergency be at q, without the Eye, the Image will occupy more ſpace on the Retina than at E, or any other Point within the Eye. I ſhall therefore ſuppoſe, that, when the Eye is naked, the Point of convergency of the Rays is in its Center; and, on that ſuppoſition, do infer, that Objects do appear in proportion to the Angles under which they are ſeen. Of which I will give an eaſy and familiar Example.

Fig. 3. 4. Let AB, CD, and FG be three Globes, at different Diſtances from the Eye, at E, and of different Magnitudes; ſeen under the Angles AEB, CED, and FEG reſpectively *.

Now, if the Diſtance of the Globe CD be equal to twice the Diſtance of AB, and FG be double of CD, four times the Diſtance of AB; then will the Angles AEB, CED, and FEG, which they ſubtend at the Eye, if the Diameters of the Globes are in the ſame proportion as their Diſtances, be equal; and conſequently, their apparent Magnitudes, to the Eye, at E, are equal.

Draw E S, to the Center of the Globe F G; draw C *D* perpendicular to E S, and equal to the diameter of the Globe C D; alſo, draw B *A* perpendicular to E S, and equal to the Diameter of the Globe A B.

* The full Diameter (FG) of a Globe cannot be ſeen; for, the Viſual Rays being Tangents to the Globe, from the ſame Point E, as E *f*, E *g*, are all equal (Cor. 2. 16. 3. El.) and, by reaſon of the convexity, towards the Eye, they cannot touch the two extremes of the ſame Diameter; unleſs the Diſtance be infinite; and then, the Viſual Rays are conſidered as being parallel; *fg* is, therefore, the apparent Diameter of the Globe FG.

DEM.

Dem. It is evident, that the Viſual Rays F E and G E, from each extreme of the Diameter of the Globe F G, will paſs through the extremes of C *D* and B *A*.

For, in the ſimilar Triangles F E G, *D* E C and B E *A*; F G : S E :: *D G* (equal C D) : K E, and, as B *A* (equal A B) : L E. — — 6. 6. El.

Then, ſince they touch the ſame Viſual Rays F E and G E ; conſequently, they are all ſeen under the ſame Angle, F E G. — — by Theorem

And, the Diameters, A B, C D and F G, being in proportion to their central Diſtances, are, therefore, all ſeen under equal Angles, A E B, C E D and F E G.

Conſequently, the Images, a b, b c, and c d, on the Retina, ſubtending equal Angles, are equal ; and therefore, the appearances of the three Globes A B, C D, and F G are equal. Q.E.D.

If the Globe C D be ſuppoſed to be moved to M, its central Diſtance, E M, being equal E S; and, being, in Diameter, equal half F G; it will ſubtend an Angle G E H, i. e. C E I, half C E D, equal F E G; as the Arks 1 2 3 4, which meaſure thoſe Angles, exemplify.

Conſequently, the Image, (h c) of the Globle G H, at that Diſtance, is, in magnitude, but one fourth part of the Image (c d) of the Globe C D, of the ſame Magnitude, and ſeen at half the Diſtance; for, they are in the duplicate ratio of the portions of the Ark 3 0 to 3 4. — — Cor. 1. 13. 6. El.

5. Yet it is manifeſt, that two or more unequal Objects, at an equal Diſtance from the Eye, and ſeen direct, do not appear exactly of the proportion of their reſpective Magnitudes; likewiſe, equal parts of the ſame Object, on the ſame ſide of that Point where it is cut by the Axe of the Eye, do not appear equal.

Let A B be an Object, of length ſimply, biſected by the Axe E C, to which it is perpendicular, in the Point C; and ſuppoſe A C and C B again biſected, in F and G. Fig. 1.

Then, the parts A F, F C, &c. are equal, and A B is double F G; but, they do not appear of that proportion, at E.

Dem. Now becauſe A F is equal to F C, the Angles A E F, F E C, under which they are ſeen, are unequal — — — — 3. 6. El.

For, if the Angle A E C be biſected, the Right Line E O, biſecting it, will cut A C, in the Point O, in the ratio of E A to E C.

But, E A is greater than E C; for, E C is perpendicular to A C.—Cor. to 12. 1. El.

Wherefore, A O is greater than O C, and the Angles A E O, O E C, under which they are ſeen, are equal; conſequently, the equal parts A F, F C, are ſeen under unequal Angles ; therefore, they appear unequal, by the Theorem.

And, becauſe the Angle C E F is greater than F E A, the equal part C F will appear greater than F A,

After the ſame manner it may be proved, that the equal part C G appears greater than G B. Conſequently, C G + C F, i. e. F G, appears greater than half A B. Q. E. D.

Cor. Hence, it is evident, that if A B be produced, and the equal diviſions, C G, G B, B P are continued, they will appear continually leſs, being ſeen under leſs Angles. For the Angle B E P is leſs than B E G, which, is leſs than G E C; as the meaſures, p o, o r, and r *C*, of thoſe Angles, ſufficiently evince.

N. B. The farther any two or more Objects are from the Eye, the nearer they will appear in the proportion of their reſpective Magnitudes.

Becauſe they are ſeen under leſs Angles; and, the Tangents of ſmall Angles deviate leſs from the ratio of the Angles, than the Tangents of greater Angles.

6. As Objects of different Magnitudes, at equal Diſtances, do not appear in the proportion of their reſpective Magnitudes, ſo neither do Objects of equal Magnitude, ſeen at different Diſtances, appear exactly in the ratio of their ſeveral Diſtances.

Let

Plate I. Fig. 4. Let A B be an Object at any Diſtance (E F) from the Eye, at E; and, let C D be another Object, parallel to A B, of equal Magnitude, as to length ſimply; alſo let C D be at twice the diſtance of A B (E G double E F). And let them be ſo ſituated, that, right lines, A C and B D, joining their extremes, are parallel to E G i. e. perpendicular to A B and C D.

By means of the Viſual Rays E A, E B, &c. thoſe Objects are ſeen under the Angles A E B and C E D. I ſay, the Angle A E B is not double of the Angle C E D.

Dem. Becauſe A B is parallel to C D, E F : E G :: E c : E C, as c F : C G. 6. 6. El.
But E F is equal to ½ E G (Hyp.) wherefore, c F = ½ C G, equal A F; that is, A F is biſected, in the point c.

Now becauſe A c is equal to c F, the Angle A E c is leſs than c E F 3. 6. El. for, if they were equal; A c would be to c F as E A to E F.

But, E F is perpend. to A B, therefore, E F is leſs than E A. Cor. 3. 12. 1. El.

But A c and c F appear (to the Eye at E) in proportion to the Angle under which they are ſeen; i. e. as A E c to c E F; (by Theo.) and c F appears equal to C G, being ſeen under the ſame Angle; conſequently C G appears greater than half A F.

By the ſame reaſoning, G D appears greater than half F B.

Therefore, C D appears greater than half A B; i. e. the Angle C E D is greater than half A E B; although C D is equal to A B, and the diſtance double.

Let I K be another Object, equal and parallel to A B. and at three times the Diſtance, from the Eye, at E.
I ſay, that I K will appear greater than one third of A B.

Produce E G to H, and I K to L, making K L equal to H K; and draw E K and E L. Then will A F be triſected, in k and l.

Dem. Becauſe A F is parallel H K, k F : K H :: E F : E H
And, becauſe K L is equal to K H, k l is equal to k F. } — 6. 6. El.

But, E F is equal to one third of E H; conſequently, F k and k l are, each, equal to one third of A F, equal K H.

Then, becauſe F k, k l, and l A are equal, the Angles F E k, k E l, and l E A are unequal; for, if the Angle F E k, was equal to k E l, F k would be to k l as E F to E l. — — — 3. 6. El.

But E F is perpendicular to A B; wherefore E F is leſs than E k; conſequently, F k would be leſs than k l.

And, for the ſame reaſon, if the Angles k E l and l E A were equal, k l would be leſs than l A; becauſe E k is leſs than E A.

But F k, k l, and l A are equal; therefore, the Angle F E k is greater than k E l, and k E l than l E A.

Therefore, F k, i. e. H K, appears greater than one third of A F.

By the ſame reaſoning, it is manifeſt, that H I appears greater than one third of F B, equal H I;

Therefore, I K, appears greater than one third of A B. Q. E. D.

Thus, I have made it appear, to Demonſtration, that Objects do not appear in that proportion to each other, as their reſpective Diſtances; in which I have been more particular, becauſe, Mr. Kirby has expreſſly ſaid, Page 13, that theſe Pictures (i. e. the appearances of Objects) will be to each other, as their ſeveral Diſtances are to each other*. Rohault ſays, nearly ſo; Page 243.

* The Appearances, i. e. the Angles under which Objects are ſeen, being in proportion to their Diſtances is, literally, abſurd; becauſe, as the diſtance of an Object is increaſed, the Angle, under which it is ſeen, decreaſes; which kind of reciprocal Proportion is always to be underſtood in this Caſe.

7. If the Eye be moved from E to O, making OF equal to FH, and OI, OK be drawn, EB will be cut in the ſame Points, c and d, as by the Viſual Rays EC, ED.

But, IK will not appear of the ſame proportion to AB, at the Point O, as CD, at E, the Objects AB, CD, and IK being equal.

DEM. Becauſe AB, CD, and IK are parallel, the Triangles EcF, ECG, alſo, OcF, and OKH are ſimilar; wherefore EF : EG :: cF : CG, and, OF : OH :: cF : KH. — — — 4. 6. El.

But, EF is equal to half EG, and OF is equal to half OH; conſequently, cF is equal to half CG, and alſo to half HK; equal CG.

By the ſame reaſoning, Fd may be proved equal to half GD or HI.

Therefore, AB is cut in the ſame Points c and d; as it was affirmed.

2. CD appears to the Eye, at E, in proportion to AB, as the Angle CED to AEB; and IK appears, at O, as the Angle IOK to AOB.

Now, the ratio of the Angle CEG to AEG is greater than KOH to AOH.

For the Angle AEG is greater than AOF, equal CEG; wherefore, the ratio of cEF to AEF is greater than cOF to AOF.

For, let Em be drawn parallel to OK; the Angle FEm is equal FOc conſequently, the ratio of FEm to FEc is equal to that, of FOc to FOA, equal FEc.

But Fc is equal to cA (proved) wherefore, Fm is equal to mc.

And, equal Parts of a Right Line ſubtend leſs Angles, the farther they are from a perpendicular to the Line, from the Eye; proved.

Therefore, the Angle FEm to FEc, i. e. FOK to FOA, has a leſs ratio than FEc to FEA, i. e. than GEC to GEA, being nearer equality.

Conſequently, HK appears leſs, in proportion to AF, at O, than CD at E.

After the ſame manner, HI may be proved to appear leſs, in proportion to FB, at O, than GD at E.

Therefore, IK appears leſs, in proportion to AB, at O, than CD at E. Q.E.D.

Hence it is manifeſt, that, notwithſtanding the ſections of the Viſual Rays EC and OK, ED and OI, with AB, are the ſame; the Diſtance of the Eye on one ſide of AB, and the Object on the other being proportional; yet, the appearances of the Objects, to each other, are varied, conſiderably, as the Diſtance is greater or leſs.

For, two Objects of equal height, at a great Diſtance from each other, and parallel between themſelves, and the Eye at the ſame Diſtance from one of them, and in a Right Line with both, will appear, in proportion to each other, nearly as their Diſtances; i. e. the fartheſt off will appear half the height and half the width of the neareſt, being at twice the Diſtance, and, one third in height and width at three times the Diſtance, nearly, i. e. ſomewhat more.

As IK appears, at O, ſomewhat more than half AB. But, CD (equal IK) appears much greater than half AB, at E. Conſequently, if the Eye be at a greater diſtance from AB, than at O, and the Object at an equal diſtance on the other ſide, they would appear nearer in proportion to their Diſtances, reciprocally; and therefore, when the diſtance of both is ſo great, that the Viſual Rays, OI, OK, approach nearly to paralleliſm (which, it is obvious muſt be infinite, before they can be perfectly ſo) they will appear nearly as their Diſtances. Notwithſtanding, on a Picture parallel to the Objects, they muſt ever be repreſented in the ratio of their reſpective Diſtances, and Magnitudes.

THEOREM II.

Parallel Right Lines, however ſituated, being produced, appear to approach towards each other; and, if produced infinitely, they will appear to meet in a Point, at an infinite Diſtance *.

Plate I. Fig. 5. Let I F and H G be two parallel Lines; and, E, an Eye, ſituated any how between them. Let K, B, D, and G repreſent various Diſtances in the Line H G, and let the Lines I K, A B, &c. be drawn perpendicular to H G, cuting the other Line (alſo perpendicularly) in the points I, A, C, and F. Alſo, let the Viſual Rays E A, E B, &c. be drawn.

Dem. The ſpace between the Lines, I F and K G, at the ſeveral Diſtances E S, E O, &c. appear, to the Eye at E, under the ſeveral Angles I E K, A E B, &c. each of which, as the Diſtance is farther from the Eye, is leſs. C. 14. 1. El. For, the Space between the parallel Lines is, every where, equal. Def. 7. Geo.

The Space, at I K, is ſeen under the Angle I E K; and the ſpace between them, at A B, under the Angle A E B; which is leſs than I E K, nearly in proportion to the Diſtances, E S to E O; C D ſubtends the Angle C E D, and F G the Angle F E G; each of which is leſs than the former; for their Subtenſes, A B, C D, &c. are equal; and the Lines E A, E B; E C, E D, &c. forming or containing the Angle, are continually longer; wherefore, the Angle F E G is leſs than C E D, which is leſs than A E B †, &c. — Cor. to 14. 1. El.

From which it is manifeſt, that at a greater Diſtance the Angle will be ſtill leſs; conſequently, it will, at laſt, become inſenſible, the interval, between the Lines, is loſt to Sight, and, the Lines appear to meet each other. Q. E. D.

Hence, the Lines, I F and H G, are ſaid to vaniſh; and conſequently, the ſpace between them vaniſhes alſo.

2. Let E R be drawn, from the Eye, parallel to I F and H G; thoſe Lines will appear to approach, continually, towards E R; and being produced, infinitely, they will all appear to meet in the ſame Point; however E R may be nearer to one than the other; for, the farther any parallel Line is from E R (which may be called its Radial) the more ſudden is its apparent approach to that Radial,

The Space I A, of the one, appears to advance from *h* to *i*; an equal Space K B of the other, advances from *p* to *o*; which is leſs than the other, in proportion to the Angle K E B to I E A; i. e. as the Ark *p o* to *h i*. The Space from A to C appears to advance from *i* to *k*; an equal Space, B D, in the other, from *o* to *n*. From C to F it advances from *k* to *l*, an equal Space, D G, from *n* to *m*; in the proportion of the Arks *k l* to *m n*.

It is evident, that if thoſe Lines were produced longer, they would ſtill appear to approach nearer to the point e, as E L, E M, evince. And, being infinitely produced, they would at laſt appear to terminate in the Point e, and be loſt to ſight.

For, it is manifeſt, that the farther any Points, L and M (conſidered as being in the parallel Lines I F and G H) are diſtant, the nearer the Viſual Rays, E L, E M, are to a coincidence with E R; which, it is evident, muſt be infinite before they can coincide perfectly; in which caſe, E L, E M, and E R will be the ſame; for they will all appear to unite.

* It is ſaid in Smith's Optics (Vol. I. Art. 156, Page 58) that parallel Lines, ſeen obliquely, appear to converge more and more as they are farther extended from the Eye; which, will ever be the caſe however the Eye is ſituated, in reſpect of the parallel Lines. For, being ſeen obliquely, means nothing more than when we look towards either of their extremes; as they are ſeen direct, only where a Line, from the Eye, cuts them both at Right Angles; on either ſide of the Perpendicular they are neceſſarily ſeen oblique.

† When the Angle, A E B or C E D, under which any Object, as A B, is ſeen, does not exceed 20 or 30 Degrees, at the moſt, and that Object is removed to the ſeveral Diſtances, from E O, to E P, E Q &c. the difference between the Tangents O A, O *C*, &c. deviates but little from the Arks O *C q*, O *C*, &c. and that ſtill leſs as the Angle is leſs; as P C, P *F z*, &c.

3. Hence

3. Hence, it is eaſy to account for the appearance of a horizontal Plane, or continued level Surface; which, being below the Eye, will gradually appear to aſcend; and, being above the Eye, it appears to deſcend; and, if they were produced infinitely, they would appear to meet in a Right Line, on a level with the Eye.

Let the Right Line H G be a ſection of a horizontal Plane, below the Eye, at E; and let I F be a ſection of one above the Eye. Alſo, let I K be a ſection of a Plane direct before the Eye* (at E) perpendicular to the other two.

Now if the Viſual Rays, E A, E B, E C, &c. be drawn, they muſt neceſſarily cut the Plane of which I K is a Section; in a, b, c, &c.

It is evident that the Space K B will appear to riſe, towards S, on the Plane I K, from K, its interſection, to b, where the Viſual Ray E B cuts that Plane; the ſpace K D will riſe to d, and K G to g.

For the ſame reaſon, the ſpace I A will appear to deſcend from I to a; I C, from I to c; and I F, from I to f; each Plane appearing gradually to approach towards S, where a Perpendicular, E S, to that Plane, cuts it.

And, if the Planes were produced infinitely, they would at laſt vaniſh, or be loſt to ſight, in a Right Line paſſing through the Point S, parallel to the interſection of the other two Planes, with the Plane I K.

And this is evidently the caſe however a Plane be ſituated; whether horizontal, vertical, or inclined, it is the ſame; for, a Plane is ſtill a Plane, in all poſitions, and has no properties peculiar to any poſition, in reſpect of the Horizon.

Therefore, there may be drawn this Concluſion, that every Plane, in which the Eye is not ſituate, will appear to approach towards, and at length to meet, another Plane, paſſing through the Eye, parallel to the former.

4. Let A B, C D, and F G repreſent three Objects of equal height, at the ſeveral Diſtances, E O, E P, and E Q, from the Eye, at E.

The Viſual Rays E A, E B, E C, &c. being drawn, thoſe Objects will appear in proportion to the Angles A E B, C E D, and F E G.

For, the portions of an ark of a Circle, whoſe Center is E, intercepted between the Viſual Rays, meaſure thoſe Angles, reſpectively, and determine the apparent proportions of the Objects A B, C D, and F G to each other; ſeeing that, the Angles which they ſubtend are in the ſame Ratio. — — 19. 6. El.

Wherefore, ſince, in Perſpective, the Repreſentation is always on a Plane, and not on a ſpherical Surface; ſuppoſe I K a ſection of a Plane, in a direct poſition; the Spaces, a b, c d, and f g, between the Points where the Viſual Rays cut and paſs through that Plane, conſidered as a Picture, are the proportions of the Repreſentations of the Objects, A B, C D, and F G; but their apparent Magnitudes are the portions *i o*, *k n*, and *l m* of the Ark *h e p*.

Let H G repreſent a ſection of the level Ground, or any other horizontal Plane; and A B, C D, and F G, Objects perpendicular thereto.

If I K be ſuppoſed a ſection of a vertical Plane, the foot or ſeat B, of the Object A B, will appear, on that Plane, at b; and conſequently, K b will repreſent the ſpace of Ground, K B, between the Picture and the Object A B; i. e. it will appear to riſe ſo high on the Picture; K d will repreſent the Space of K D, and K g of K G. So likewiſe, a, c, and f ſhew how much the tops of thoſe Objects appear to deſcend, on the Picture.

5. Now, if the poſition of the Picture be inclined to the Horizon, as *a* K, the Objects and the Eye remaining as before; the Repreſentations *a b*, *c d*, and *f g*, on that Plane, are very different from the proportions of a b, c d, &c. on the Plane I K.

* By being direct before the Eye, I would be underſtood to mean, in a vertical Poſition; and when a Right Line, from the Eye, falling in the middle, between its Bounds, cuts the Plane perpendicularly.

For, if the Eye be not in the Plane, or in a continuation of it, the Eye, being conſidered as a Point, can have but one poſition, either to a Plane or to a Right Line, being produced; however the Plane may be ſituated, in reſpect of the Horizon.

But,

Plate I. Fig. 5. But, the apparent proportions, of both, are the ſame, at E; *viz.* the portions on the Ark *b e p*, which meaſure the Angles A E B, C E D, and F E G.

Nor, is it poſſible to be otherwiſe; ſeeing that, the Objects, A B, C D, &c. remain the ſame, in reſpect of each other; and the ſituation of the Eye, is not varied; conſequently, the Angles A E B, C E D, &c. are not altered, and the Objects A B, C D, &c. the Subtenſes of thoſe Angles, muſt, neceſſarily, appear the ſame, repreſented on any Surface whatever, and in any Poſition, cuting the Viſual Rays E A, E B, &c.

N. B. The Proportions *C D*, and *F G*, &c. on A B, are the ſame, as c d and f g on I K (A B being parallel to I K); wherefore, the Triangles A E B, a E b, alſo C E D, *C E D*, and c E d, &c. are ſimilar; and therefore, as f g : c d, or to a b, :: *F G* : *C D*, or to A B. — — 4. 6. El.

Conſequently, Viſual Rays, being cut by parallel Planes, in any poſition, are not only themſelves cut in equal Ratio, but alſo the Sections, or the Projections on the Planes, by their ſections with the Rays. — — — — — — 1. 8. El.

And, their Proportions are as their Diſtances E S, E O, E P; i. e. as f g : E S :: *F G* : E O as *F 2 G 2* : E P, and, as F G : E Q; (6. 6. El.) and ſo of any other.

Hence, it is manifeſt, that the Sections of any ſyſtem of Rays, by parallel Planes, are ſimilar.

PROBLEM.

Fig. 6. A Right Line (A B) being obliquely ſituated in reſpect of the Eye, at E, to determine the Point D, to which if the Axe of the Eye be directed, the two Parts, A D and D B, ſhall be ſeen under equal Angles.

Without drawing the Viſual Rays, or biſecting the Angle made by the Rays.

Make A D to A B, as the diſtance of the Point A, from the Eye (E) is to the diſtance of the ſame Point A, added to the diſtance of B. — Pr. 32. El.
Then, A D : A B :: E A : E A + E B; and, A D : D B :: E A : E B.

Dem. The Viſual Rays E A, E D, E B being drawn, the Angle A E B is biſected by E D; wherefore, A D and D B are ſeen under equal Angles, at the Point E,
For, if the Optic Angle A E B be biſected by the Right Line E D;
then is A D : D B :: E A : E B. — — — 3. 6. El.

2. If the Axe of the Eye be directed to C, the middle Point of A B, then will the Point B be more diſtinctly ſeen; becauſe its Image falls nearer to the Image of C than the Point A. And, if the Ark *c f* be made equal *c a*, and E *f* be drawn, meeting A B produced, in F; A C will then be to C F as E A to E F; and, notwithſtanding the difference between A C and C F, their Images on the Retina, and conſequently their apparent Proportions, are equal, being ſeen under equal Angles.

Now, if thoſe Viſual Rays are cut by a Plane in the poſition I K; the part a c, the greater portion of a f, will repreſent A C, the leſſer Segment of A F; and c f, the leſſer portion, repreſents C F, the greater Segment;
For, ſince the Optic Angle, A E F, is biſected, by E C; a c : c f : : E a : E f; 3. 6.
alſo, as A C : C F :: E A : E F; conſequently, a c, c f repreſent A C and C F.

3. If the Right Line A B be produced, at the extreme A, cuting the Plane I K, at I; the whole indefinite repreſentation of that Line, on the Plane I K, is the ſection I K, of a Plane paſſing through the Line A B and the Eye, and terminated by a parallel Line from the Eye (E K) to that Plane.

For, becauſe E K is parallel to A B, they will appear to meet in a Point at an
† Theo. 2. infinite Diſtance; † which is repreſented by the Point K, on the Plane I K; for, ſeeing that the Line E K paſſes through the Eye, its whole appearance is but a Point; conſequently, the Point K repreſents the whole Line E K, produced infinitely; and A B, infinitely produced, will appear to meet E K in that Point.

If the Line AB be produced on the other Side of the Plane IK, and any Point (G) be aſſumed, between the Eye and the Plane; its repreſentation, on that Plane, is the Point g, in which the Right Line EG, produced, cuts KI produced; the part Ig, beyond the Interſection, I, repreſents IG, the portion of the Line, AB, lying on the ſide of the Plane, IK, towards the Eye; as Ia or Ib repreſents the part IA or IB, on the other Side; all which is ſo very obvious, from inſpection of the Diagram, it is needleſs to ſay more about it.

4. After the ſame manner as the apparent magnitudes of Objects are determined, ſo are their apparent Diſtances, or Bearings, in reſpect of each other; *viz.* by the Degrees on the Ark of a Circle, which meaſure the Angles they make at the Eye.

Let C, D, and F, be three Objects, and E the place of the Eye. Fig. 7.

Draw the Viſual Rays CE, DE, FE; then, the Angles DEC, CEF are the Optic Angles of their apparent Diſtances, or Bearings at the Station E.

Notwithſtanding their real Diſtances from each other, CF is nearly double that of CD, yet, the Angle CEF, is much leſs than the Angle DEC; as it is evident, from the portions, cf and cd, of the Ark df.

If *fd* be ſuppoſed the ſection of a Plane, the appearances, or places of thoſe Objects thereon, are at *f*, *c*, and *d*.

5. Suppoſe C, D, and F to be Stars, in the unbounded Expanſe of the Heavens, at an immenſe diſtance from each other and from the Eye, at E.

It is impoſſible to form an Idea of their real Diſtances or Situations, in reſpect of each other; for, if the Star C was at B, and F at G, or any where elſe in the direction EG, their apparent places are ſtill at c and f, in an imaginary Arch of the Heavens, as it appears to us, equally diſtant in every part.

Now, ſince the whole Diameter of the Earth's Orbit is not ſufficient to make any ſenſible difference in their Bearings, and, conſequently, of their apparent Places, in the Arch df, there cannot be any poſitive Idea formed of their real Diſtances; for the portions fc and cd, of the Arch fd, are the meaſures of their apparent Diſtances, only; i. e. of the Angles dEc, cEf, or the Originals of theſe Angles DEC, CEF under which they are ſeen.

Hence is conſtructed the Celeſtial Globe or Sphere, which is a true Picture of the Heavens, of Stars, &c. divided into various Figures called Conſtellations.

If dcf be ſuppoſed a portion of a Sphere; or, let HI be an entire Globe.

It is evident, that from its Center, E, which is alſo the Center of the Arch df (E being ſuppoſed the Eye of a Spectator) the Star C will appear on the Globe at o, and D at s, &c. in the ſame poſition, ſituation, and diſtance, in proportion to the Radius, EH or Eo, to Ec, as in the imaginary Arch of the Heavens, df. So that, whether the real Star be at A, B, or C, or any where elſe in the Right Line EC; its apparent or repreſentative Place, on the Globe HI, will ſtill be at o; and conſequently, can make no difference, in its true Place thereon; but its apparent Magnitude will be in proportion to its Diſtance, nearly.

6. Let AB, DF, and GH be three Objects, promiſcuouſly ſituated in reſpect of the Eye, at E. It is manifeſt, that, whilſt the Eye remains at E, the three Objects muſt neceſſarily appear the ſame, upon any Surface whatever, cuting the Viſual Rays EA, EB, &c. Fig. 8.

Suppoſe aha ſection of the Rays, made by a Plane, and ECa Perpendicular from the Eye to the Plane, cuting it at C. Then, the Viſual Rays EA, EB, which are neareſt to the Point C, cut that Plane leſs oblique than EF; and EG, EH are ſtill more oblique, as they are more remote from C. Whereas, on a ſpherical Surface of which a *dfh* may be ſuppoſed a Section, being an ark of a Circle, deſcribed on the center E, the Rays EA, EF, EH, &c. are all perpen-

Plate I. Fig. 8. dicular to its Surface, being perpendicular to the Ark C *f h*. Wherefore, ſince E a, E *d*, E *g*, &c. are equal, for, they are all Radii; conſequently, the ſurface of a Sphere is equidiſtant from its Center, in every part; and, the Repreſentations, a b, *d f*, and *g h*, of the Objects AB, DF and GH, on its Surface, are alſo their true Appearances, from the Point of View E.

But, on the Plane, or its Section, a h, the Object A B, has its Repreſentation, a b, nearly equal to a b on the Curve; whereas, d f, it is obvious and demonſtrable, is larger than *d f*; and g h ſtill larger than *g h*, as it is more remote from the Point C; nevertheleſs, its true appearance is *g h* on the Curve, or ſpherical Surface; for, they both repreſent the ſame Object, from the ſame Point of View, which cannot vary in its Appearance.

Therefore, the Repreſentations of Objects, on various Surfaces, and in various Situations, although they may differ greatly in Figure, will, if truly repreſented, appear the ſame in the true Point of View.

The three Objects A B, D F and G H, though of various dimenſions, appear equal; not owing to their Diſtances, but to their Poſitions in reſpect of the Eye, being ſeen under equal Angles, A E B, D E F and G E H; as the portions *a b*, *d f*, and *g h*, of the Ark C *f h*, indicate. Whereas, their Repreſentations on the Plane, of which a h is ſuppoſed a Section, are unequal; for, a b, the repreſentation of A B, being neareſt to the Center (C) is the leaſt; d f is greater than a b, and g h than d f.

Yet, it is manifeſt that the Repreſentations, a b, d f, and g h, have the ſame appearance, to the Eye at E, as their correſponding Images, a b, *d f* and *g h*, on the ſpherical Surface. For, the Eye being in the true point of View (at E) they are both ſeen by the ſame Viſual Rays as the original Objects themſelves; and conſequently, under the ſame Optic Angles. Wherefore, their Appearances are the ſame on either Surface; by Theorem 1ſt.

From what has been advanced in this Section, it is evident that there is a manifeſt difference betwen the Repreſentation of an Object, on a Plane, and its true Appearance; which difference is the greater, as the Eye is nearer to the Object. For, ſince the Viſual Rays muſt neceſſarily cut any Plane, on which they fall, more and more oblique, the farther their Interſections are from that Point, in which a perpendicular Line from the Eye would cut the Plane; conſequently, the Repreſentations of Objects, on a Plane, cannot be in proportion to their true Appearances, but deviate continually, more and more, as they fall farther from the Point deſcribed. Whereas, on a ſpherical Surface, which is every where equally diſtant from its Center, every Right Line, and in every direction, which tends to its Center, is perpendicular to its Surface, and none of them can be ſaid to cut it oblique; conſequently, the true Repreſentations, and alſo the true Appearances of Objects, can only be depictured on the Surface of a Sphere; that is, the Repreſentation on its Surface and the true Appearance of the Object, from the Center, as the Point of View, are the ſame.

SECTION IV.

Containing ſome objections to the received Opinion of the Cauſe of VISION.

IN the laſt Section I have explained, demonſtrated, and briefly illuſtrated the whole Theory of Direct Viſion; or ſo much, at leaſt, as is eſſential, to give a clear and juſt Idea of Perſpective. To dwell longer on it, in this place, is unneceſſary; ſeeing that, Perſpective itſelf, in Theory, is nothing more than a continuation of the ſame Science. But, before I enter upon the ſubject of Perſpective, I ſhall, in this Section, give my objections to the received Opinion of the cauſe of Viſion

It

It is not an eaſy matter to remove Prejudices, which are deep rooted; eſpecially if they are ſtrengthened with the ſanction of Men of known abilities. I am at a loſs to conceive why it is, that the generality of mankind ſo willingly impoſe on their own judgments, or are ſo ready to be impoſed on by others; eager of being amuſed with deep and profound Diſcourſes, pretending to penetrate into the myſteries of Nature; which, neither themſelves nor their Authors know any thing of. In fact, we ſee and know nothing further than the ſurfaces of Bodies; the formation of them, or their conſtituent parts, we are, and ever ſhall be, totally ignorant of. We daily ſee the effects of one Body acting on another, and, by repeated Experiments, we find their full force; that they are regularly and uniformly the ſame; but, the true cauſes of thoſe effects are to us unknown; we cannot form an adequate Idea, how, and in what manner they act, and produce ſuch wondrous effects, although ſo ſimple in their Nature.

We have not the leaſt Idea how a Solid can become a Fluid and a Fluid become again a Solid, by different degrees of Heat; what Fire is we know not, nor how it acts on Bodies, ſo as to diſſolve ſome, to change the nature of others, and, in a great meaſure to annihilate them. We know as little of the Element of the Air we breathe in; or, by what means it is often ſo violently agitated in one place (as in high Winds) whilſt it remains at reſt, or has a contrary motion in another. We know not how a Solid is diſſolved by a Fluid, and the Solid become perfectly liquidated; how two different Fluids mix and incorporate, ſo as to be beyond the power or art of Man to ſeparate again, if they have an equal ſpecific Gravity; nay two Solids, as Metals, &c. may be ſo incorporated as to be quite another kind of Body; and the conſtituent parts, of each, beyond the power of Art to diſſever, or to diſtinguiſh them apart. Some again may be ſeparated, if they have not the ſame ſpecific gravity, as Gold or Silver, from any other baſer Metal. Some Fluids can never incorporate; as Oil or Spirits with Water, &c.

How a Fluid is compoſed, or what form its minuteſt particles are of, we are totally ignorant. Is it not ridiculous to ſuppoſe Particles, of any Figure? It is the common Opinion that they are globular; becauſe, the leaſt drop of Water or other Fluid aſſumes that form, than which, nothing is more abſurd; becauſe, perfect Globes may be piled into a Pyramid, as firm and permanent, almoſt, as Cubes, with very little lateral preſſure; conſequently, a Fluid might be ſuſpended in the ſame Figure, if its conſtituent Particles were ſolid Globes. For, on the ſuppoſition of Particles, they muſt neceſſarily be ſolid, or each Particle muſt, again, be compoſed of Particles.

To compare a Fluid with fine Sand is truely ridiculous. Sand will ſupport itſelf in a pyramidal form; and, if the grains were perfectly globular, it would always aſſume one, of the figure of its Baſe, terminating in a ſingle grain at the Vertex. It is not ſo with Fluids; a Fluid cannot reſt till it recovers an equilibrium in every part, preſently acquiring a perfectly level Surface. Beſides, if its Particles were globular, there muſt be interſtices between them filled with Air, which cannot poſſibly be incorporated with Water, as, in that caſe, it would be, to all ſenſe. Alſo, if Metals, &c. when in a fluid ſtate, by means of Fire, were compoſed of globular Particles, it is reaſonable to ſuppoſe they would remain ſo when fixed; conſequently, they would be extremely porous, and admit Air to paſs freely through, which is repugnant to common experience. No, ſay thoſe acute reaſoners, it is by reaſon of the Particles changing their Figure that they become fixed. Amazing truely! that the leaſt particles of Matter, which I ſhould ſuppoſe (on their Hypotheſis) indiviſible, conſequently impenetrable and immutable, by any means, can be varied in their Shape or Figure. It is alſo amazing, that, by being globular, they are ſo eaſily ſeparated, when in a Fluid ſtate, and, when fixed, are ſo extremely difficult to diſſever.

If the Particles of ſolid Bodies are kept together only by coheſion, why does not the ſame power act on them in their fluid ſtate? or why, when the particles of a ſolid

ſolid Body are, by any means, ſeparated, do they not cohere together again? or, what reaſon can be given, why they ſhould cohere more ſtrongly one way than another; as in Wood of all kinds? It is downright Sophiſtry, all; a language without meaning. That Bodies may be ſeparated into Parts or Particles is undoubted; but, that they are compoſed of Particles, is what I cannot give them credit for. Every Body, whether Solid or Fluid, before any of its parts are ſeparated, is one entire Maſs of Matter.

Again, if Fluids are compoſed of Particles, and allowing them to be tranſparent in their nature, their globular form would render them, in ſome degree, opake; ſo that, a Fluid would not admit of diſtinct Viſion through it. Now, I am clearly of opinion that all the parts of a Fluid lie perfectly cloſe to each other, without any Cavities interſperſed; it is impoſſible there can be any; conſequently, there is not the leaſt particle of Air contained in Fluids. If any diſſoluble Body be immerſed, as Sugar, &c. in Water; does not the Air, which is contained in its cavities, immediately aſcend to the Surface, as ſoon as it is at liberty, by the diſſolution of the immerſed Body?

The cauſe of Tranſparency, either in Fluids, or, more particularly, in ſolid Bodies, is, to us, an impenetrable myſtery. It is aſtoniſhing that there ſhould be diſtinct and perfect Viſion through Glaſs, or pellucid Stones; and, with all the knowledge and penetration human wiſdom can pretend to, we cannot form an Idea tending to comprehend it. Yet, this aſpiring, preſumptuous, all-ſufficient Man, would pretend to account for the cauſe of Viſion, itſelf, and for all the other Senſes; nay (if we will take their Words for it) for every operation in Nature. But I ſhall make free to tell them, plainly, that they cannot account for one. Firſt, let them account for their own Exiſtence; can they tell where the firſt ſpring of ſelf-motion begins? no, not ſo much as for the motion of a Finger.

As for the generality of Mankind, they neither do nor are capable of judging for themſelves, in many caſes; and only give their aſſent, or diſſent as they are influenced, or depending on the judgment of others, whom they either know or believe to have Judgment. That this is the caſe, in general, is certain; and is the reaſon why the opinions of ſome great Men are, though ever ſo arbitrary, adopted as orthodox; becauſe, there are but few who dare venture to contradict them, and think or judge for themſelves. And, becauſe thoſe Perſons have advanced ſome received and eſtabliſhed truths, theſe therefore conclude, that all which they preſume to advance muſt neceſſarily be ſo.

Thoſe profound Gentlemen knowing their own importance, and the deference which the World pays to their Opinions, are but too apt to be intoxicated with it, and often take too much Conſequence to themſelves; imagining, that they are really endowed with a penetration, into the myſteries of Nature, beyond the reſt of Mankind. Or, from the knowledge they have of Mankind, the great deference they pay to their own ſuperiority over them, and the implicit aſſent which is given to all their productions, make them forget, that they, as well as the reſt, are fallible.

Reſpecting Viſion. It is ſaid in Smith's Optices, Page the firſt, and that from. the Authority of Sir Iſaac Newton, " that we ſhall find it difficult to conceive, " Light to be any thing elſe but very ſmall and diſtinct particles of Matter, inceſ- " ſantly thrown out from ſhining Subſtances; and every way diſperſed by Reflec- " tion from all others." I freely own, that I am at a loſs to conceive how it can be Matter, and that for ſeveral reaſons which may be given.

Firſt; it is an indiſputable Maxim, univerſally allowed by all Men of common underſtandings, that two Bodies cannot occupy or fill the ſame Space at the ſame Time; even Air cannot be where there is already any other Body. Steam, or Water expanded, almoſt, if not totally, excludes Air; conſequently, no other Body can occupy the ſame Space where there is Air. Now, ſince the whole of Space, within our Atmoſphere, at leaſt, is filled with Air, how can it be alſo filled with Light, if Light be, alſo, a Body, compoſed of Matter? for, ſince it is manifeſt, to ocular

conviction, that, in full Day Light, there is no part of the Space around us but what is wholly illumined; conſequently, being wholly, and in every part, filled with Light, if it be a Body, Air muſt be totally excluded; agreeable to the general Maxim, that two Bodies cannot fill the ſame Space. But, I preſume, we do not find that to be the Caſe; for, the Space is filled with Air, and cannot poſſibly be excluded by Light.

I expect it will be alledged, that Air, being a Fluid, is compoſed of globular Particles; and that, Light fills up the interſtices between them. Of what ſhape and magnitude, then, are the particles of Light, compared with the particles of Air? If Light be an exiſting Medium it muſt alſo be a Fluid; conſequently, its Particles are alſo globular. And, if it be ſuppoſed to proceed from the Luminary, progreſſively, in Right Lines, in all directions, I own I am at a loſs to conceive how that can poſſibly be effected, through innumerable globular particles of Air; to ſay nothing of the inconceivable velocity of its motion, being ſuppoſed to proceed from the Sun to the Earth in ſeven minutes and a half. For which extremely nice calculation we are indepted to Roemer; who firſt obſerved, that the Eclipſes of the Satellites of Jupiter happen, when in Oppoſition, that is, when the Earth is in a Right Line between the Sun and Jupiter, fifteen minutes ſooner than when in Conjunction, or the Sun in a Right Line between the Earth and Jupiter *. The difference of the diſtance of Jupiter from the Earth, at theſe two periods, is the whole Diameter of the Earth's Orbit; computed at more than 160 millions of Miles; half of which is the Diſtance of the Sun from the Earth. From which curious obſervation it is conjectured, that, Light is projected, from the Sun to the Earth, in ſeven or eight minutes; that is, the Sun being ſuppoſed to be continually emiting Light, it fills a concave Sphere, equal in Diameter to the Earth's Orbit, in that Time. Quere; how is the Sun ſupplied? and, being Matter, what becomes of it?

Now, admiting this extraordinay obſervation to be truely calculated, it undoubtedly proves that Viſion is not inſtantaneous, at any diſtance; which ſeems reaſonable; but I cannot ſee that it proves the progreſſion of Matter from Jupiter to us. Beſides, the Light which proceeds from Jupiter, or his Satellites, is but reflected, at ſecond Hand; perhaps, Light, coming immediately from the Sun, may be much ſwifter in its motion. I am almoſt of opinion, that if it were poſſible to ſcreen the Sun from our ſight, by the interpoſition of an opake Body, cloſe to its Surface, being removed in an inſtant, that we ſhould ſee the Luminary inſtantaneouſly; and, the whole Space between be inſtantly illumined.

The Law of reflected Bodies, that is, the velocity with which an elaſtic Body rebounds, according to the force by which it is firſt impelled, I am not acquainted with. But, of this I am well convinced, that the velocity of any impelled Body is not uniformly regular, but is continually varying, and the Body, in motion, tending to reſt; conſequently, it cannot acquire an additional force by ſtriking againſt another Body, at reſt, ſo as to rebound with the ſame velocity by which it was at firſt impelled; and, if Light, as a Body, acts on the ſame Principles, it muſt come, immediately from the Sun to us, with a greater velocity than when reflected from any other Body.

The Diſtance of Jupiter from the Sun is computed at 424 millions of Miles; and the Earth but 81 millions; which, is not one fifth part of the Diſtance of Jupiter. Conſequently, the velocity, with which Light is firſt impelled, muſt be greatly diminiſhed before it reaches Jupiter, and more ſo in being reflected back from Jupiter to the Earth. I ſhall endeavour to illuſtrate what I advance.

* It very rarely, if ever, happens, that they are in a Right Line, in a ſtrict ſenſe; becauſe, the Plane of the Orbit of Jupiter is inclined to the Ecliptic, that is, to the Plane of the Earth's Orbit, in an Angle of one Degree and 20 Minutes. But, the Line of the Nodes, that is, the Line of interſection of the Planes of the two Orbits, is ſuppoſed to have a revolution (as the Moon's Orbit in 19 Years) but in what time is not, I believe, as yet determined. Conſequently, they can never be perfectly in a Right Line, but only when the Line of the Nodes paſſes through the Centers of all the three; viz. the Sun, the Earth, and Jupiter. Wherefore, at a Conjunction, Jupiter will appear either above or below the Sun, according as he is in the upper or lower part of this Orbit; and, at an Oppoſition, the Earth will be either above or below a Right Line paſſing through the Centers of the Sun and Jupiter.

Plate II. Fig. 9. Let S repreſent the Sun, in the center of our Syſtem, of Planets, &c. and C D E F the Earth's Orbit, whoſe ſemidiameter, S E, is 81,000,000, of Miles.

Let A B be a portion of the Orbit of Jupiter, and J the place of Jupiter therein; whoſe Diſtance, S J, is above five times S E or S C. Let a b be the Orbit of one of Jupiter's Satellites, in which the Satellite is ſuppoſed to be, at a, emerging from behind Jupiter, after an Eclipſe.

2. It is affirmed that we ſee Objects, only by means of material Rays of Light, reflected from them to the Eye; which impreſs on that Organ the perception of Viſion.

Suppoſe the Satellite, at c, to be in a direct Line behind Jupiter; it is plain that it could not be ſeen at C or E, becauſe it would be hid by the Body of Jupiter; but, as ſoon as it emerges, that is, as ſoon as it can appear, as at a, in the Right Line C a or E a, it becomes viſible; not, merely, becauſe there is no other Body between, but becauſe it becomes illumined from the Sun, in the Right Line S a; which, whilſt behind Jupiter, from a to b was deprived of Light.

Let S a repreſent a Ray of Light from the Sun to the Satellite at a; the Earth, at C, being directly between. The ſame individual particles of Light, which paſs by the Sun to Jupiter, paſs by the Earth to Jupiter, and back again, before either Jupiter or the Satellite can be ſeen; according to their Hypotheſis.

It is reaſonable to ſuppoſe, that the velocity with which thoſe particles of Matter firſt ſet off, from the Sun, is greater than when they arrive at the Earth; how much, then, muſt it be decreaſed before they reach Jupiter? Which Opinion, Rohault (Chap. 27, Art. 34,) endeavours, to confirm, by a compariſon with a conical Veſſel filled with Water, being raiſed, at its Surface, by ejecting in more Water at the Vertex. The compariſon has indeed ſome affinity, though a very groſs one, becauſe Light is ſuppoſed to expand as it ſpreads; by which means, its direct motion may be equible.

Suppoſe Light loſes one fourth of its velocity before it reaches Jupiter, it can have loſt but one twentieth part at the Earth; and it is reaſonable to ſuppoſe, that the force with which it returns from Jupiter cannot be equal to what it came with. Suppoſe then it has loſt a fourth part more of its force, before it reaches the Earth again, at C; which, conſidering how languid its Light is, compared with the Light of the Sun, is very moderate. According then to this ſuppoſition, Light comes from Jupiter to the Earth, with little more than half the velocity with which it comes from the Sun, directly.

C repreſents the place of the Earth, at an Oppoſition and E at the Conjunction; the difference in the Diſtance of thoſe two ſituations, from Jupiter to the Earth, is C E, the Diameter of the Earth's Orbit.

Now the motion of a Particle of Light (for a Ray, or continued Right Line of Light, I have no conception of) in its paſſage from the Sun to the Earth, is, I preſume, at is firſt and greateſt velocity, always the ſame; but, at J, it has loſt one fourth of its force; and, from J to C again, it has loſt a fourth more; and conſequently, in its paſſage from C to E, which is nearly half J C, it muſt neceſſarily have loſt one eighth of the velocity it had, at its return to C; at which time the calculation is made.

Wherefore, the motion of Light, from C to E, has, at moſt, but half the velocity it has from S, to C or E, directly; and, it paſſes, according to Roemer's calculation, from C to E in 15 Minutes; conſequently, it paſſes from S to E (with double velocity) in a fourth part of the time, that is in 3 minutes and 3 quarters.

But, if the velocity, to and again, be regular and uniform (which I believe is contrary to the laws of motion, of Bodies impelled and reflected again) being 15 minutes in paſſing from C to E, half of which, S E, muſt conſequently take 7 minutes and a half. For a Satellite, emerging at a, when the Earth is at C, appears 7 minutes and a half ſooner, and at E 7 minutes and a half later, than at D or F, where the diſtance of Jupiter from the Sun and the Earth is equal, according to the calculations made by the ableſt Men. But ſince it is not poſſible, or, at

leaſt, probable that the velocity of Light can be uniform and equal, at all Diſtances from the Sun (if it has any motion at all) the greateſt velocity muſt be at its firſt emiſſion from the Luminary; and conſequently, it takes leſs time in paſſing from the Sun to the Earth than in its return from Jupiter.

3. It is certain that no opake Body, I mean, ſuch as are not luminous, can be ſeen at a great Diſtance; unleſs its magnitude be ſufficient to obſtruct a great quantity of Light, when oppoſed to a luminous Body; or, when it is ſo ſituated, in reſpect of a luminous Body, that the Light, it receives from the Luminary, is reflected again*; of which, the various appearances of the Moon is ſufficient proof. Conſequently we could not diſcern either Jupiter or his Satellites (whoſe immenſe Diſtance is more than ſeventeen hundred times the Moon's Diſtance) if they were not illumined by the Sun.

Jupiter being a ſuperior Planet, that is, he moves in an Orbit beyond, yet concentric with the Earth's, he is, conſequently, always illumined on that Face towards the Earth; and conſequently (being of immenſe magnitude) may always be ſeen, when above the Horizon, by the naked Eye; except when in near vicinity with the Sun. But the Satellites are ſecondary Planets or Moons (of which Jupiter has four) accompanying the primary ones, as the Moon does the Earth; and are ſo very ſmall, in compariſon of their primaries, that they cannot be ſeen at that diſtance without a good Teleſcope.

Now, if thoſe Satellites cannot be ſeen, after they emerge from behind Jupiter, until the Light (which is always ready, at hand, to illumine them) is reflected, back again, from the Satellite to us; and, ſuppoſe this Light to be a material Body, I am firmly perſuaded that they would never be ſeen at all, by us; ſuch an immenſe Space, from a to E, above 500 millions of Miles, for Matter to be projected in about 45 minutes, is beyond the power of my reaſoning faculties to find credit for (with God nothing is impoſſible.) Beſides, they not only reflect the Light directly, but alſo obliquely, in all Directions, filling a Hemiſphere; which, I ſhould ſuppoſe, is too groſs an improbability (being Matter) to gain credit with a thinking rational Being; let thoſe, who can, find belief and give credit for it.

The whole of this Phænomenon I ſuppoſe is this; (for I have never ſeen the Experiment; as it muſt require extraordinary Teleſcopes, which magnify to a great degree, to diſcern the emerſion diſtinctly) either it is impoſſible to perfect Teleſcopes ſufficient to diſcern the emerſion, till after the Satellite is conſiderably advanced from behind Jupiter; and alſo, from the near vicinity of the Sun, when in Conjunction with the Earth, at E, whoſe ſuperior ſplendor muſt hinder the Sattellite from being ſeen, for ſome time; together with the ſo much greater Diſtance, than when at C; which, I know from my own obſervation, renders them ſcarcely viſible, with a common reflector; all which, may concur to render it inviſible for 15 minutes, after it might be viſible from C. For I make not the leaſt doubt, that their motions and revolutions are as regular and equal as the motion of the Earth itſelf; which, to us, is the only ſtandard meaſure of Time.

4. If we ſee Objects, only by means of material Rays of Light paſſing from the Object to the Eye; by what means are opake Objects, which are immerſed in Shade, ſeen at all? as they do not receive Light, immediately, from any luminous Body; nor, perhaps, from any other, oppoſed to them, by reflection. But, with ſuch reaſoners, who can give what properties they pleaſe to Matter, there is no arguing; ſeeing that, they can make Light reflect and rebound from one Object to another, at pleaſure. But, are not thoſe ideal properties of Light of their own creating, entirely? That Light is reflected from one opake Object to another is beyond a doubt; but, that real Matter is projected, to and again, in every direction, I cannot acquieſce in.

* By Light being reflected, I would not be underſtood to mean, that there is any kind of real Matter projected from the reflecting Body; but only, by being illumined, itſelf, it becomes, in ſome degree, luminous, ſo as to ſhine with its borrowed luſtre, and illumine others.

Let

Plate II. Fig. 10. Let A B repreſent, what is called, a Ray of Light, from ſome luminous Object, falling on any Surface, as CD.

I ſhall ſuppoſe this Ray of Light to be a phyſical Right Line, the leaſt in its dimenſions of breadth and thickneſs, that can be conceived; conſequently, this phyſical Ray of Light can ſtrike any Surface, on which it falls, in a phyſical Point only, the ſmalleſt that can be conceived.

Now by the laws of Reflection, according to all the writers on Optics, whenever a Ray of Light ſtrikes or impinges on any Body, it is reflected again from that Body, making an equal Angle with a Right Line (B F) perpendicular to the Surface, at the Point in which it ſtrikes the Surface *. Or, if it be a Plane Surface, it makes equal Angles with the Plane (A B I = E B K) both, the Original Ray, A B, which is called the incident Ray, and the reflected Ray, B E, being in a Plane, which is vertical to the other Plane.

It is reaſonable to conclude, if this Ray of Light, A B, be material, and it is by means of material Rays ſtriking the organs of Sight, that Viſion is performed, or the ſenſe of Seeing is effected, that, the Point B could only be ſeen by an Eye placed, any where, in the direction of B E, as at E; B E being conſidered as the ſame individual Ray, A B, reflected or broken at the Surface, in the Point B†. It is certain, that if the ſurface C D be Water, or a poliſhed Mirrour, the Image of any Object, at A, will be ſeen at B, by the Eye at E, or ſome where in the direction B E, only.

But experience convinces us, that the Point B, or any other Point in the Surface C D, may be ſeen as perfectly in a thouſand other Directions; as at F, G, H, &c. Conſequently, if Eyes were placed all around, within the compaſs of a Hemiſphere, of which the Plane C D may be conſidered as the Baſe, or being elevated but a few Degrees above the Plane C D, the Point B may be ſeen by them all, at the ſame moment of Time.

Now, ſince the Point B can receive but one Ray of Light from the luminous Body, how can that identical Ray be reflected in ten thouſand Directions, in all Directions above the Plane C D, and thoſe Rays to be material? at the ſame Time, the whole Space is filled with Air in every part. But, the ſame Eyes can alſo ſee every other phyſical Point in the whole Surface C D, as C, D, &c. conſequently, every Point in the Surface muſt alſo reflect Rays in all Directions, croſſing and cuting the former in every Point; which, unleſs, not only two, but, an infinite number of Bodies may occupy the ſame Space, and at the ſame Time, cannot poſſibly be, on the ſuppoſition that Light is a material Body.

5. Again. Since, as I have obſerved before, it is aſſerted that Viſion is performed by means of thoſe material Rays of Light, which enter the Aperture of the Eye, and, impinging on the Retina, affects the Optic Nerves with the perception

* It is advanced by Sir Iſaac Newton, (Prop. 8, Part 3. of the Second Book of his Optics) that Light is reflected from Bodies before it impinges on their Surfaces. But I muſt own, that in all the ſeven reaſons he gives for that Opinion, I cannot find one of weight, nor all of them together, ſufficient to prove it. Which, Smith, from that authority, and from Sir Iſaac's firſt and fourth Quere, endeavours to prove, is not reflected immediately in an Angle, but makes a regular parabolic kind of curve, at a very ſmall diſtance from the Body, which he calls the ſpace of activity; and which ſpace, he ſays, is ſo extremely ſmall, that, conſequently, in phyſical Experiments, the incurvation of a Ray of Light may be conſidered as performed in a phyſical Point. Emerſon ſays (in a Scholium to Prop. 1. of his Catoptrics) that the Curve, deſcribed by a Ray of Light, is ſo extremely ſmall, that it may be looked upon as a ſingle Point.

It is, to me, aſtoniſhing, how any Perſon dare preſume to advance ſuch Opinions! and, for what purpoſe they do advance them, except it be to make the World have a high veneration for their extraordinary ſagacity and penetration. Can they, either by experiment or otherwiſe, prove the truth of their Hypotheſis? or do they ſuppoſe mankind ſo credulous as to give them credit for it, from the ridiculous experiment of a Hair, or of the Knives, and fringes of Colours produced by them? And yet it is certain, that ſome (not to be behind them in penetration) either do or pretend to do; for I have heard the ſame thing advanced, verbally, by a Perſon who has a tolerable ſhare of mathematical knowledge.

† See Fig. 10. No. 2. A B is ſuppoſed to be an incident Ray, and C D the reflected Ray; making a Curve, B E C; before it touches the Surface G H; which Curve is performed in ſo ſmall a Space, that the Points, B and C, are ſuppoſed to coincide. Quere; how is the Curve to be aſcertained, and determined? or, how ſhall we have conviction that it does not touch the Surface, G H?

of the Objects from which they flow, how ſhall we account for Viſion through Glaſs, or much harder diaphanous Subſtances, as Cryſtal, or Adamant? which is impenetrable, by any tool made of the hardeſt Steel.

Let any object, as A B, be ſeen, by an Eye at E, by means of the Viſual Rays A E, B E, &c. and, let any tranſparent uncoloured Stone, as Cryſtal, &c. be interpoſed directly between the Eye and the Object, as C D, of any thickneſs; having its oppoſite Surfaces parallel Planes, and well poliſhed. The Object, A B, will be ſeen through it inſtantaneouſly, as repreſented at a b, with every circumſtance of Colour, &c.

Now, I ſhould be glad to know, how or by what means thoſe Rays of Light, tranſmited or reflected from the Object (A B) to the Eye (at E) paſs in an inſtant through this ſolid Body of Cryſtal or Adamant, if they are material? and firſt, I ſhould be glad to know, what is the cauſe of Tranſparency? or, in what the difference conſiſts, between tranſparent and opake Bodies?

The reaſons given by Sir Iſaac Newton, in the third part of the ſecond Book, and which, Smith, in his Optics (Ch. 8, P. 95) has very wiſely copied, word for word, concerning the cauſe of Tranſparency, Opacity, and Colours, in Bodies, is but little to the purpoſe; it does not convey, to me, the leaſt Idea of the cauſe of Tranſparency, and how Viſion is conveyed through tranſparent Mediums, in every direction, inſtantaneouſly.

The learned and reverend Dr. Harris, in his Lexicon, tells us (under the article Tranſparency) that, a diaphanous or tranſparent Body is one which, probably, has its Pores all Right or Direct, and nearly perpendicular to its Surface.

By the Pores being all right or direct is meant, I preſume, that they go directly through, in Right Lines, without any obſtruction; that they are perpendicular, or nearly ſo, to the Surface, I muſt needs ſay, he was extremely ſagacious who firſt found out that extraordinary quality. But how ſhall we interpret the Word, probably? is it not indeed a tacit acknowledgement, that it is all nothing more than Conjecture? I cannot ſay that it is even a probable Conjecture.

I am afraid that the Doctor was not very happy in his memory; for, in another place he tells us (under the article Diaphaneity) that the Pores of a diaphanous Body are ſo ranged and diſpoſed, that the Beams of Light can paſs freely through them, every way; which plainly contradicts the former aſſertion, that they are nearly perpendicular to its Surface; and which, in reality, is ſaying nothing, becauſe, a Surface may be made at pleaſure; and we cannot ſuppoſe that Pores can change their Poſition, as a Surface may be altered, at diſcretion; or how a Ray of Light, in paſſing through a triangular Priſm, can be nearly perpendicular to both Planes, I am at a loſs to deviſe.

For my part, I look on it as ridiculous and preſuming, to tell us that Glaſs and all tranſparent Bodies are full of Pores and minute Interſtices, through which the Rays of Light have a free paſſage; becauſe, theſe Pores muſt be in every part of it, and in all Directions; neither can there be any ſolid part, between one Pore and the contiguous or adjoining Pores; for, if there be, it muſt neceſſarily impede, or entirely ſtop the progreſs of ſome Rays of Light in their paſſage to the Eye, and, conſequently, will prevent the Viſion of the Object from being perfect. Wherefore, ſince the Pores are ſo very numerous, have no ſolid part between them, and that, in all Directions; for it is plain, that they are not confined to one poſition, but muſt lie in all directions, which is evident; for, turn the Glaſs or Cryſtal (C D) as you pleaſe, the perception of the Object is as diſtinct as before; from which it is clear, that the Pores muſt be the ſame in all Directions, and then, I would aſk, what becomes of the ſolid Body? for, in reality, it muſt be all Pores.

It is an inſult on our Underſtandings, on our Reaſon, and on common Senſe, to ſuppoſe, or for any perſon to attempt to perſuade us, that Glaſs, or Adamant, the moſt compact of Stones, is more pervious, or that the Pores are more direct than in ſoft Wood, or in any Wood; which every one, who has conſidered it,

Plate II. knows to be full of Pores (like Veins in the animal Body) through which the Juices paſs for its nutriment; and, in many kinds of Wood, they are perceivable by the naked Eye, and lie in direct Lines. Can any Perſon perſuade himſelf, that the Pores are more numerous, or more direct, or that they are more capacious, ſo as to admit a freer paſſage, through Adamant, for the moſt ſubtle Fluid whatever (which I will ſuppoſe Light to be) than through a thin piece of Wood; in which the Pores are obvious, and clearly viſible? Air, which is much denſer than Light, or Water, ſtill denſer than Air, has a free paſſage through; but not through Glaſs, I preſume. Yet, if you oppoſe it to a Candle (being cut very thin acroſs the Pores) you may perceive, indeed, a few ſcattered Rays of Light paſs through, but very far from having diſtinct Viſion of the Candle, or even the out Line of the Flame, only. The reaſon is very obvious; becauſe, the parts, between the viſible Pores, being more condenſed and compact, receive the Light which falls on them, and either abſorb or reflect it; which, therefore, does not paſs through to the Eye; conſequently, the parts of the Object, which are in a direct line to the Eye, cannot be ſeen; therefore, the Viſion of the Object is imperfect.

As this is evidently the caſe when it is oppoſed to a luminous Body, which is ſeen by its own Light; what will be the conſequence when it is oppoſed to an opake Body, which (as we are told) is ſeen only by means of reflected Light? Why, that there is not the leaſt appearance of it to be ſeen at all; not even through fine oiled Paper, which, I muſt needs ſuppoſe, is infinitely more porous than any Glaſs or Stone whatever.

Whereas, through perfectly pellucid Subſtances, every viſible Point, in any Object whether luminous or opake, is diſtinctly ſeen, without the leaſt impediment; conſequently, if Light be material, and a Ray is tranſmited from every Point, in a viſible Surface, to the Eye (for every phyſical point may be ſeen) and if thoſe Rays are conveyed through tranſparent ſolid Bodies, they muſt neceſſarily paſs through Pores, direct in all poſitions.

Now I am fully convinced, that the Pores in Glaſs, &c. are of thoſe ingenious Gentlemen's own creating; who, when they are at a loſs for proof of certain Hypotheſes (for want of better) they imagine Bodies to poſſeſs ſuch and ſuch qualities as may beſt anſwer their purpoſe. But, are thoſe Chimeras, of their own fertile imaginations, to paſs on the World for real exiſtencies? are the Concluſions drawn from ſuch Premiſes candid? by no means, they are very diſingenuous, inſomuch that, I deny it to be in the power of any Man, to give ocular or other demonſtrative proof that there are Pores in Glaſs or tranſparent Stones; and, I do believe that the moſt pellucid ſubſtances are the freeſt from Pores; for all porous Bodies are compreſſible into leſs compaſs, which neither Glaſs nor Stones can poſſibly be; nor Water, which is perfectly tranſparent.

Pores are, in my opinion, rather the cauſe of Opacity than of Tranſparency in Bodies, ſeeing that, they abſorb the Light in their receſſes. Yet, I do not ſuppoſe that all Bodies, as Wood or Metals, which are the freeſt of the kind from Pores, have any degeee of tranſparency, but when they are exceeding thin; as leaf Gold, &c. but, they approach nearer to tranſparency than the more groſs and porous kinds. The real cauſe of Tranſparency, and how Viſion is conveyed through tranſparent Bodies, are (I am firmly perſuaded) among the hidden myſteries of Nature, which is not given Man to explore.

6. It is, I believe, a Paradox, not eaſily accounted for; that, if two triangular Priſms, of Glaſs or other diaphanous Subſtance, having equal Angles, are ſo placed
Fig. 12. together (as A B C, B C D) that the outſide Faces (A B and C D) are parallel, we have direct Viſion through them both (from the Eye, at E, to an Object at F); whereas, if either of them (as B C D) be taken away, the Object is loſt to ſight from that Point of View, by reaſon of the ſuppoſed refraction of the Rays of Light, in paſſing through the Priſm A B C.

Now,

Now, if the Pores, through which the Rays E a, E b, &c. paſs, go directly through both Priſms (from a to c, and from b to d) how can they be varied by the removal of one of them? the taking away of one Priſm can certainly have no effect on the Pores of the other, ſo as to alter their Direction; yet (whatever be the Cauſe) it is certain, if either Priſm (as B C D) be removed, that the Viſion, of the Object F, is turned aſide out of a Right Line, and totally loſt to ſight, from that Station; for, it will be ſeen by an Eye at *E*, and not at E, and appear to be in the direction *E F*. Or, the Object, at F, being removed to G; will appear to be at F, in the direction of E F; and F will be its apparent place.

Again; if the Pores go direct through Glaſs, &c. the Rays of Light do not ſtop at the Surface, and conſequently, they cannot ſuffer either Reflection or Refraction by it. For, I preſume, the true definition of a Pore is a ſmall Cavity or Interſtice, which admits a free paſſage for Fluids. If, therefore, they do enter, and paſs freely through, how can the ſurface affect them? or how can the Rays of Light, if Light is a Body, be turned aſide, within the Glaſs, in any other direction than that of the Pores?

It is affirmed and allowed, that the Angle of Refraction is always to that of Incidence in a certain Ratio or Proportion; and ſince the Angle of Incidence may be any Angle at pleaſure, it neceſſarily follows, that Light paſſes freely through in all Directions; which (according to the eſtabliſhed Hypotheſis, that it is corporeal) implies Pores in all Directions, according to one of the Doctor's Definitions; and, if it were poſſible for Pores to go right or direct through, in all Directions (which is repugnant to reaſon and abſurd to ſuppoſe) the whole Priſm muſt be all Pores; which omniporous quality, being attributed to any kind of bodily Subſtance, I am perſuaded, no Man, in his ſenſes, will acquieſce in.

7. I ſhall juſt give one more objection to the materiality of Light, and conclude this Section, and Subject.

Suppoſe the Eye, at E, viewing an Object, A B. There is ſuppoſed to be Rays of Light, A E, B E, &c. tranſmited from every point to the Eye, forming an Image of the Object on the Retina, which is generally allowed to be inverted; although a late writer has given ſome reaſons to the contrary. Be that as it may, it is certain, that if thoſe Rays of Light enter the Eye, they muſt paſs through the Aperture or Pupil, and converge to a Point (E) within, before they can diverge againto form an Image of the Object; at a c b. Fig. 13.

Now if the Eye be ſo ſituated (in reſpect of Diſtance) to the Object, that the extreme Rays, A E and B E, incline to each other in an Angle (A E B) not exceeding 40 Degrees; it is certain, that the Eye is capable of taking in the whole of that Object at one View; although, every part cannot be diſtinctly ſeen at once. Every phyſical Point, in the ſurface of the Object, as C, D, F, &c. is ſuppoſed to tranſmit a Ray of Light to the Eye, as well as in all other Directions, at the ſame moment of Time. Conſequently, the whole ſyſtem of Rays generate a Cone or Pyramid of Rays, cloſe wedged together in every part; which, all enter the Eye at the ſame inſtant, paſſing through a Point of the ſame dimenſion as one ſingle Ray, at E, the Vertex of the Pryamid or Cone; which Circumſtance is ſo egregiouſly abſurd, that it is ſufficient, in my opinion, to refure all that can be alledged concerning the Rays of Light being material.

For, can a great quantity of Air (the rareſt Fluid that we are acquainted with) paſs through a Pin-hole in an inſtant? Will it not (like Water) require more or leſs time, according to the dimenſion of the Hole, to paſs through? can any force drive the ſame quantity of Air through a Hole of half an inch in Diameter, in the ſame Time as through one of an Inch? No; the ratio of the Time required, with the ſame force, will ever be in proportion to the area of the Aperture; that is, the ratio of different Apertures, to each other, is equal to the ratio of the Time required, with equal force.

And

And yet, a Pyramid of Light, whoſe Baſe is equal to St. Paul's Cathedral, nay, millions of times greater (for the Eye can take in, not only the Sun, but, a great part of the Hemiſphere, at once) and its Altitude of any dimenſion, as far as the fixed Stars; yet, I ſay, this prodigious Pyramid of Light, a material Body, a Fluid, can paſs (without any known impulſe) through ten thouſand imaginary Pores in the Cornea, in their paſſage to an imaginary Point within the Eye; through which, the whole is conveyed in an inſtant, to the ſeat of Viſion; and what becomes of it after? the Eye muſt be very capacious to contain it all. Nay, more; the very ſame Baſe, or Object, can ſend forth millions of Pyramids of Light, to other Eyes all around, at the ſame Time, and in all Directions. Impoſſible! being Matter. Can any Perſon form an Idea, to comprehend how the ſame Body, or Surface, ſimply, can emit, or reflect what it receives, this inſtant, from any other Body, in innumerable directions at the ſame Inſtant? Let thoſe, who can, find belief and give credit for it.

Much more might be ſaid in ſupport of this Argument; but, as it is not directly to the purpoſe of Perſpective, I ſhall not treſpaſs any longer on the Readers time and patience, in this Digreſſion from the Subject. I ſhall only beg leave to draw from it this Concluſion; that Light is not an exiſting Medium compoſed of Particles; which, being reflected from Objects, in all Directions, and ſtriking on the Organs of Sight, conveys the Viſion of them to the Mind, and occaſions the ſenſe of Seeing; intimating, by means of the different qualities of its heterogeneous Rays, not merely the exiſtence of the ſurrounding Objects, but, they are alſo ſuppoſed to excite, in the Mind, the idea of Colours on their Surfaces; which, otherwiſe, have no exiſtence: ſtrange Doctrine!

But, with ſubmiſſion, I think, that the perpetual exiſtence of ſuch a Medium is repugnant to the notion of luminous Bodies emiting Light, inceſſantly; which, proceeding from them, progreſſively, in Right Lines, excites Viſion; not inſtantaneouſly, but propagated in Time: which, opinion, is more conſiſtent with the notion of its being reflected from other Bodies. For, how a ſtagnating Medium, a Fluid, can be ſo actuated, as to be reflected, from Objects, in all Directions within itſelf, and conſequently, in direct Oppoſition to its firſt, or incident motion, and with ſuch amazing velocity, is beyond the reach of human reaſon to conceive; much leſs to comprehend and explain.

It is the diſtinguiſhing Property of lucid or luminous Bodies to diſpenſe Light all around them; but how, or in what manner, it is not my intention to enquire into; being well aſſured, that the attempt would be as fruitleſs as preſumptuous. Such Phænomena are, and ever will be, to Man, impenetrable and inſcrutable; Myſteries not to be unfolded but by infinite Wiſdom itſelf. To us, there is a large field of knowledge, open and in view, whereon to exerciſe our reaſoning faculties, and which lies within our reach; let us not, then, ſtep aſide into intricate Mazes and Labyrinths, out of which it is impoſſible to extricate ourſelves; in which, the farther we wander the more we are bewildered; till, wearied with the vain purſuit, we are, at laſt, obliged to own, that all our boaſted knowledge is but to know how little can be known.

SECT.

SECTION V.

Of Refracted Vision.

THE laſt Section touched, though very ſlightly, on that part of Optics called Catoptrics, or reflected Viſion. In this I ſhall briefly touch on Dioptrics or refracted Viſion; both which, I look on, in many reſpects, as Deceptions in Viſion. For it is evident, that, in the caſe of Reflection, on the ſurface of Water, &c. or poliſhed Mirrours, we do not ſee the Object, but only its Image or Appearance. So likewiſe, in the caſe of Refraction through tranſparent Mediums, ſolid or fluid, we do not ſee the real Object, but its Image or Repreſentation only; which is likewiſe a Deception; ſeeing that, the Object appears, in ſome caſes, larger, in others, ſmaller than it really is, and always appears to be where it really is not; although, we imagine that we are looking directly at the Object.

Perſpective Repreſentations, are alſo manifeſt Deceptions in Viſion. The buſineſs of Perſpective is to repreſent Objects on a Plane Surface, by the rules of Optics; which, in the true Point of View, will give the Idea of a real ſolid Object (being properly and judiciouſly ſhaded) having the appearance of projectures and recedings, of one part before or behind another; and having alſo the ſame hue or teint of Colour, the Deception becomes ſtronger; inſomuch, that it is poſſible ſo to deceive the Eye, as to imagine the Repreſentation to be real and ſubſtantial.

In looking at Objects through a Teleſcope, either reflecting or refracting; although we level it exactly to the Object, in a Right Line, and imagine we look directly at the Object, through the Tube; yet, it is plain that we do not ſee the Object. For, in a Reflector, we look directly at a concave Mirrour, which is placed direct between the Object and the Eye; conſequently, in that caſe, it is impoſſible to ſee the Object, but only its reflected Image on the Mirrour, which is firſt received on another Mirrour, at the other end through which we look, and reflected again to the former, oppoſite to the Eye, (a moſt curious and ingenious invention; which was firſt conſtructed by Dr. Gregory, and thence called the Gregorian Teleſcope.) The Image, of the Object received on the Mirrour, is magnified, by means of convex glaſſes in the ſmall Tube; ſo that, we do not ſee even the reflected Image on the Speculum, but the Image of that, only, magnified, to a great degree. In reſpect of the rafracting Teleſcope, the Object itſelf is magnified, in the ſame manner as the Image on the Mirrour, in the Reflector; ſo that, we do not, in either caſe, ſee the real Object, but only its magnified Image, between the Object Glaſſes and the Eye. I ſhall illuſtrate it by a ſingle magnifying Lens.

1. Suppoſe an Object, as AB, and a convex Lens, CD, placed between the Object and the Eye, at E. It is manifeſt, ſince Viſion is conveyed in Right Lines from the Object to the Eye, that, if we ſaw the real Object, by the Viſual Rays AE and BE &c. it would appear, in proportion to the Lens CD, of the magnitude *ab*, only; but we find, that it appears larger than its real magnitude, in proportion to the Lens.

For, if the Rays Aa and Bb fall perpendicularly on the convex Surface, CHD towards the Object, they will paſs directly to the other Surface, which is concave towards the Object, cuting it at a and b, and are thence refracted, to the Eye at E, as aE, bE; then, if A*A* and B*B* be drawn parallel to the Axe of the Lens (EI) and Ea, Eb are produced, cuting A*A* and B*B* in *A* and *B*, then, is *AB* the apparent place of AB, or its Image; which, being ſeen under a larger Angle *AEB* than the real Object AB, if the Lens was removed, or Plane, it will conſequently appear larger; by Theorem 1ſt.

Plate II. Fig. 14. Hence it is manifeſt, that the real Object, A B, is not ſeen; for it is not poſſible that the Point, A, can be ſeen under the refracted Lines A a, a E; and if E a, E b be produced, it is evident that the Object A B, in that place, muſt be larger, to appear equal to a b, on the Lens.

This way of determining the Image or the apparent place of the Object, is according to Smith, P. 51. Art. 139. which, in ſome reſpects, ſeems right (He does not indeed determine the Refraction by drawing a right Line, from A or B, to the Center of either Surface; nor does he give any certain Rule to determine the Refraction. It is impoſſible to aſcertain the Point where any incident Ray, from an Object, cuts the Lens; ſeeing that, the inclination of its Surfaces is continually varying, (from the Center to its Extremes) but I find there are Caſes in which it is very exceptionable. For, according to this method of determining the apparent magnitude, or place of the Object, it can never appear larger than at the diſtance of the Lens. But it is certain, that the Eye and the Object may be ſo ſituated, in reſpect of the Lens, that notwithſtanding, the Object, A B, is conſiderably leſs than the Lens, it will appear larger; and conſequently, its apparent place is on this Side (in the focus of the incident Rays) between the Eye and the Lens, as at *a* E *b* the Eye being removed to *E*; for *a b*, the Image of A B, appears, to the Eye at *E*, under a greater Angle (*a E b*) than the Lens C D; and therefore, A B, or its Image *a b*, appears, to the Eye at *E*, larger than the Lens C D, which it never could do, if its apparent place was on the other ſide of the Lens.

In this Caſe, the Object A B appears erect; but, if another Object, F G, be placed beyond the Focus of the Lens, and Right Lines F *f*, G *g*, be drawn, through the Center, I, of the oppoſite Surface, conſequently perpendicular to it; and if F *G*, G *F* be drawn parallel to the Axe cuting F *f* and G *g* in *G* and *F*; then is *F G* the Image, or apparent place of F G, which, in this Caſe, is inverted; and conſequently, the real Object F G is not ſeen; for the extreme F is ſeen at f, and G at g, on the ſurface of the Lens, in a contrary direction to their true places.

The contrary effect is produced through a concave Lens, in which, the Object always appears leſs than its real magnitude, according to its Diſtance; which, being but the reverſe of the other, 'tis needleſs to illuſtrate it.

It is evident, that Viſion is conveyed through any tranſparent Medium whatever, obliquely ſituated, in refracted or broken Lines; conſequently, in ſuch Caſe, the Object never is where it appears to be. For, the Point A, which is ſuppoſed to be ſeen by the Eye at E, appears to be at *A* or a, and not at A where it really is; and the Point S, which is ſeen directly through the Center of the Lens, although it be ſeen in the Right Line E S (for there is no refraction perpendicularly thro' parallel Surfaces of any kind) yet, its apparent place is at *S*; and, if either the Eye or the Lens be removed, the apparent place of the Object is varied.

2. Lenſes of all kinds, excepting ſuch as are plane on both Sides, or ſuch Meniſcus, (hollow on one Side and convex on the other) as being portions of concentric Spheres, conſequently parallel Surfaces, partake of the nature and properties of Priſms, whoſe Surfaces are inclined to each other. Wherefore, the Rays, in paſſing through them, are more or leſs refracted, according as the Surfaces are more or leſs inclined; that is, the leſs the radius of the Sphere, to which the Lens is formed, the greater is the magnifying power; becauſe, the refracting Angle is greater; and, conſequently, projects the Rays to a greater diſtance from the Center.

Hence it becomes a Paradox, how a large Object (I mean one, which, according to its Diſtance, occupies, or appears equal to the whole ſurface of the Lens) can be regularly magnified; for ſince it is manifeſt, that through parallel Surfaces there is no ſenſible Refraction (except the Surfaces are conſiderably diſtant from each other and ſeen through obliquely, as *E* b c e, Fig. 12.) and the ſurfaces of every Lens are parallel at the Vertex, and nearly ſo at a ſmall Diſtance from it; and, the greater the angle of Inclination, the Refraction is greater; wherefore (the ſurfaces of a convex Lens, being leaſt inclined at its extremes, i. e. the refracting Angle is the

the greateſt; and, ſeeing it is continually varying, from the Vertex to the Extremes) it ſeems reaſonable, to conclude, that the Object would be more diſtorted about the edges of the Lens than near the middle; which I do not find to be the Caſe. Query; by what means is the Object magnified equally?

3. There is another circumſtance, which I do not remember to have ſeen noticed, by thoſe who treat of the properties of Priſms, in reſpect of Refraction; and which, I do not ſee a reaſon for. Right Lines, ſeen through a triangular Priſm, or any two inclined plane Surfaces, being parallel to the interſecting Line of the two Planes, that is, to the refracting Angle of the Priſm, do not appear Right Lines, but, curved; and they are more or leſs curved, as the Angle of Refraction is greater or leſs; the Curve being concave towards the refracting Angle.

Now it is reaſonable to ſuppoſe, that the ſyſtem of Rays from any Right Line to the ſurface of the Priſm, which is a Plane, would alſo form a Plane; which, by its interſection with the ſurface of the Priſm, muſt neceſſarily generate a Right Line, in any poſition of the Priſm; which, would be the Repreſentation of the original Line; and, when the Angle of Refraction is parallel to the original Line, they will alſo cut the other Surface, towards the Eye, in a Right Line, parallel to the oppoſite; how is it then that it appears curved? The vertex of the Curve, whatever it be, is where it would be cut by a Plane, paſſing through the Eye, perpendicular to the Original Line and the Planes of the Priſm.

I have already deſcribed a Priſm, in the laſt Section, and the nature of Refraction by it, as much as is my deſign, in reſpect of the deception of Viſion through a Priſm. Thoſe who would be more acquainted with the properties of Priſms and Lenſes, in general, I refer to Sir Iſaac Newton's, or, where they are treated more at large, to Smith's Optics; where, if he has patience to go through it, he will find enough to exerciſe his patience on. I do not mean to diſparage the Work, for I believe it to be the beſt of the kind, in many reſpects; yet, I think it deficient, in ſome Caſes; but, in reſpect of Colour, &c. it is of a piece with the reſt, and, being dwelt too long on, becomes tedious, if not trifling.

4. As to the Phænomenon of Colours ſeen through a Priſm it is really ſurpriſing; but, it is only the Edges of Objects that are tinged or produce the Colours. A perfectly plane Surface is not at all varied in its Colour, when ſeen through a Priſm; except at the Edges, when it is joined or oppoſed to a darker Body; and thoſe Edges are moſt coloured when they are parallel to the refracting Angle; for, when they are perpendicular to it, and ſeen direct through, they have no Colour at all, but the natural Colour of the Body, whatever it be. Alſo, the Colours ſeen through the Priſm, do not always follow in the ſame order as when a Beam of the Sun's Rays paſſes through; if a ſmall Object be oppoſed to the Light (as, a Bar in a Saſh-Window) being parallel to the Priſm, the Object is wholly loſt in the Colours, between the Red and Blue; the Red being towards the refracting Angle; an intenſe Red always follows after any Line or Edge of an Object which is oppoſed to, or which obſtructs the Light in one poſition of the Priſm, the brighter Colours ſucceeding; but, being reverſed, there is a deep Purple, and Blue following; the other Colours are very faint. The Colours are more intenſe and vivid the brighter the Object, or the greater the oppoſition and obſtruction of the Light by an opake Object. If a ſlip of White Paper be laid on any dark Body, being ſeen through a Priſm, its Edges are bordered, one with Red, the other Blue, according to the refracting Angle; but being laid on another, of the ſame Colour, none is perceived, except it makes a Shade on the other, not lying cloſe, which Shade occaſions Colour. Alſo, if a ſtrong, black Line be drawn thereon, the Red and Blue proceed contraryways from the Line, which is quite loſt between them; the Colours being produced, on any Surface, only where there is an oppoſition with a darker Body. How, then, it is poſſible to deduce or to draw a concluſion, from ſuch Experiments, that all Objects, which we perceive, are by Nature of the ſame, or have no Colour at all, ſave what is effected by the Light reflected from their Surfaces?

5. As,

Plate II. 5. As in Nature, there is neceſſarily Refraction at the common Surface of any two different Mediums, whether ſolid or fluid, except when we look perpendicularly through the Surface, or Surfaces, at the Object, although the true reaſon of it be to us unknown; ſo it would be ſuperfluous and foreign to my Deſign to multiply Caſes wherein it happens. I ſhall, therefore, only take notice of one common Caſe, which is ſo very common, though but little attended to, as to be obvious to every Perſon who is bleſſed with the faculty of Seeing.

Let any Perſon put a ſtreight Stick into Water, ſlanting in any direction, and it immediately appears to be broken, at the ſurface of the Water; the part within, appearing to take a different direction to that which is out of the Water; and, the more it deviates from a Perpendicular, the more it is refracted or broken, till the Stick makes an Angle, with the ſurface of the Water, of about 45 Degrees; and then, the Refraction is continually leſs, which is evident; for, when the inclination of the Stick, to the Surface, is ſuch, that the Angle it makes with the Surface is very ſmall, conſequently, the Refraction cannot be great; and being immerſed perpendicularly, there is no Refraction; which is the ſame thing as looking directly into the Medium, at an Object in the Water, or through a Body of Glaſs, very thick; in which Caſe, the Object appears to be conſiderably nearer, but, it is in the ſame Direction as we ſee it. Alſo, when we look at the Heavens of Stars, &c. thoſe which are in the Zenith are ſeen in a Right Line; and, the farther they are from that Point, the more the line of direction is refracted or broken, ſo that, they are ſuppoſed not to be in that place in the Heavens where they appear to be; which Refraction, is ſaid to be owing to the Atmoſphere of Air and Vapours, which ſurrounds the Earth, being denſer than the upper Regions.

6. It is a common, though curious and entertaining, Experiment, to immerſe a piece of Money or other Object in a Baſon full of Water; which will appear to be raiſed conſiderably higher than when the Veſſel is empty.

Fig. 15. Let ABCD repreſent a Veſſel, ſuppoſe of Glaſs; and ſuppoſe E an Eye looking at an Object, at F, at the bottom of the Veſſel, being empty. Whilſt the Eye remains fixed, at E, ſo that the Object at F, can juſt be ſeen over the edge, at C, let there be Water poured gently into the Veſſel; as the Veſſel fills, the Object will appear, to riſe, gradually, to G, ſo as to be ſeen quite clear of the edge of the Veſſel, in the direction EG.

Now it is certain, that the Object is at F, which appears to be at G; conſequently, if the real Object be ſeen at all, it is ſeen in the refracted Lines FI, IE, where the Right Line EG, to its apparent place, cuts the ſurface of the Water; but, more probably, the Image of the Object, only, is ſeen at G. Conſequently, a ſtreight Stick or Wire being put into the Water, from E to F, will appear broken at the Surface (at C) and appear to go in the Direction CG.

Again. Let the Eye be removed to *E*, the Water remaining in the Veſſel, ſo that the Object is apparently ſeen over the edge, at C; and, whilſt the Eye remains fixed, let the Water be drawn gently off, by means of a Cock, or otherwiſe, at the bottom; the Object, apparently at G, will gradually ſink lower in the Veſſel, and totally diſappear from that Station.

Hence it is manifeſt, that when the Water is in the Veſſel, the real Object, at F, is not ſeen by an Eye at *E*; for, if it was poſſible, it muſt be ſeen through the Side of the Veſſel, at H, which it is plain is not the Caſe; and, hence it is plain, that the Object being ſeen, apparently at G, is a manifeſt Deception in Viſion. The Caſe is the very ſame, in Objects ſeen oblique through Glaſs, or any other pellucid Subſtance; excepting ſome ſmall variations in the degree of Refraction.

7. Refraction in Water, according to all writers on Optics, is ſubject to one invariable Law, without any ſenſible error, but it is not to be demonſtrated mathematically. I have made the Experiment myſelf, as accurately as it will admit of, and find it to be nearly as follows.

If AB be ſuppoſed the ſurface of Water, and CE, DE or FE an incident Ray of Light falling on it, at E; on which Point, ſuppoſe a Circle be deſcribed, and the Perpendicular, EG, drawn. Then, if CE be produced to c, and ac be drawn, parallel to AB; take ab equal to three fourths of ac, and draw bd perpendicular to ac, cuting the Circumference in d; the Ray CE is ſuppoſed to be refracted, at E, and go in the direction Ed, into the Water. By the ſame Rule, DE goes in the direction Ee, and FE in Eg. Fig. 16.

But, the ſame Rays CE, DE, &c. will alſo be reflected in equal Angles, GEH equal DEG, &c. Can the ſame Ray be both reflected and refracted? impoſſible; yet, an Object (☉) may be ſeen at H, by the reflected Ray EH, as well as at e, by the refracted Ray Ee. It is however certain, that if a Circle be deſcribed on the Plane AGBg, and Lines are drawn thereon, as in this Figure; being immerſed perpendicularly in Water, to the Diameter AB; CEd, DEe, and FEg will appear Right Lines; which is very ſurpriſing. But yet, I do not ſee which way this Experiment proves, that Rays of Light go in thoſe directions into the Water; the whole Maſs being illumined in every part. If the Eye, being immerſed in Water, at e, ſees the Object ☉ over the edge of the Veſſel, or any other obſtacle, at E, on the Surface (which, I preſume is the Caſe) it indicates, that the Object is not ſeen in a Right Line; or, rather, that its Image is ſeen at E, on the ſurface of the Water.

8. Before I conclude this Section and Subject, I cannot help taking notice of an extraordinary paſſage; which is in the Concluſion drawn from Prop. 8, Part 3, of the ſecond Book of Newton's Optics, Page 69; concerning the extraordinary poroſity of Water; which is 19 times lighter, and conſequently, he ſays, 19 times rarer than Gold; and Gold is ſo rare, that Water may be forced through its Pores. For, as he was informed, by an Eye witneſs, a Globe of Gold being filled with Water and ſodered up (but of what thickneſs he does not tell us; I have heard ſay above a quarter of an Inch) and, being preſſed with great force, the Water ſqueezed through its Pores; and ſtood, all over its Surface, in multitudes of ſmall drops, like Dew. From which, he concludes that Gold has more Pores than ſolid Parts; but how ſuch a concluſion can poſſibly be drawn I cannot conceive. I ſhould ſuppoſe, that, in ſuch Caſe, the Water would ſpout out in ſtreams, rather than ſtand on the Surface like Dew. Yet Gold will not admit either Air or Light through its Pores, though much rather Fluids than Water; which, he alſo concludes, from the ſame Experiment, has above forty times more Pores than Parts; and conſequently, Gold (the compacteſt Metal we know of) according to that Ratio, contains above twice as much empty Space as Matter.

It is no wonder that Water is pellucid, being ſo extremely porous, and admits Light ſo freely through its Pores; but, it is ſomewhat ſurprizing that it is not compreſſible, which all porous ſubſtances muſt be. And I think it alſo ſurprizing, that a Man of Sir Iſaac's ſagacity ſhould advance ſo much on the credit of any Experiment he never tried himſelf; which, he certainly had opportunity enough for, being Maſter of the Mint near thirty years.

If Water be ſo full of Pores, as Sir Iſaac imagines, it might be compreſſed into leſs Space; which, the ſame Experiment abſolutely proves that it cannot. If it be compoſed of globular Particles, they may certainly be ſqueezed ſo cloſe together, that (ſuppoſing every Globe inſcribed in a Cube) the difference between the quantity of Matter and of Space, would be no more than the difference between a Globe and a Cube, whoſe Diameter and Side are equal, which is nearly double; I mean, that the quantity of Matter contained in Cubes, which admit of no Space between them, is nearly double of that which is contained in the inſcribed Globes; and conſequently, the Space between Globes, lying in that poſition, is nearly equal to the Globes; for, the ſame Ratio muſt continue *ad infinitum*.

But it is manifeſt, that Globes will lie conſiderably cloſer together; as Circles leave only triangular ſpaces between them; ſo Globes of equal Diameters, will lie ſo together as to leave a kind of triangular pyramidal or priſmatic Space between,

Plate II. joining each other. The difference between the Globes and Space may be very nearly aſcertained, by puting a quantity of equal Globes or Bullets into a hollow triangular Pyramid, which may eaſily be made, the true figure of a pyramid of Globes (for it is a regular Tetrahedron) and, having placed them regularly, pour in Water juſt to cover them, filling up the Interſtices between; then take out the Globes and fill the Box equally with Water. I have tried the Experiment, and find the difference between the Globes and Space to be, as 2 to 1, as near as can be; in the former Caſe they are nearly equal. Query; where is the remaining 40 times to be found? are the leaſt particles of Matter, which cannot be conceived of any dimenſions, to contain, in each, 40 times more Space than Matter? for, as I have obſerved before, the ratio, of the Globes to the Space between them, will continue to infinity; and what other regular Figure can there be, to contain more Space between them than Globes?

The Queſtion then is, whether we are to give credit to ſuch Aſſertions, or are we to credit the evidence of our own Senſes and Reaſon? I ſhall truſt the latter, before the bare aſſertion of the greateſt Man that ever lived.

9. As I have had neither time nor inclination to go through the whole of Smith's Optics; I ſhall juſt take notice of one paſſage in it, in which his geometrical reaſoning is ſomewhat erroneous; but, whether it be of Conſequence, in the following part of the Work, I have not enquired; as the Subject is not to my preſent purpoſe. I ſhall, literally, quote his own words, as follow.

Fig. 17. In Vol. 1. Page 59. Art. 157, he ſays, "The apparent magnitude of a given "line, AB, ſeen very obliquely at a given diſtance, OA, increaſes and decreaſes "in proportion to the increaſe and decreaſe of OP, the perpendicular diſtance of "the Eye from the line AB produced; provided the diſtance AO be very large in "compariſon to AB. For let the Ray BO cut a line AC perpendicular to AB "in C; and while the Eye is raiſed or depreſſed in the perpendicular OP, the "line AC will increaſe and decreaſe as OP does; and ſo will the Angle AOC, "ſubtended by AC, and this Angle meaſures the apparent magnitude of AB."

Firſt, he ſays, that the apparent magnitude of AB increaſes and decreaſes in proportion, as the Eye is raiſed or depreſſed, in the perpendicular OP; and the Angle, it ſubtends, meaſures the apparent magnitude of AB.

Let PO be produced to T; in which, take ſeveral Diviſions, each equal to PO, at Q, R, S, and T; and draw QA, QB and RA, RB, &c.

Now, becauſe PQ is double PO, and PR is triple, the Angle BQA, (according to him) will be double BOA, and BRA triple; for, AG being drawn perpendicular to AB, it is certain, that the Diviſions AC, CD, &c. are equal, AG being parallel to PO†; conſequently, AC increaſes as OP does; and ſo, he ſays, does the Angle AOC, ſubtended by AC; wherefore, the Angle at Q is double, and, at R triple, of the Angle AOB. But, the Perpendiculars AC, AD, and AE, are not the meaſures of thoſe Angles; for they are truly meaſured by an Ark of a Circle, of the ſame or an equal Radius, only.

† Cor. 6. 6. El.

According, then, to his Words, the Angle AOB increaſes as PO increaſes; which I deny, and will prove to the contrary.

It is obvious, that the Angle AQB is conſiderably larger than AOB, but it is not double, although PQ is double PO; but, ARB is very little larger than AQB; and notwithſtanding, PR is triple PO, yet the Angle ARB is not double AOB; as the Arks *ab*, *cd*, and *ef*, evince.

Now, by the ſame reaſon, the Angles ASB, ATB are continually greater than ARB; but, they are continually leſs. For, if PR be a mean Proportional between PA and PB, the Angle ARB is the greateſt that can be made, by Right Lines drawn from the Extremes of AB, and touching the Perpendicular PT.

Dem. Deſcribe a Circle through the three Points R, A, and B; by Prob. 40 Geo.
Then, becauſe PR is a mean Proportional between PA and PB (by Hyp.)
the ſquare of PR is equal to a Rectangle under PA and PB;
i. e. PB × PA = PR□ — — — — — — } Cor. 9. 6. El.
Conſequently, PR will touch, or be a Tangent to that Circle, drawn through
the Points R, A, and B — — — P. 16. 3. El.
But, the Angle ARB is at the Circumference of that Circle; conſ. ASB and
ATB, which are beyond the Circumference, are leſs than ARB. — 20. 6.
Therefore, ARB is the greateſt Angle, that can be, touching the Tangent
PT, ſubtended by AB; and conſequently, the Angle ASB is not encreaſed,
as the Eye is raiſed in the Perpendicular OP.

10. Again. He ſaies, in the following Paragraph, as deduced from the foregoing, that "the apparent magnitudes of equal parts, AB, ab, of a Line, PAa, ſeen very "obliquely, at great diſtances from the Eye, are, reciprocally, in a duplicate propor- "tion of thoſe diſtances. For example; let Ob be double of OB, and the angle "OBP will be double of ObP; and accordingly, ſince AB, ab are equal, the "perpendicular AC will be double of ac; and, being ſeen twice as near as ac, will "appear four times bigger than ac. Again, if Ob be triple of OB, the line AC "will be triple of ac, and being ſeen three times nearer than ac, will appear nine "times bigger than ac; and ſo on.

Let Ob be double OB; I ſay, the Angle OBP is greater than twice ObP; Fig. 17.
and he expreſſly ſaies that OBP is double ObP.

Dem. Becauſe Ob is equal 2OB, bB is greater than OB. — P. 13. 1. El.
wherefore, the Angle bOB is greater than ObP. — — 12. 1.
But, the Angle OBP, being external, in reſpect of the Triangle bOB, is
equal to bOB + ObP; wherefore, it is greater than double ObP. 10. 1.
But, if the Angle abc be equal half ABC, ac will not be half AC. 3. 6. El.
Conſequently, ſince the Angle ABC is more than double abc (and AB is
equal to ab) AC is ſtill more than double ac.
Therefore, the apparent magnitude of AC to ac is more than duplicate;
ſeeing it is more than half ac, and is alſo ſeen at leſs than half the diſtance.

Now I am far from ſuppoſing, that this Author was ſo deficient in Geometry as theſe Examples ſeem to indicate; nay, I am well convinced he was not, as is evident in the next Book; and his Proviſos are ſome extenuation of the errors; he ought, then, to have told us that they are nearly ſo, by approximation, and not that they are ſo, in expreſs Terms. But, in any Caſe whatever, there is not an equal ratio, or nearly, between the increaſe and decreaſe of the Perpendicular AC, and the Angle AOC.

As this and the preceding Article are the only paſſages which any way tend to advance the Theory of Perſpective (having been quoted, by Mr. Kirby, for that, though but little, purpoſe) I have, therefore, been more particular in my remarks on them. And, as what I have advanced in the two laſt Sections, and part of the firſt, is not directly to the purpoſe of Perſpective, it will, I know, by ſome, be deemed impertinent and foreign to the Deſign of this Treatiſe; let them, if they pleaſe, paſs it over, and proceed immediately to the Subject, which is not vitiated by it. As I do not intend to publiſh a Treatiſe on Optics, and as Perſpective has a near affinity to that Science, what exceptions I have always had, to ſundry paſſages in the works of optical writers, I thought proper to give here, where I was treating on an eſſential part of the Theory of Perſpective, and a branch of the Science of Optics.

I hope I have not treſpaſſed, too much; on the time and patience of the candid and unprejudiced Reader. I ſhall now proceed to Book the ſecond, which treats on the Theory of Perſpective; where, I ſhall endeavour to make ſome amends for his time ſpent, I hope not loſt, in peruſing this Digreſſion.

BOOK

BOOK II.

Of the Theory of Perſpective, rectilinear and curvilinear.

SECTION I,

Containing a general INTRODUCTION to PERSPECTIVE.

TO define the Terms of Art peculiar to any Science, with brevity and perſpicuity, I have always looked on as a particular excellence in the Work; but I have frequently found, that an affectation of brevity has left the Term, intended to be explained, rather obſcure, at leaſt doubtful; whilſt others, endeavouring to render it clear by a multiplicity of words, have, at laſt, involved it in perplexity.

Although my deſign is to be as brief as the nature of the Subject will admit of, yet I am afraid I ſhall rather be thought prolix, than otherwiſe, in ſome of the following Definitions or explanation of the Terms; but certain I am, that, the Time ſpent in acquiring a perfect knowledge of all the Terms I have defined, will not be loſt, as they contain many uſeful Leſſons to a young Student. I have endeavoured to explain every Term in the moſt familiar manner; not ſaying more than was neceſſary, yet, enough to be clearly underſtood. To be too brief is worſe than prolixity, ſo it be not tedious and trifling; the one leaves us doubtful, the other, probably, makes it clear at laſt.

I may, perhaps, be particular and ſingular in my opinions; but, I think it better to define each Term ſeparately, than to include ſeveral in one Definition, as is very frequent; and I always chooſe to name the Term, I mean to define, firſt, rather than end the Definition with it, or name it promiſcuouſly. My reaſon for which is, that 'tis much eaſier to find when refered to; or, when there is occaſion to look for any particular Definition, without reference, the Number not being known.

The Order in which the Terms are defined is, with me, a material circumſtance; begining at the Foundation, and going gradually on in a regular ſucceſſion; never uſing any Term, if poſſible to avoid it, in the explanation of another, which has not already been defined. In general, I find them promiſcuouſly jumbled together, without Order; going, as it were, from one end of the Science to the other, to and again; too haſtily introducing, perhaps, ſome new or favourite Term, before others which ſeem neceſſary to be firſt known, in order to prepare the way, by removing ſome impediment. How I have ſucceeded, muſt be left to the deciſion of the candid and impartial Reader.

I am no favourer or admirer of new and uncommon Terms; nor, indeed, do I think that every writer, on any Science, has a right to impoſe new Terms; unleſs he has found out ſome new Principles, on which it was not poſſible for him to expatiate, by the Terms already known and in uſe; as Dr. Brook Taylor has done in Perſpective; on which, I have ſpoke more largely in another place. It was impoſſible for him to convey the Ideas he intended to inculcate by the old Terms, and therefore, he was under the neceſſity of inventing and enforcing new ones; which are extremely expreſſive of the thing meant. But, if every writer, on that or any other Subject (who, becauſe he knows ſomething of it, imagines he knows more

than any who have wrote before him) was to take the liberty to impoſe new and unmeaning Terms, of his own, ſuited only to his own trifling Ideas of the Subject, the Science would, by that means, become perplexed and intricate; each Perſon, who happened to receive his knowledge of it from different Books, would, conſequently, underſtand and call the ſame Thing by different Names; than which, eſpecially when they are abſurdly or falſly named, nothing tends more to perplex, and involve the Science in obſcurity.

I have already, in the Preface, given my reaſons for omiting, in this Treatiſe, geometrical Definitions and Problems; becauſe, I ſuppoſe the Reader already tolerably verſed in Geometry; if not, I adviſe him firſt to ſtudy it, at leaſt Practical Geometry; without which, it is uſeleſs to attempt, and impoſſible to ſucceed in the Study of Perſpective: the better he is acquainted with Euclid, the greater progreſs will he make in Perſpective; of which, Geomety is the foundation. To aſſiſt him in it, I have compiled and compoſed a Volume (mentioned in the Preface) which may be called an abridgment; it, nevertheleſs, contains all that is eſſential. My chief aim, in that work, was to make it uſeful, the Study of it pleaſant, and attainable by any tolerable Capacity, and applicable to various uſes in Life; particularly ſubſervient to this Treatiſe of Perſpective, as I always refer to it for Demonſtration: in which caſe, it may be deemed a part of this Work, and ought always go together. But, although I have, there, fully defined a Plane (Def. 6th.) yet, as it is ſo very eſſential in Perſpective, it was by no means proper to omit it here; ſeeing that, on it the whole Theory of Perſpective is built.

A PLANE is a perfectly even, ſtreight, and regular Surface, which is neither convex nor concave in any part; but agrees, in every part, with a Right Line or ſtreight Ruler, applied, any how, to the Surface.

2. By the motion of a Right Line, a Plane may be conceived to be generated; either by a direct, lateral motion, or by ſuppoſing it whirled around (ſo as not to generate a Cone) on any Point in it.

3. It is eaſy to conceive, that, if the Eye of a Perſon be in a Plane, or in a continuation of it, the neareſt extremes or limits, towards the Eye, hide all the reſt of the Plane; for, the whole Plane vaniſhes, and is loſt to an Eye in the Plane; which appears but a Right Line, extended in length to the apparent dimenſions of the Plane, conſidered as having only length and breadth: thickneſs or ſubſtance is the property of Solids, a Plane has none; 'tis the Surface, only, that is conſidered.

N. B. The Picture (in Perſpective) is always underſtood to be a Plane. Therefore, the Board, Canvas, or Paper, on which we draw, is the Plane of the Picture.

N. B. 2. A Plane may be of any Figure; and, in Perſpective, it is frequently conſidered as being infinitely extended, without regard to Figure, or to its limits.

A CUBE is bounded by ſix Planes, which are all Squares; as A, B, X. Fig. 1. Plate III.
Every other Parallelopipid has alſo ſix Planes, which are all Parallelograms; either right angled, as Fig. 2, or acute angled, as Fig. 3.

A PRISM is a Solid, whoſe Baſe, A, and Top, B, are either ſimilar Triangles *, Quadrangles †, or Poligons ‡ of any Number of Sides. The other Planes or Surfaces are all Parallelograms. (As X, X.)

* Fig. 4.
† Fig. 1, 2, & 3.
‡ Fig. 5.

PYRAMIDS have their Baſes (A) of any Number of Sides. The other Planes, of the Sides, are all Triangles. (As Y, Y.) Fig. 6.

N. B. No Solid can be formed of Planes, having leſs than four; and that muſt be a Pyramid.

PlateIII. Of PLANES and their POSITIONS, in GENERAL.

Fig. 7. 1. HORIZONTAL is the firſt and moſt natural poſition of Planes; ſuch, are all Planes which are parallel to the Horizon; conſequently, all horizontal Planes are parallel amongſt themſelves. As Z, H, H.

2. VERTICAL. All Planes which are perpendicular to, or which cut the Horizon at right Angles, are called vertical Planes. As V. Fig. 7, 8, and 9.

Fig. 8. Vertical Planes may be in all poſitions, in reſpect of each other, *viz.* parallel, perpendicular or inclined; as may be conceived, by revolving a vertical Plane, on a Right Line, AB, perpendicular to the Plane of the Horizon; conſequently, they will, if produced, all paſs through the Zenith and Nadir * of our Horizon.

3. INCLINED. All Planes whatever, which are neither parallel nor perpendicular to the Horizon, are Inclined Planes.

For, if a Plane cut the Horizon, or would if produced, in an acute Angle, it is not vertical or perpendicular; conſequently, it inclines to the Horizon on one ſide more than the other; which, Inclination, is always meaſured on that ſide making the acute Angle; as, for Example.

Fig. 9. The Plane X inclines to the Horizon, in the Angle BCA; and it is alſo ſaid to incline to a vertical Plane, in the Angle BCE; which Angle, if they have the ſame Interſection, CD, or parallel Interſections with the Horizon, is the Complement of it sinclination to the Horizon. See N. B. Def. 13th. Geo.

4. It may not be improper, here, to obſerve (for it is neceſſary to know and underſtand well) that the Angle of Inclination, of one Plane to another, can be meaſured, only in a Plane to which both the other are perpendicular; or, which is the ſame thing, if a Line be drawn in each Plane, from the ſame Point in their common Interſection, and perpendicular to it, an acute Angle made by theſe Lines is the Inclination of the Planes; for it is evident, that a Plane paſſing through thoſe two
† 2. 7. El. Lines will be perpendicular to the common Interſection † and conſequently to both Planes. To illuſtrate it.

Suppoſe the horizontal Plane Z raiſed up into the Poſition of X, inclined to the Horizon; the line CD, on which it was ſuppoſed to turn, may be conſidered as the common Interſection of the two Planes, X and Z.

It is evident, that the point A will, in that motion, have deſcribed the Ark AB; and, if the Angle ACD be a Right one (as it is ſuppoſed) BCD is ſtill a Right Angle;
§ Def. 11. Geom. wherefore, AC and CB are both perpendicular to CD; § and the Plane ABC, de-
† 2. 7. El. ſcribed by the motion of CB, is alſo perpendicular to CD, † and conſequently, to
‡ 9. 7. the two Planes X and Z.‡ Therefore, the Angle ACB, in the Plane ABC, is the Angle of Inclination of thoſe two Planes.

Draw AD, at pleaſure, cuting the common Interſection, CD, in D; and, from the ſame point D, draw DB, in the inclined Plane X.

DEM. Now, if ACD be a Right Angle, ADC is acute — Cor. 3. 10. 1. El.
wherefore, AD is longer than AC; and alſo, BD than BC — P. 12. 1. El.
But, the Chord, or Subtenſe, AB, ſubtends both the Angles, ACB and ADB;
conſequently, the angle ADB is leſs than ACB. — — Cor. to 14. 1. El.

Or, ſuppoſe BF perpendicular to the Plane Z; and let FD be drawn.

Then, a Plane DBF, paſſing through BF, is perpendicular to the Plane Z, but not to X. And, becauſe FD is longer than FC, and BD than BC, it is manifeſt, that the Angle BCF is greater than BDF.

But, the Plane CBF is perpendicular to both the Planes, X and Z (as before) and, conſequently, to their Interſection CD.

* Imaginary Points in the Heavens, diametrically oppoſite to each other; the one perpendicular over our Heads, the other under our Feet, in the lower Hemiſphere.

From which it is clear, that the Angle made by a Plane cuting two other Planes, perpendicular to their common Interſection, is the Angle of Inclination of thoſe two Planes; ſeeing that, the Angle made by any other Plane, paſſing through A B or B F, will neceſſarily be leſs, the greater the Inclination of A D, or F D, to C D.

To ſet this matter in the cleareſt Light poſſible, it being ſo very eſſential in Perſpective, as well as in other Sciences and Arts, I have added the following Figure.

Let A B C and C B D be two rectangular Planes, cuting each other in B C, their common Interſection. Let E F and F H be two Right Lines, one in each Plane, perpendicular to their Interſection B C, at the ſame Point, F. Fig. 10.

Wherefore, the Plane E I F G, paſſing through thoſe Lines, is perpendicular to both Planes, A C and C D; § and, the Angle E F G, made by that Section, is the largeſt that can poſſibly be made by a Plane which is perpendicular to either of them. § 2. 7. El.

For, ſuppoſe the Line E H perpendicular to the Plane A C B, only; and, a Plane E I K L to paſs through that Line, it will be perpendicular to the Plane AC;† and becauſe the Plane E F G is perpendicular to both the Planes A C and C D, and paſſes through the ſame Point E, it will, alſo neceſſarily, paſs thro' the Line E H; wherefore, E H is the common Interſection of thoſe two Planes,* E I F G and E I K L produced. I ſay, that the Angle E F H, made by the Plane E I F G, is greater than E K H. † 9. 7. El.

Dem. Now, E H, the common Interſection of the two Planes I F G and I K L, together with the Interſections E K and K H, of the Plane I K L with the two, Planes A C and C D, form a Triangle; and ſo does the ſame line E H with the two Interſections E F and F H, made by the Plane I F G, with the ſame Planes A C and C D.

But H F K is a Triangle, and, the Angle H F K is preſumed to be a Right one; wherefore, H K is longer than H F;‡ and, for the ſame reaſon, E K is longer than E F; and conſequently, the Triangle E F H, having one Side (E H) common with the other Triangle E K H, and, having the other two Sides, E F and F H, leſs than the two Sides E K and K H, reſpectively, they, therefore, contain a larger Angle, *viz.* E F H than E K H. — — Cor. to 14. 1. El. ‡ 12. 1. El.

But, the Plane I F G is perpendicular to both the Planes, A C and C D; and the Plane I K L is perpendicular to one of them (A C) only.

Therefore, the Angle E F G, made by the ſection of a Plane which is perpendicular to both the other, is larger than any Angle made by any other Section, of a Plane perpendicular to one of them only; and conſequently, the Section E F G meaſures the true Angle of Inclination of the Planes A B C and C B D. Fig. 10.

N.B. The Angle made by the ſection of a Plane inclined to both Planes, A C and C D, may be either greater or leſs than E F G.

5. Although it is not poſſible to conceive an Idea of a Plane abſtracted from one or other of the three Poſitions I have explained, yet, in the application of Planes, in Perſpective, as in Geometry, no particular regard is had to them; for one Plane is ſaid to be perpendicular to another, if it makes right Angles with the other Plane, as H to V: each of which is ſaid to be perpendicular to the other, notwithſtanding one of them (H) is horizontal. Fig. 11.

The Planes X and Y are alſo ſaid to be perpendicular to each other, although both are inclined to the Horizon; and, whatever their Inclination to the Horizon may be, it matters not, if they make right Angles with each other, as at C. For, if the Plane Y was turned up, on A B, its interſection with the Horizon, into the vertical Poſition W, and, along with it, the Plane X, into the horizontal Poſition Z; their Poſition, in reſpect of each other, is not altered, if the Angle, at D, be ſtill a Right one, as before, at C.

* E I is the true Interſection, perpendicular to the Plane A C; and conſequently, E H would be I E produced. But, as it would run into the Figure below, I thought it beſt to diſpenſe with it, as the Demonſtration is the ſame.

Plate III. 6. So likewiſe, one Plane is ſaid to incline to another, if they do not interſect at right Angles, as H and X, or would not if produced, as X and V.

The Plane X being inclined to both H and V, (Art. 3. of Planes) they are, for the ſame reaſon, both inclined to X; yet, one is horizontal and the other vertical; for, the inclination of two Planes is mutual. So that, when it is ſaid that one Plane is perpendicular or inclined to another, it means nothing more, than, that they are at right Angles, or otherwiſe with each other; no regard being had to the horizontal or vertical Poſition of either; except the Poſition of one is previouſly known or determined, to which, the other is ſaid to be perpendicular or inclined.

7. In Perſpective, it is alſo frequently ſaid, that, Lines are perpendicular to certain Planes; whereas, if the Plane be vertical, it is eaſy to conceive, from what has been ſaid, that all Lines, which are perpendicular to a vertical Plane, are horizontal,
Fig. 12. and parallel amongſt themſelves; as AB, EF, and CD, to the Plane GIK. Yet, the Planes, in which theſe Lines are, may be either horizontal, as ABFE; vertical, as EFCD; or inclined to the Horizon, as ABCD.

If the Plane be horizontal, the Lines perpendicular to it are really perpendicular i. e. to the Horizon; as AJ, ED and FC. But, if the Plane be inclined to the Horizon, then, the Lines, which are perpendicular to it, are alſo inclined to the Horizon, yet parallel amongſt themſelves.

Suppoſe the Plane ILMN vertical, and perpendicular to the inclined Plane GH; the Lines LI and MN, which are at right Angles with its Interſection, IN, are perpendicular to the Plane GH. But the line IP, which is perpendicular to the Horizon, is inclined to the Plane GH, in the Angle LIO. And OI, at right Angles with IP, is horizontal; but, it is alſo inclined to the Plane GH, in the Angle OIN equal LIP.

Dem. Let NI be produced, towards H. LI is perpendicular to NH.
Then, becauſe LIN is a Right Angle, LIH is, alſo, a Right one - C. 2. 1. El.
Conſequently, LIO, equal LIN—OIN, is equal to PIH, or LIH—LIP.

Every Right Line, therefore, which is neither parallel nor perpendicular to a Plane, is conſequently, inclined to that Plane; and its Inclination may be known, by drawing a Perpendicular from the extreme, or any other Point, as M, in the Line IM, to the Plane GH; the Complement of the Angle IMN, i. e. IML
* 4. 1. El. equal MIN*, is the Inclination of IM to the Plane GH.

Or, If a Plane (ILMN) be drawn, through the Line IM, perpendicular to the Plane GH; the Angle MIN, which the line IM makes with IN, the interſection of the perpendicular Plane with GH, is the Angle of its inclination to the Plane GH.

N. B. IN is the Seat of the Line IM or IO, and alſo of ML, or, of the Plane NL, on GH; produced by the Interſection of a perpendicular Plane paſſing through the Line, as above. Therefore, the Angle which any Line makes with its Seat, on a Plane, is the Angle of its inclination to that Plane.

8. PQRS is a horizontal Plane, cuting the inclined Plane KPQ in the Line PQ. But, PQ is not the Seat of the Plane PR, on KQ, it being inclined to KPQ, in the Angle SPT; for, if ST and RU be drawn, perpendicular to the Plane KPQ, the Lines PT and QU (joining the Points, T and U, where the perpendiculars cut the Plane, with P and Q) are the Seats of the Lines PS and QR; TU is the Seat of the Line RS, or of the Plane TURS; and conſequently, PQUT is the Seat of the Plane PQRS, on KPQ. Alſo, turs is the Seat of that part which is over the Ground Plane.

I would adviſe the young Student, who is not well verſed in theſe things, to make them familiar to him; for which, the reading over a ſecond time, with due attention to the Figures, will be ſufficient.

I have been more particular on this Subject, becauſe I have frequently known Pupils to be miſlead, by calling Lines and Planes perpendicular, imagining them to be really ſo, i. e. to the Horizon; from the common acceptation of the Term, Perpendicular, (to hang down, as a plumb Line) not conſidering the Poſition of that Plane or Line, to which, the other Planes or Lines are ſaid to be perpendicular; and to which, due attention muſt firſt be given.

9. Suppoſe the Object AIKC to be, in the lower part, a right angled Paral- Fig. 13.
lelopiped (the moſt general form for Buildings, or the ſeveral Parts of a Building) the Planes BEDC, and AIB, of the Front and End, and their oppoſites, are vertical; for they are perpendicular to the Horizon, or Ground (conſidered as a Plane) on which it ſtands.

Now, the Lines AB and FE, in the Plane AIB, are perpendicular to the Plane BEDC; and the Lines BC, ED, and IK, are all perpendicular to the Plane AIB (i. e. the originals of thoſe Lines are ſo, in the real Object) yet, they are all parallel to the Horizon, and the three laſt, parallel between themſelves, though in different Planes; § for, a Plane may paſs through any two Lines which are parallel. (Ax. 5.) § 4. 7. El.

10. In the Practice of Perſpective, it is often neceſſary to ſuppoſe the Object, we are delineating, tranſparent, as if the whole Object was Glaſs; and, the Planes or parts of the Building, which are adjacent, are ſuppoſed to be ſeen through the hither Planes; as ABCH, the ground Plane, and FEDG parallel to it; AFGH parallel (or otherwiſe) to BEDC, and HGDC oppoſite to AFEB.

Theſe ſix Planes, forming a right angled Parallelopiped, compoſe the Body of the Building; every Angle of which, A, B, E, D, &c. is a ſolid Right Angle; each being compoſed of three plane Right Angles, as ABC, ABE, and EBC, of the Angle B; conſequently, AB, BC, and BE are each perpendicular to the other.

By means of this ſuppoſed tranſparency, the connection of the ſeveral parts of an Object are diſtinctly ſeen, and delineated with greater accuracy; which, in ſome Caſes, could not, without that expedient, be ſo certainly aſcertained.

11. The Roof, FIKDGE, is a triangular Priſm, in its conſtruction. The Planes, EIKD on this ſide, and FIKG on the other ſide, are inclined to the Horizon; yet, theſe Planes may be perpendicular to each other, if the Angles, FIE, GKD, are Right. (See Art. 5.)

They are alſo inclined to the Front and its oppoſite, which are vertical Planes; but obſerve, that, their Inclination is not the internal Angle IEB, or IFA, for thoſe Angles are obtuſe; but, if either of the Planes be produced, as BEDC to LM, there is made an acute Angle (IEL) with that Plane, equal to the Complement of IEB to two Right Angles; which, is the Angle of Inclination of thoſe Planes.

The inclined Planes, of the Roof, as well as the horizontal and vertical Planes AC, EC, &c. are perpendicular to the Plane AIB, and to HCK, its oppoſite.

This, I hope, is ſufficient to make all I have ſaid, relative to Planes and Lines (in reſpect of their poſitions, and affinity to each other) clearly underſtood; if ſo, it is a very material point gained towards underſtanding Perſpective clearly; which will be found a moſt neceſſary Introduction to the Theory, and will, alſo, greatly facilitate the Practice.

Plate IV.

SECTION II.

Of the ſeveral kinds of PROJECTION, and other introductory Matters.

PROJECTION is the deſcription or delineation of Objects in Plano, or on a Plane, according to a certain Law; by means of Right Lines, called Rays, ſuppoſed to be drawn from every Angle of the Object to ſome Plane.

Fig. 14. As A a, B b, C c, &c. which, taken altogether, are called the SYSTEM OF RAYS.

When thoſe Rays are all united in a Point (as A O, B O, &c.) it is called a CONE or PYRAMID OF RAYS; and that Point (O) being ſuppoſed an Eye, and the Right Lines O A, O B, &c. Viſual Rays, the Syſtem of Rays is then called, the OPTIC CONE, or PYRAMID (See the 4th Definition, in Optics; Book I. Page 9.)

The Figure deſcribed, as a e f b, on the Plane V, a b c e, on the Plane Z, or, a b d e, on the Plane X, is the Projection of the Object A B C.

N. B. When the Object to be delineated is circular or globular, the Syſtem of Rays is then properly called a Cone, for it is really ſo; but when the Object is right lined, it is more properly called a Pyramid, as in Fig. 6. Plate 3; the Baſe, A, only, being conſidered as the Object, and a B, b B, &c. as Viſual Rays to the Eye, at B, in the Vertex of the Pyramid, a B d, formed by the Rays.

ICHNOGRAPHY, or Ichnographic Projection, is that which is deſcribed by Right Rines, parallel amongſt themſelves and perpendicular to the Horizon, from every Angle of the Object, on a Plane parallel to the Horizon.

Fig. 14. No. 1. As A a, B b, C c, &c. from the Angles A, B, C, &c. of the Object, to the Plane Z, which is horizontal. The points, a, b, c, d, &c. where the perpendicular Lines, or Rays, cut that Plane, being joined by Right Lines, a b, b c, c d, &c. and diagonal wiſe, as a c, b d, &c. is the Ichnographic Projection of the Object, A C B.

The Figure, a e c b, projected on the horizontal Plane Z, is likewiſe called the PLAN, or SEAT, of that Object on the Ground Plane. The Points a, b, c, &c. are the Seats of the Angles, A, B, C, of the Object. The Line a b is the Seat of the Side A B, and b c of B C, &c. alſo the Diagonal a c is the Seat of A C; by which means, the Plan of the whole Figure is formed.

ORTHOGRAPHY. If the projecting Rays are parallel to the Horizon, and fall perpendicularly on a Plane (conſequently vertical) the Figure deſcribed, on that Plane, by the interſection of the Rays, is the Orthographic Projection.

Fig. 14. No. 2. The Lines A a, B b, &c. are parallel to the Horizon and to each other; which, falling perpendicular on the vertical Plane V, deſcribe the Orthography of the Object A B C, on that Plane.

Of this kind of Projection there may be infinite variety; for, if either the Plane or the Object be turned, though ever ſo little, the Figure will be varied on it.

The Figure, a e f b, thus projected, may likewiſe be called the SEAT of the Object A B C on that Plane; as a e c b on the Ground Plane.

a is the Seat of the Angle A, and b of the Angle B, &c; a b is the Seat of the Side A B, a e of A E; and, the Diagonal a f of the Side A F.

a e may alſo be conſidered as the Projection, or Seat, of the Plane A E C, and a b of A B C; for, if they were produced, they would cut the Plane V in thoſe Lines.

N. B. The Seat of a Point, Line, or Plane, may be had on any Plane in whatever Poſition; by drawing Right Lines from each extreme of the Line, &c. perpendicular to the Plane.

b f on the Plane Z, and b f on the Plane V, are the Seats of the ſame Line B F, in the Object A B C; and ſo of the reſt.

Ortho-

Orthographic Projection is uſually called the ELEVATION. But, when it exhibits the End of a Building, or part of the End only, ſhewing the Projectures of the ſeveral parts, from the main Body of the Building, the Contour of the curves of Moulding, &c. it is called a PROFILE. And, when the Building, or other Object, is ſuppoſed to be cut by a vertical Plane, in any direction through the Building, the hither part being ſuppoſed to be removed, and the inſide expoſed to view, ſhewing the thickneſs, &c. of the Walls and Floors, the ſtructure of the Roof and proportion of the Timbers, &c. it is called, a SECTION.

All theſe different kinds of Projection are entirely geometrical, and ſuppoſes the Eye of the Spectator at an infinite Diſtance; the projecting Rays being parallel.

SCENOGRAPHY is the Projection made by a Cone or Pyramid of Rays, Right Lines, from every Angle of the Object, converging to a Point. As O A, OB, &c. Fig. 14. No. 3.

Thoſe Rays being cut by a Plane (X) paſſing, in any direction, between the Object and the Vertex (O) the Figure projected, by the Rays, on that Plane, is called the Scenographic Projection of the Object A B C.

If the Vertex, (O) of this Pyramid of Rays, be conſidered as the Eye of a Spectator, and the Right Lines O A, OB, &c. as Viſual Rays, the optic Pyramid being cut by the Plane X, conſidered as a Picture, the Figure, a b d e, projected thereon, is the PERSPECTIVE REPRESENTATION of the Object A B C.

STEREOGRAPHY comprehends the whole Art of repreſenting Solids * on a Plane; which differs from geometrical Projection, in what was explained in the preceding Definition; that, the projecting Rays, ſuppoſed to proceed from every part of the Object, terminate in a point, at any diſtance from the Object; which Point, is always conſidered as the Eye of a Spectator, and the interſecting Plane as the Picture. The ſection of the Rays is the Repreſentation.

By this kind of Projection, the ſeveral dimenſions of Bodies, *viz.* length, breadth and thickneſs, are all repreſented at one View, in the greateſt degree of perfection that Art is capable of, or Nature exhibits.

As the Stereographic Projection (of Objects) is various, I ſhall explain it, more fully, under the three ſeveral Heads, Perſpective, Projected Perſpective or Projection, and Tranſprojection. And firſt, of PERSPECTIVE, which is, more immediately, the Subject of the following Treatiſe; that part of Stereography being more particularly adapted to the purpoſe of delineating all kinds of regular Objects, than either of the other; which are, therefore, but ſeldom practiſed; and indeed, becauſe theyare but little underſtood, or the difference between them known; yet, they have their ſeveral uſes in delineating, particularly in projecting ſhadows.

PERSPECTIVE is a Science founded on Geometry; which teaches how to delineate, or draw, on a plane Superficies (or, ſimply, on a Plane) the true repreſentations of Objects, according to their Diſtance and Situation, and Bearings of the Objects to each other; from any Station, at pleaſure, real or imaginary.

To give a clear and perfect Idea of the Principles on which the Science of Perſpective is built, it is neceſſary that the Objects, to be repreſented, are conſidered as being beyond, or on the other ſide of the Picture; that is, having fixed on the Station, from which you intend to delineate an Object, imagine a tranſparent Plane interpoſed between the Eye and the Object, or Objects; through which you have diſtinct Viſion, of the Objects on the other Side.

It is evident, to all who have conſidered it, that if, with a ſteady hand, you trace every Line, of Objects, accurately, as they appear on the tranſparent Plane (which may be ſuppoſed a Window, or a perfectly regular Plate of Glaſs) keeping the Eye fixed in a Point, there will be true, linear perſpective, Repreſentation of all the Objects, on that Plane; which, is conſidered as the Picture. Now, the performance of this, by geometrical Rules, is what is properly called PERSPECTIVE.

* Every Object, having length, breadth, and thickneſs or depth, comes under the Denomination, geometrically, of a Solid; the concavity, (as of a Box, or Houſe, &c.) not being conſidered; but only the external Form, conſiſting of Planes or other Surfaces, variouſly diſpoſed.

Plate V.

N.B. Every Reference, to Figure 15, refers likewiſe to the Apparatus.

Fig. 15.

Let B F I K L be an Object having the three Dimenſions, length, breadth, and thickneſs; which may be ſuppoſed a Building, or what you pleaſe, to be repreſented on the Plane, or Picture, M N O P, ſtanding perpendicular on the Ground Plane, S R Z; whoſe Interſection with it, (M P) is at right Angles with the line of Station, (S L.) The Picture, M N O P, is, therefore, direct, between the Object and the Spectator, E S. E A, E B, E F, &c. may be conſidered as Viſual Rays, in which the viſion of the Object, B F I L, is conveyed to the Eye, at E.

2. It is obvious, that if this Plane, or Picture, be tranſparent, the Repreſentation, a i f c, on that Plane, of the Object, B F I L, on the other ſide, would, by means of the Right Lines E A, E I, E F, &c. from each Angle of the Object to the Eye, exactly coincide, and agree in every part, with the original Object.

For, as it is not poſſible for Viſion to be conveyed but in Right Lines to the Eye (except through denſe, refracting Mediums) the Angle A, of the Object, muſt neceſſarily appear at a, on the Picture, B will appear at b, and F at f, &c. where the Right Lines, from the Angles of the Object to the Eye, paſs through the Picture; and is the ſuppoſed reaſon for the geneſis of a Point on the Picture, in Theory. The Right Line a b, or b g, on the Picture, joining the repreſentations of the Angles, or Points, A and B, or B and G, in the Object, will alſo coincide with, and hide the Original Line, A B, or B G, from the Eye, at E; and is, therefore, its Repreſentation; and ſo of all the other Lines on the Picture.

This, I think, needs no other Demonſtration, for it is ocular, and evident, that the Angle, made by the Pyramid of Rays forming a ſolid Angle (A E F, F E C) at E, is, not only equal, but, the ſame, under which, both the Object and its Repreſentation are ſeen.

So likewiſe, the repreſentation, f i or f d, on the Picture, of any Line, F I or F D, in the Object, is ſeen under the ſame Plane Angle, I E F or F E D. Conſequently, f i coincides with F I, and f d with F D; the repreſentations, a b g h of the Plane A B G H, f g h i of F G H I, and b f c of the Plane B F C, coincide with each other, reſpectively; and conſequently, the whole Repreſentation, a i f c, or Projection of the Object, A I F C, perfectly coincides with the Original, in the Point of View E, in that Poſition and Situation of the Picture and Object.

Wherefore, ſince the Eye is affected, in the ſame manner, by the Lines and Angles on the Picture, as by the correſponding Lines and Angles of the Object, it is evident, that, if the Picture had the ſame degree of Light and Shade, and alſo the true teint of Colour, as the Object, it would be impoſſible for the Eye, at E, to diſtinguiſh whether it was a Picture, delineated on the Plane M N O P, or the real Object, on the others ſide, that was perceived.

3. Hence it is manifeſt, that there is, and may be, great deception in Viſion; and alſo, that there may be various Repreſentations, of the ſame Object, from the ſame Station or Point of View; which, notwithſtanding their difference in figure and dimenſions, will have the ſame Appearance in the true Point of View.

For, if any other Plane, as M N *O P*, be placed between the Eye and the Object, not parallel to M N O P, the Repreſentation, of the ſame Object, projected on that Plane, by its interſection with the Viſual Rays, will not only be ſmaller, but, alſo, very different in Figure and Proportion. For, having drawn the Right Lines S A, S B, S C, from the Foot of the Spectator, at S, to each angle of the Object, towards the Picture, on the Ground Plane, they will determine the extreme width of the repreſentation of that Object on each Picture; and alſo the proportion, of the repreſentation of the Plane A G to that of G C, which is, as k l to l m, on the Picture M N O P, where the lines S A, S B, and S C cut the bottom edge of the Picture; and, on M N *O P*, the Proportion is as n o to o p; but, M *P* is not parallel to M P; conſequently, the proportion, of n o to o p, is not as k l to l m.

If the Picture was placed on M Q, it is obvious that the Repreſentation would be ſtill leſs, and alſo different in its Figure and Proportion; as the parts Q r, r q, intercepted between the lines S A, S B, and S C, ſufficiently evince.

Thus, may there be as many different Repreſentations, of the ſame Object, as you pleaſe, and from the ſame Point of View; from the different Poſition and Diſtance of the Picture; all which, will affect the Eye alike, at the Point E, in the Vertex of the Optic Cone or Pyramid of Rays.

PROJECTED PERSPECTIVE, or PROJECTION.

If the Viſual Rays E B, E C, E D, &c. are ſuppoſed to be produced, or projected beyond the Object, A B C D (a quadrangular Pyramid) and there fall on, or are cut by a Plane (V) the Repreſentation (a b d c) of the Object, projected on that Plane, is the Projection of the Object A B C. Fig. 14. No. 4.

Projected Perſpective is the very ſame, except in the operation, as common Perſpective, i. e. when the Rays are cut by a Plane paſſing between the Object and the Eye, with only this difference, that, in common Perſpective, the Repreſentation is always leſs than the Object; becauſe the ſection of the Rays, by the Plane X, is on this ſide, towards the Eye; in projected Perſpective, the Repreſentation muſt neceſſarily be larger than the Object, becauſe the Plane of the Section (V) is beyond the Object; but, if the Planes are parallel between themſelves, whether the Rays are cut on this or on the other ſide of the Object, or both, the Repreſentations will be perfectly ſimilar. No. 5. No. 4.

a, b, c, and d are the projective Repreſentations of the ſeveral Angles of the Pyramid, A, B, C, D; which, joined by Right Lines, is the Projection of that Object on the Plane V.

F G H I is the Plan or Seat of the Object A B C D, on the Ground Plane, to which the Baſe, A C d D, is parallel.

By means of the Seat and the Station Point, S, the Repreſentation, a b c d, on the Plane V, may be projected.

The difference between Perſpective and Projection is very obvious.

In Perſpective, the Object, to be repreſented, is always ſuppoſed to be beyond the Picture; in Projection, the Picture is beyond the Object; which is projected, or ſuppoſed to be thrown forward to the Picture, and which is full as rational to ſuppoſe. Nor is there occaſion, in this Caſe, to ſuppoſe the Picture tranſparent, as in the former, when the Picture is ſuppoſed to be on this ſide of the Object, and ſeen through; which indicates the meaning of the term Perſpective.

The difference in the operation is very little, and is illuſtrated by frequent uſe in the practical part of this Work.

TRANSPROJECTION. If the Viſual Rays, from the Object (A B C D) are ſuppoſed to paſs through the Eye (at E) forming an oppoſite Pyramid of Rays (a E b) and there fall on, or are cut by a Plane (Y) the Figure (a c b d) projected on that plane, by its interſection with the Rays, is the Tranſprojection of the Object (A B C D). Fig. 14. No. 6.

It is evident, that, in this kind of Projection, the Repreſentation may be either larger or ſmaller, or equal to the Object, as the Plane, Y, is removed farther from or nearer to the point E; or, as the point E is removed nearer to, or further from the Object. For, if the point E be in the middle, between the Object and the Plane of the Section, the Repreſentation will be equal to the Object; and, whether it be nearer to, or further from the Object, the Repreſentation will have that Proportion, to the Original, as their Diſtances from the point E.

It is alſo evident, that Projections of this kind muſt neceſſarily be inverted; as the Rays all paſs through one common Point (a is the tranſprojected place of the Angle A, on the Plane Y, c of C, and b of the Vertex, B, of the Pyramid) in the ſame manner as optical Philoſophers endeavour to account for Viſion; by ſuppoſing an Image, of every Object perceived, formed in the back part of the Eye,

Plate IV. on the Retina, in the ſame inverted Poſition. The Eye being in this reſpect like a *Camera Obſcura*, which is a kind of artificial Eye; in which, the Picture is always inverted. (See Page 10, 1ſt and 2nd Par. Alſo, ſee Fig. 13, Plate II.)

Notwithſtanding, if the Plane Y, of the tranſprojective Picture, be parallel to either the perſpective, (X) or the projective, (V) the Repreſentation thereon will be ſimilar to the other, or to both, if they are all parallel amongſt themſelves.

An ORIGINAL OBJECT is any Object whatever, which is the Subject of the Picture we are delineating.

Fig 15. BFIL is an original Object, of which, the Projection, aifc, on the Plane MNOP, or *aifc*, on the Plane MNO*P*, are Repreſentations.

Alſo, parts of an entire Object, as a Column, a Chimney, &c. are the Originals of their ſeparate Repreſentations.

By ORIGINAL PLANE is meant, not only the Ground, or other Planes upon which Objects are ſeated, but alſo, the plane Surfaces of original Objects to be delineated.

ABGH and BFC. &c. in the Original Object BFIL, are Original Planes, as well as the Ground Plane (Z) on which it ſtands.

It may be neceſſary to make a diſtinction between Figures and Objects, although every Plane Figure may be called an Object; but, I think, that Term is more properly, applicable to Solids than to Plane Figures.

By ORIGINAL FIGURE, I ſhall therefore mean, only, Figures in Original Planes, as Doors, Windows, &c. in Original Objects; or the Figure of the Original Plane itſelf. As FGHI, or BGFDC, &c.

Any geometrical Plane Figure whatever, in Original Objects, is, therefore, an ORIGINAL FIGURE.

ORIGINAL LINE is any Line in an Original Object, whether Right Line or curved; as the bounds or limits of all Original Planes, or Figures, are Lines.

AB, BG, FG, FD, &c. are Original Lines, in the Original Planes ABGH and BFC, each, of which, is likewiſe in another Plane; for, each is the common Interſection of two Planes; conſequently, each Line is in two Planes.

By Th. 1.7. El. the common Interſection of two Planes is a Right Line, 3. 11. Euc.

AB is the common Interſection of the Ground Plane and the Plane ABGH, and is, therefore, in both Planes; and, BG is, for the ſame reaſon, in both the Planes ABGH and BFC; and ſo of all the reſt.

The Interſection of two Lines is a Point; and the extremes of Lines are alſo Points; wherefore, the Angles F, G, H, &c. of the Object BFIL, are Points; for they are the Interſections, and alſo the extremes of Lines; and are called ORIGINAL POINTS.

All Objects, whatever, that are formed by Art, as Buildings of all kinds, or other regular Objects, (which are the fiteſt ſubject for Perſpective) are compoſed either of Planes or of curved Surfaces, or both. Every Building and parts of Buildings come under the denomination of ſome one or other geometrical Solid; as Parallelopiped, Priſm, Pyramid, Cylinder, Cone, or Sphere; or they are compounded of ſeveral, together. For, the Body of the Building is either one Parallelopiped, or it is compoſed of ſeveral, variouſly diſpoſed, at the diſcretion of the Architect. The Roofs are, generally, either triangular Priſms, or Pyramids. The Planes, of which Roofs are chiefly compoſed, are either Triangles, Parallelograms, or Trapezia. The Planes of the Fronts and Ends of a Building are, for the moſt part, Rectangles; ſometimes Pentagons (as BGFDC) or other Poligons; which contain other geometrical Figures, variouſly diſpoſed, as Doors, Windows, &c.

which are generally Rectangles, ſometimes Circles, Ellipſes, or mixed Figures.

Temples, in Gardens, Cupolas, &c. are either cylindrical, or poligonal Priſms; the Roofs, of which, are either pyramidal, or ſpherical, or mix'd curved Surfaces.

Columns approach nearly to Cylinders; in the lower part they are perfectly ſo.

Thus, may every part of a Building, or other regular Object, be reduced to ſome geometrical Solid or other. Solids are compoſed of Planes or other Surfaces, all which, may again be reduced to their firſt Principle, Lines; for, as the boundaries of Solids are Planes, or other Surfaces, ſo the bounds of Planes, &c. are Lines.† Alſo, the Figures in Planes, whether Doors, Windows, or other Figures, are formed of Lines; all which, are ORIGINAL LINES.

† Def. 55, Geo.

So likewiſe, Mouldings, Steps, &c. are repreſented by Lines; which, in the Originals, are generated either by Planes, only, as ſtreight Steps, or by Planes and curved Surfaces, as in Mouldings and circular Steps. Each Moulding, if it be right lined, is compoſed of Planes and cylindrical Surfaces; which, by their parallel interſections, generate Right Lines. Circular Mouldings, in Cornices, &c. are compoſed of Planes, with cylindrical and other curved Surfaces; which, by their regular interſections, generate circular Lines.

The Edges of Columns and other cylindrical Objects, which are repreſented by Lines, on the Picture, have no real exiſtence in the Originals, but are only apparent; for there is no real Line; as it is but one continued Surface, which returns again into itſelf, without cuting or interſecting, by which Lines are generated. The Lines which form the repreſentations of the Baſes, &c. of Columns are, in the Originals, ſome of them real and ſome only apparent; as the Contour of the curve of the Torus, &c. and have various forms, on the Picture, according to the ſituation of the Eye, or Picture.

Having thus reduced compound Original Objects into their firſt Principles or Elements, *viz.* Planes and Lines; the next thing, to be conſidered, is the Poſition and Situation of thoſe Planes and Lines, in reſpect of the Picture and of each other; which being premiſed and well conſidered, the whole myſtery of linear Perſpective will be found compriſed in a ſmall Compaſs, both in Theory and Practice. The Principles on which the Theory is built are few, but they are general, and applicable in all poſitions of the Picture and ſituation of the Object, or of the Eye. Wherefore, the Diſtance and Situation of the Object being determined; that is, the Station being fixed, from which an Object is to be delineated, and the Poſition of the Picture determined, the Repreſentation is alſo determinable; which, Repreſentation, is in proportion to the Diſtance of the Picture.

Thus far I have proceeded, by way of Introduction. I have called it an Introduction to Perſpective, becauſe, it cannot be called a part or branch of that Science; ſeeing, all which it contains may be, and is, known to ſeveral, who are not acquainted with one Theorem, or any Rules for the practice of Perſpective. Nevertheleſs, as this Work is intended to be a perfect Tutor for young Students, I am well convinced, that, the knowledge they will acquire by this Introduction is by no means to be diſpenſed with; being as eſſential to be previouſly known, as the Definitions of the Terms uſed in Perſpective; which are elementary. It is certainly poſſible, by Rules laid down, for a Perſon to practice Perſpective without knowing what Perſpective means; but that is not compatible with my Deſign, having entitled this Work a Compleat Treatiſe, which could not poſſibly be, without accounting for the Rules given; which I ſhall do, as briefly as is conſiſtent with the Subject, not dwelling on any unneceſſary part of the Science; but certainly, every neceſſary knowledge, previous thereto (which being, perhaps, no part of any other diſtinct Science) ſhould firſt be inculcated.

SECTION

Plate IV.

SECTION III.

Containing the ELEMENTS of PERSPECTIVE.

IN order to inveſtigate the Theory of Perſpective, with clearneſs and preciſion, it is neceſſary to have recourſe to certain imaginary Planes, which may be conceived to paſs through the Eye of a Spectator, or Point of View, from which an Object is ſuppoſed to be delineated, in all Poſitions as occaſions require.

The Picture, as it has already been obſerved (under the Article Perſpective) is ſuppoſed to be between the Eye and the Object.

Three of thoſe imaginary Planes, together with the Picture and any Original Plane whatever, are ſuppoſed to be conſtructed as they are repreſented in Plate IV, Fig. 16, 17, 18, 19, 20 and 21. The Planes, thus conſtructed, I ſhall firſt define; afterwards, the Lines generated by their Interſections; and laſtly, the Points produced by the interſections of the Lines; all, which, are ſo eſſentially neceſſary, that, in ſhort, without the aſſiſtance of theſe five fundamental Planes, and the Lines and Points generated by their Interſections with each other, Perſpective would be a very intricate and perplexed Study; and, in Theory, a moſt imperfect Science.

I would particularly advertiſe young Students, not to pay the leaſt regard to the general poſitions of the Planes I am about to define, but only their poſitions in reſpect of each other; for which reaſon I have given them in various Poſitions, and adviſe the Reader not to give particular attention to the firſt. For, in the Theory of Perſpective, the poſition they have to the Horizon is not conſidered, at all; as the Theorems are general, and applicable in all poſitions and ſituations of the Picture, whatever; ſince (as Dr. Brook Taylor, in the Preface to his ſecond Treatiſe, juſtly obſerves) all Planes, ſimply as Planes, are alike in Geometry, and have the ſame properties however ſituated.

DEFINITIONS.

Fig. 16. 17, 18, 19, 20 and 21.

I. Let ABGH be conſidered as a part, or a continuation of an ORIGINAL PLANE; being ſuppoſed to be produced (if neceſſary) from any Original Object to the Picture; and, till it cuts a Plane, paſſing through the Eye of a Spectator, parallel to the Picture.

DEFINITION II.

The PLANE of the PICTURE is conſidered as the Board, Paper, or Canvas, on which is to be delineated the repreſentation of ſome Original Object, or Plane Figure. As ABLM, or the Plane X, is the Picture.

The PICTURE is, generally, and for the moſt part, vertical, and direct between the Eye and the Object to be delineated; as in Fig. 16, 18 and 19; but may be in any Poſition whatever; as in Fig. 17, 20 and 21.

DEFINITION III.

VANISHING PLANE. If a Plane be imagined to paſs through the Eye, parallel to any Original Plane, it is called the VANISHING PLANE of that Original Plane; or, ſimply, the PARALLEL of the Original Plane.

As V, or IKLM, parallel to Z or ABGH, Fig. 16, 17, &c. E is the Eye.

And, a Perpendicular (EC) from the Eye to its Interſection with the Picture, is the RADIAL of the Vaniſhing Plane.

DEFINITION IV.

The DIRECTING PLANE is an imaginary Plane, ſuppoſed to paſs through the Eye of a Spectator, parallel to the Picture.

G H I K, or the Plane Y, parallel to the Picture (X) is the Directing Plane; the Eye of a Spectator being ſuppoſed at E, in the Directing Plane.

N. B. The Diſtance of the Directing Plane, from the Picture, is always equal to the Diſtance of the Eye (as E C) and being parallel to the Picture, it makes equal Angles with the Original Plane, as the Picture. (I H A equal M A N).

DEFINITION V.

The VERTICAL PLANE (E C D F) is ſuppoſed to paſs through the Eye, perpendicular to the Original Plane and to the Picture. Wherefore, it cuts all the four preceding Planes at right Angles, in all poſitions of the Picture whatever.

DEFINITION VI.

RADIAL PLANE is an imaginary Plane, paſſing through the Eye and any original Right Line, whatever.

As E V *I D*, or E *C P D*; which, being produced, would paſs through the original Line O N, or Q P. Fig. 20.

In Fig. 16. the Original Plane (A G) is ſuppoſed horizontal, and the Picture (A L) vertical; conſequently, the Varniſhing Plane (M K) and Directing Plane (K H) have the ſame Poſitions, and cut each other at right Angles.

In Fig. 17. ſuppoſe the Picture (A B L M) and the Directing Plane (H I K G) in the ſame vertical Poſition; and ſuppoſe the four Planes, *viz.* the Original Plane, the Picture, the Vaniſhing and the Directing Planes, ſo fixed together, as to be moveable on their Interſections, A B, G H, I K, and L M, as on hinges; and then, let us ſuppoſe, the Picture and Directing Plane (and along with them the Vaniſhing Plane) puſhed into an inclined poſition, on either ſide of the original, vertical Poſition; the Interſections, A B and G H, remaining as they were, unmoved.

It is evident, that the paralleliſm of the Planes is not deſtroyed by this motion; for, the Vaniſhing Plane (I K k M) is ſtill horizontal as before, though removed lower, to i k l m; and, the Directing Plane, H i k G or H *I K* G, is ſtill parallel to the Picture, A m l B or A *M* B, both being inclined to the Original Plane and its Vaniſhing Plane, in equal Angles.

In Fig. 18. the Picture (X) and the Directing Plane (Y) are ſtill vertical, but the Original Plane (Z) and its Vaniſhing Plane (V) are inclined to them, and to the Horizon; making equal Angles with each other, as in the former Caſe.

Nor is there underſtood to be any difference, in their Poſition, between this and the preceding Figure; for, either Plane (A L or H K) may be ſuppoſed the Picture, and the other the Directing Plane; by which means, it has both the Poſitions of the former; only, ſuppoſing the Original Plane horizontal, the Picture and Directing Plane are conſequently inclined.

In Fig. 19. both the Original Plane and Picture (N B H, and A B L M) are ſuppoſed vertical; therefore, the Vaniſhing and Directing Planes are alſo vertical; and the vertical Plane (C D F E) in this Caſe, is conſequently horizontal.

In Fig. 20. the Planes are all inclined to the Horizon, and alſo to each other; excepting the Vertical Plane; which always cuts the other four at right Angles, and is, therefore, perpendicular to them all.

This Figure I recommend, more particularly than any of the other four, to be contemplated by the young Student (although it is the ſame in all) in order to diveſt him, entirely, of partiality to any particular Poſition, reſpecting the Horizon.

In Fig. 21. The four primary Planes are all moveable, and may be put into all the Poſitions of the former; either at right Angles, as Fig. 16; or inclined, on either ſide, as in Fig. 17, in any Angle at pleaſure.

Plate IV. As theſe Planes are all marked with the ſame Characters, as in the five preceding Figures, it would be ſuperfluous to particularize them here; and, if a Plane be ſuppoſed to paſs through the four Points, E, C, D, and F, it will be vertical or perpendicular to them all, in every Poſition.

N. B. Any one of the four Planes may be the Original Plane, the oppoſite one is, conſequently, its Vaniſhing Plane; either of the other may be the Picture, and the oppoſite to it is the Directing Plane; E C D F is ſtill the Vertical Plane.

Of LINES, generated by the Interſections of the five elementary Planes.

DEFINITION VII.

INTERSECTION of the PICTURE, with an Original Plane, is the Line in which any Original Plane cuts the Picture; or, in which an Original Plane, being produced, would cut the Picture.

Fig. 16, 17, 18, 19, 20 and 21. AB; is the Interſection of the Picture ABLM. with the Original Plane ABGH.

DEFINITION VIII.

VANISHING LINE is a Line produced by the Interſection of an imaginary Plane, paſſing through the Eye parallel to any original Plane, with the Picture.

LM, the Interſection of the Vaniſhing Plane, I K L M or V, with the Picture A B L M or X, is the Vaniſhing Line of the Original Plane, A B G H or Z.

DEFINITION IX.

PARALLEL of the EYE is the Line I K, in which the Vaniſhing Plane (V) and the Directing Plane (Y) interſect each other.

As both theſe Planes are imagined to paſs through the Eye (at E) conſequently, their Interſection (I K) paſſes through the Eye; and, becauſe the Directing Plane is parallel to the Picture, the Parallel of the Eye is parallel to the Picture.

DEFINITION X.

DIRECTING LINE is the Line G H, in which, an Original Plane (Z) cuts, or would, if produced, cut the Directing Plane (Y).

DEFINITION XI.

Fig. 16, &c. The VERTICAL LINE is the Line C D, in which, the Vertical Plane (E C D F) cuts the Picture; at right Angles with the Vaniſhing Line and Interſection of the Original Plane.

DEFINITION XII.

The DIRECTOR of an Original Line. If an Original Line be produced till it cuts the Directing Plane, a Right Line paſſing through the Eye and that Point is the Director of the Original Line.

E *D* (Fig. 20 and 21) is the Director of the Line N O, being produced to *D*.

DEFINITION XIII.

VISUAL RAY. With optical writers, this Term ſignifies an imaginary Ray of Light; by which, Viſion is ſuppoſed to be conveyed from the Object to the Eye; therefore, in Perſpective, it is a Right Line imagined to be drawn from any Point, in an Object, to the Eye.

E A, E I, E F, &c. (Fig. 15.) and E N, E O, &c. (20 and 21.) are Viſual Rays.

DEFINITION XIV.

RADIAL LINE is the parallel of any Original Line, producing its Vaniſhing Point. (See Def. XXII.) As E V (Fig. 20, and 21) parallel to N O.

DEFI-

DEFINITION XV.

DIRECT RADIAL is a Right Line, from the Eye or Point of View, perpendicular to the Picture. As EC.

N. B. If the Original Plane be at right Angles with the Picture (as in Fig. 16.) the Direct Radial (EC) is the common Interſection of the Vaniſhing Plane, (IKLM) and the Vertical Plane (ECDF); and is, always, in the Vertical Plane.

Of POINTS, and their Diſtance from the Eye.

DEFINITION XVI.

The POINT of SIGHT is that Point where the Eye, of a Spectator, ought to be placed to look at a Picture; for, in that Point, only, can a perſpective Picture be ſeen perfectly.

E is the place of the Eye, or Point of View, to look on the Picture ABLM.

It is the Point where the three imaginary Planes, *viz.* the Vaniſhing, the Directing, and the Vertical Planes, interſect; conſequently, the Eye is in all the three. Or, it is the Point of Interſection between the Parallel of the Eye (IK) and the PRIME DIRECTOR (EF)

N. B. It is the Vertex of the optic Pyramid of Rays (EA, EB, EF, &c.) the only Point in which the Images, or Repreſentations (a i f c,) on the Planes or Pictures, MNOP or *OP*, can exhibit a true Appearance of the Original Object (BFIL) on the other ſide. Fig. 15.

DEFINITION XVII.

The CENTER of the PICTURE is the Point C, in which a perpendicular Line from the Eye, or Point of Sight, is cut by the Picture.

The Direct Radial (EC) being perpendicular to the Picture, is, therefore, in the Vertical Plane (ECDF) conſequently, the Center of the Picture, produced by the Perpendicular EC, is in the Vertical Line (CD) and, when the Original Plane is perpendicular to the Picture, it is the Interſection of the Vertical Line (CD) and its Vaniſhing Line (LM). Fig. 16.

DEFINITION XVIII.

DISTANCE of the PICTURE, or principal Diſtance, is the Direct Radial, or perpendicular Line (EC) from the Eye to the Picture, or to its Center.

N. B. The Center and Diſtance of the Picture are moſt eſſential, and ought to be well underſtood; for, except the Interſecting and Directing Points, all the reſt are dependant on them.

DEFINITION XIX.

CENTER of a VANISHING LINE is the Point where it is cut by a perpendicular Line from the Eye or Point of Sight.

EC (Fig. 15 and 16) or E*C* (Fig. 18, 19 and 20) being a Perpendicular from the Eye (E) to the Vaniſhing Line (LM) C or *C* is, therefore, its Center.

DEFINITION XX.

DISTANCE of a VANISHING LINE, is the Perpendicular, EC or E*C*, from theEye to its Center; the ſhorteſt Line that can be drawn to the Vaniſhing Line.

DEFINITION XXI.

POINT of INTERSECTION is that Point in which any Original Line (being produced) cuts the Picture; or the Plane of the Picture produced, if neceſſary. Fig. 20, and 21.

I is the Interſecting Point of the Line NO, produced to the Picture; *B* and *F* are the Interſecting Points, of the Lines AB and FI, on the Pictures MNOP. Fig. 15.

DEFINITION XXII.

VANISHING POINT. If a Line be drawn from the Eye, parallel to any original Right Line, the Point, where it cuts the Picture, is the Vaniſhing Point of that Original Line. (See Theo. 2. Sect. 3. Book 1.)

Plate IV. Fig. 20, and 21. NO is an Original Line, in the Original Plane HNBG; EV is a Line from the Eye, parallel to NO, cuting the Picture; V is, therefore, the Vaniſhing Point of NO, and EV is its RADIAL, or PARALLEL of the Original Line; conſequently it is in the Vaniſhing Plane, or parallel of that Plane the Original Line is in.

Fig. 15. EC or EV being parallel to the Original Lines AB, GH, FI, &c. C or V, where it cuts the Picture, is, therefore, the Vaniſhing Point of thoſe Lines.

N. B. The RADIAL EC, or EV, is the Diſtance of the Vaniſhing Point, C or V.

DEFINITION XXIII.

DIRECTING POINT is that Point in which any Original Line, being produced, would cut the Directing Plane.

Fig. 20. and 21. *D* is the Directing Point of the Original Line NO, or PQ, being produced to the Directing Plane (GHIK). It is the Interſection of the Original Line and the Directing Line, of the Plane the Original Line is in.

In Fig. 23. O, P, and Q are the Directing Points of the Lines CB, AB, and AD, where thoſe Lines cut the Directing Plane, *GIKH*, produced.

DEFINITION XXIV.

STATION POINT is the Point in which a perpendicular Line, from the Eye or Point of Sight, cuts the Ground, conſidered as a horizontal Plane; or any other horizontal Plane, on which an Object, to be delineated, is ſeated.

Fig. 15. S, is the real Point of Station, at the foot of a Spectator (ES.)

In Theory, it is that Point in which a Right Line, from the Eye, cutst he Directing Line, perpendicularly.

Fig. 16, 17, &c. As F, in the Directing Line GH; EF being, ſuppoſed, perpendicular to GH.

AXIOMS.

I. One part of a Right Line cannot be in any Plane, and another part of the Line out of that Plane, being produced.

II. Two Planes which are parallel cannot interſect.

III. The common Interſection of two Planes is a Right Line.

IV. The Interſection of two Lines is a Point.

V. Two Right Lines, that are parallel, are in the ſame Plane: i. e, a Plane may paſs through both Lines.

VI. Two Right Lines, meeting in a Point, or which, if produced, would interſect, may be in the ſame Plane.

VII. Two Right Lines, being parallel, and both cut by another Right Line, are all in the ſame Plane.

VIII. Three Right Lines, meeting or interſecting each other, are all in the ſame Plane; conſequently, every right lined Triangle is in a Plane.

Several of theſe Axioms are demonſtrable Propoſitions, in the eleventh Book of Euclid's Elements; but, having a true Idea of a Plane, they are ſo very evident, that it is but trifling, to little purpoſe, to attempt to prove them. Nevertheleſs, as the Student is ſuppoſed to underſtand Geometry, they may, without ſcruple, paſs for Axioms here.

SECTION

SECTION IV.

Containing the THEORY of Rectilinear PERSPECTIVE.

THEOREM I.

AN Original Plane, which is parallel to the Picture, has no Interſection with the Picture, nor Vaniſhing Line.

DEM. The Original Plane being parallel to the Picture cannot interſect it. Ax. 2.
And, a Plane paſſing through the Eye parallel to the Original Plane, which ſhould produce the Vaniſhing Line, by its Interſection with the Picture, is, in this Caſe, alſo parallel to the Picture, and the ſame with the Directing Plane; conſequently, it can never cut the Picture, and therefore, cannot produce a Vaniſhing Line. Q. E. D.

EX. Let MN*O*P be the Plane of the Picture, parallel to the Plane BFC, of the Original Object (BFIL.) Fig. 15.

Then, becauſe the Plane BFC is parallel to the Picture, it cannot cut the Picture (though produced infinitely) therefore can have no Interſection with it. Ax. 2.
And a Plane (RSTU) paſſing through the Eye (at E) parallel to the Original Plane (BFC) is alſo parallel to the Picture; therefore, it cannot cut the Picture and produce a Vaniſhing Line; (agreeable to Def. 8.) for, it is the Directing Plane of that Picture; which is parallel to the Picture. — Def. 4.

COR. *Original Lines, which are parallel to the Picture, have neither Vaniſhing Point, Interſecting, nor Directing Points.*

For, an Original Line, parallel to the Picture, is, or may be, in a Plane that is parallel to the Picture; and, a Line paſſing through the Eye parallel to the Original Line, which ſhould produce its Vaniſhing Point ‡, is, in this Caſe, alſo parallel to the Picture; and is, therefore, in the Directing Plane, which is parallel to the Picture. Wherefore, it can never cut the Picture and produce a Vaniſhing Point; or, it may be ſuppoſed at an infinite Diſtance. ‡ Def. 22.
Neither can the Original Line, for the ſame reaſon, cut the Picture or Directing Plane; therefore, it has neither Interſecting nor Directing Point.

EX. Becauſe the Plane BFC is parallel to the Picture, every Line in that Plane is alſo parallel to the Picture; therefore, BG, GD, &c.
Conſequently, ſince one part of a Right Line cannot be in any Plane, and another Part of it out of the Plane †, the Line BG, or GD, &c. can never cut the Picture and Directing Plane, and produce an Interſecting and Directing Point; agreeable to Definitions 21 and 23. † Ax. 1.
For the ſame reaſon, a Line paſſing through the Eye parallel to the Original Line (BG, GF, or GD, &c.) muſt neceſſarily be in the Directing Plane (RSTU) conſequently, it can never cut the Picture and produce a Vaniſhing Point; agreeable to Definition 22.

Plate IV.

THEOREM II.

The Vaniſhing Line, the Parallel of the Eye, and the Directing Line of any Original Plane, not parallel to the Picture, are parallel to the Interſection of that Plane with the Picture.

Fig. 16. The Conſtruction of the five elementary Planes, with their uſes and relation to each other (as explained or defined) 'tis preſumed, is well underſtood.

Suppoſe, then, the firſt four, *viz.* the Original Plane (Z) its Parallel, or Vaniſhing Plane (V) the Picture (X) and Directing Plane (Y) forming a right angled Parallelopiped; or that part of them, only, which lies between their Interſections (AB, ML, KI, and GH) with each other.

DEM. Now, the Picture and the Directing Plane are parallel, and they are cut by an Original Plane and its Vaniſhing Plane, which are alſo parallel. Def. 3 & 4.

But, if two Planes, being parallel, are cut by another Plane, their common Sections are parallel. - - - - P. 8. 7. El.

Conſequently, if two parallel Planes are cut by two parallel Planes, their Interſections are all parallel amongſt themſelves.

But, AB, the ſection of the Original Plane (Z) and the Picture (X) is the Interſection of the Picture. - - - - Def. 7.

ML, the ſection of the Picture (X) and the Vaniſhing Plane (V) is the Vaniſhing Line of the Original Plane (Z) - - - Def. 8.

KI, the ſection of the Vaniſhing Plane (V) and the Directing Plane (Y) is the Parallel of the Eye. - - - - - Def. 9.

And, GH, the ſection of the Directing Plane (Y) and the Original Plane, (Z) is the Directing Line. - - - - - Def. 10.

Therefore, the Vaniſhing Line (ML) the Directing Line (GH) and the Parallel of the Eye (KI) are all parallel amongſt themſelves, and to the Interſection (AB) of the Original Plane with the Picture. Q. E. D. 4. 7. El.

N. B. Whether the Original Plane be at right Angles, or otherwiſe, with the Picture, the Demonſtration holds good, in all poſitions whatever; ſince, the Directing Plane is always parallel to the Picture, and the Vaniſhing Plane to the Original Plane; conſequently, their Interſections with each other are ſtill parallel amongſt themſelves.

Fig. 21. To aſſiſt the Imagination, I have given a Figure with the four principal Planes, moveable; which may be raiſed up, to a Right Angle or any other, at pleaſure.

In every Poſition, it is evident, that the parallelism of the Planes, and conſequently of their Interſections, remains; and the diſtance of the Interſections between each two, which are in the ſame Plane, is invariable.

The Vertical Plane may be ſuppoſed to paſs through the middle, and perpendicular to them all; cuting the Vaniſhing and Directing Plane, in EC and EF; the Picture and Original Plane, in CD and DF.

COR. 1. *If any one of the four Lines be given or determined, and one Point, in any of the other, be alſo given or found, the whole Line is determined.*

For, it is a Right Line, drawn through that Point, parallel to the given Line; ſeeing, they are all parallel amongſt themſelves, by the Theorem.

COR. 2. *If the Original Plane paſſes through the Eye, its Interſection and Vaniſhing Line is the ſame; and, the Parallel of the Eye is alſo the Directing Line.*

For,

For, seeing that the Original Plane (U, or a b c d, being produced) passes through the Eye (E) there can be no other Plane pass through the Eye parallel to it; but, in this Case, the Original Plane and its Parallel coincide; consequently, its Intersection (L M) with the Picture (A L M B) is also its Vanishing Line. And, for the same reason, the Parallel of the Eye (I K) coincides with the Directing Line; being produced by the same Original Plane. In which case, no Figure in the Original Plane can be represented. Plate VI. Fig. 22.

For, as the Original Plane passes through the Eye, the whole Representation of that Plane, and every Figure in it, is its Intersection with the Picture; and is the reason why, Figures, in Horizontal Planes, which are on a level with the Eye, cannot be represented; for they are all in the Vanishing Line of horizontal Planes, and, therefore, cannot appear in the Picture.

THEOREM III.

All Original Planes, which are parallel amongst themselves, but not to the Picture, have the same Vanishing Line.

DEM. Vanishing Lines, are produced, in Theory, by imaginary Planes, passing through the Eye or Point of View, parallel to the Original Planes. Def. 8.

Wherefore, a Plane passing through the Eye parallel to any Original Plane, producing its Vanishing Line, is also parallel to every Plane which is parallel to the Original Plane; and since this parallel Plane can produce but one Line on the Picture, by its Intersection with it, that Line is, consequently, the Vanishing Line of all the Planes to which it is parallel. Q. E. D.

EX. V, U, and W, are three Original Planes, parallel amongst themselves; A L M B is the Picture, and E the Eye, in the Directing Plane (I K G H.) Fig. 22.

I K L M passing through the Eye at E, is a continuation of the Plane U; consequently, it is parallel to the Planes V and W; and, seeing that no other Plane can be drawn through the Eye parallel to them, therefore L M is the Vanishing Line of the three parallel Planes V, U and W. Def. 8.

This may be further illustrated by Fig. 15, and by the Apparatus.

Suppose a Plane (R S T U) to pass through the Eye (E) parallel to the Planes B F C and A I L in the Original Object B F I L, cuting the Picture, M N O P, produced, in R U; which is, therefore, the Vanishing Line of those parallel Planes; and, consequently, of all other Planes parallel to them. Plate V.

For the same reason, a Plane (T O P S) passing through the Eye, parallel to the Planes A B G H and C D K L, cuting the Picture M N O P in O P; and the Picture R U O P in O P; O P, or O P, is the Vanishing Line of those Planes, on each Picture, respectively.

SCHOL. Horizontal Planes have horizontal Vanishing Lines; except, the Picture be also horizontal; in which Case they have no Vanishing Line, the Picture being parallel to the Original Plane.

It is also evident, that all vertical Planes have, when the Picture is also vertical, perpendicular or vertical Vanishing Lines; i. e. the Vanishing Lines are, in such case, perpendicular Lines on the Picture, and at right angles with such as are horizontal.

For, 1st. The Intersection of a horizontal Plane, with any other Plane, is necessarily a horizontal Line, seeing it is produced by a horizontal Plane, in which the intersecting Line must always be.

2nd. The common Intersection of two vertical Planes, is a Right Line perpendicular to the Horizon; seeing that, the Line of section, is, in both Planes.

Plate V. COR. *All Original Lines which are parallel amongſt themſelves, and not to the Picture, have the ſame Vaniſhing Point.*

For, by the ſame reaſoning as in the Theorem, it is evident, ſince the Vaniſhing Point of an Original Line is produced, in Theory, by an imaginary Line paſſing through the Eye parallel to the Original Line †, it neceſſarily follows, that, a Line paſſing through the Eye, producing the Vaniſhing Point of any Original Line, by its Interſection with the Picture, being parallel to the Original Line, is alſo parallel to all Lines which are parallel to the Original Line; and, ſince it can produce but one Point on the Picture, it is, conſequently, the Vaniſhing Point of them all. Theor. 2. Book 1ſt.

† Def. 22.

Fig. 15. In the ſame Figure, the Original Lines A B, G H, and F I, are parallel amongſt themſelves; E C V is a Right Line from the Eye, parallel to them all, cuting the Picture R U O P in V, and M N *O P*, in C, its Center; the Points C and V, on each Picture reſpectively, are, therefore, the Vaniſhing Points of the Original Lines A B, G H, and F I.

The ſame may be ſaid in reſpect of the parallel Lines G F and H I, whoſe Vaniſhing Point, on the Picture R U O P, produced, is W.

Alſo, of G D and B C, whoſe Vaniſhing Point is Y, on the Picture R U O P; but, being parallel to M N *O P*, they have, conſequently, no Vaniſhing Point on that Picture, but are parallel to their Repreſentations.

N. B. It may be neceſſary to obſerve here, that, in this laſt, and in moſt of the Schemes or Diagrams made uſe of in this Theory, (being drawn perſpectively) whenever any two, or more, Lines are ſaid to be parallel, if they are not really ſo, they tend to the ſame Point, agreeable to Coroll. 1. For, I think it unpardonable, in a Treatiſe on Perſpective, to give an Appearance of Objects, or Schemes, not truly and perſpectively drawn; merely, for the ſake of preſerving the paralleliſm of certain Lines, as ſeveral Authors have done. Unluckily, for them, they cannot alſo preſerve the true Angles; which are worſe repreſented by that means.

Therefore, whenever Angles are ſaid to be Right, or otherwiſe; and ſuch or ſuch Lines, to be parallel or perpendicular; if they are not really ſo, they muſt be underſtood to be ſo; and, I am perſuaded, that they will always appear to be ſo, when truly delineated, perſpectively.

THEOREM IV.

The Vaniſhing Lines of all Planes, which are perpendicular to the Picture, paſs through the Center of the Picture.

DEM. For, ſince the Vaniſhing Line of a Plane is produced by an imaginary Plane paſſing through the Eye parallel to the Original Plane †, and the Center of the Picture is determined by a Perpendicular, from the Eye to the Picture §, the Original Plane being, in this Caſe, perpendicular to the Picture, it follows, that its Parallel or Vaniſhing Plane (producing the Vaniſhing Line) ſeeing it paſſes through the Eye, muſt, neceſſarily, paſs through the Direct Radial, which is perpendicular to the Picture ‡, and conſequently, through the Center of the Picture.

† Def. 8.

§ Def. 17.

‡ Def. 15.

Therefore, the Vaniſhing Line, produced by the Interſection of every ſuch Plane with the Picture, muſt paſs through the Center of the Picture. Q. E. D.

EX. 1. All horizontal Planes are perpendicular to a vertical Picture; wherefore, the Plane G H K D, and alſo the Ground Plane (Z) are perpendicular to both Pictures, M N O P or *O P*; their Vaniſhing Plane, therefore, paſſes through the Direct Radial (E C) of each Picture, and conſequently, through C, their Centers.

Fig. 15.

C X and Y V (on each Picture, reſpectively) parallel to thoſe Planes, drawn through the Centers (C) of both Pictures, are the Vaniſhing Lines of thoſe Planes. For, by Theo. 3d. the Vaniſhing Line of every Plane is parallel to its Interſection; and conſequently to the Original Plane.

EX. 2. The Planes AHGB and CDKL are perpendicular to MNO*P*, only; their Vaniſhing Plane (STO*P*) therefore, paſſes through its Radial (EC) and their Vaniſhing Line (O*P*) through the Center (C) of that Picture.

As theſe two Examples are particular Caſes; *viz.* when the Original Plane is either horizontal or vertical, I ſhall give another general Caſe, which will illuſtrate the Theorem univerſally. Raiſe up the Planes of No. 2. AONB is the Picture, GEH the Directing Plane, and E the Eye.

An Original Plane, *IKLM*, is moveable on the Line TU, its common Section with the Ground, or other horizontal Plane, at right Angles with AB, the Interſection of the Picture with the Plane Z. Fig. 15. No. 2.

It is evident that, as the Original Plane, *IKLM*, is turned (on TU) it is always perpendicular to the Plane or Picture (AONB)† for, in every poſition of the Plane, in that revolution, the Line TU is ſtill perpendicular to the Picture. † 9. 7. El.

Now, if the Plane IKLM, revolving, paſs through the Eye (at E) and, being all the while parallel to the Original Plane, it muſt revolve on EC, the Direct Radial, which is parallel to TU; and, in every Poſition, its Interſection with the Picture muſt neceſſarily paſs through C, the Center of the Picture; produced by the Perpendicular EC.

Let the Original Plane (*IKLM*) be raiſed, making the Angle PQR.

If the Plane IKLM, paſſing through the Eye, makes the ſame Angle with the Horizon, i. e. if it be parallel to the Plane *IKLM*, it will cut the Picture in ON, the Vaniſhing Line of that Plane, and paſs through C, the Center of the Picture; making the Angle, NCM, with LM, the Vaniſhing Line of horizontal Planes, equal to the Angle PQR, of the Original Plane with the Ground Plane.

After the ſame manner, the Vaniſhing Lines of all Planes, which are perpendicular to the Picture, may be aſcertained and drawn on the Picture; knowing the Angle which the Original Plane makes with any horizontal Plane; or with a vertical Plane, which is perpendicular to the Picture.

N.B. The Center of the Picture is the Center of all Vaniſhing Lines, of Planes perpendicular to the Picture; and they have all the ſame Radial and Diſtance (EC) by Def. 19. and 20.

COR. *Hence it is evident, that the Center of the Picture is the Vaniſhing Point of all Lines which are perpendicular to the Picture.*

For, EC, a perpendicular Line from the Eye, producing the Center of the Picture, is the common Radial of all Vaniſhing Planes, that are perpendicular to the Picture; and conſequently, parallel to all Lines that are perpendicular to the Picture (for, they are all parallel amongſt themſelves, and to the Direct Radial, EC) as AB, GH, FI, &c. Fig. 15.

Therefore, ſince EC is parallel to all ſuch Lines, the Point (C) which it produces on the Picture, is their Vaniſhing Point; by Def. 22.

But, C is the Center of the Picture; conſequently, the Center of the Picture is the Vaniſhing Point of all Lines that are perpendicular to the Picture.

In this ſingle and particular Vaniſhing Point, many Artiſts ſeem to reſt all their knowledge of Perſpective (commonly called the Point of Sight) and, whenever a number of Lines tend to the ſame Point, it is, by them, called a Point of Sight; not conſidering, or perhaps knowing, that the Center of the Picture is a Vaniſhing Point, in common with all other, only by virtue of the general Definition (Def. 22.) And, becauſe the knowledge of it is more general, and, as they imagine, the practice much eaſier, we find it more uſed than any other; as moſt regular Objects, particularly Buildings, are right-angled; ſo that, having one Side or Front parallel to the Picture, (as is uſually the caſe) all the horizontal Lines in the adjoining Sides are, conſequently, perpendicular to the Picture, and therefore vaniſh in its Center; which, from the certainty of the Poſition of ſuch Lines, is determined and fixed at pleaſure, though often very injudiciouſly.

Plate VI.

THEOREM V.

Original Planes, whoſe common Interſection is parallel to the Picture, have parallel Interſections with the Picture, and parallel Vaniſhing Lines.

DEM. The common Interſection of the Original Planes being parallel to the Picture, a Plane may be ſuppoſed to paſs through that Interſection which is alſo parallel to the Picture.

Now, the Original Planes being produced, through their common Section, to the Picture, each Plane will cut the Picture in a Line parallel to their common Section (8. 7. El.) Wherefore, ſince the Interſection of each Original Plane, with the Picture, is parallel to the common Section of both Planes, they are, conſequently, parallel between themſelves. Q. E. D. 4. 7. El.

2. But the Vaniſhing Line of each Original Plane is parallel to its Interſection with the Picture. - - - - - Theorem 3.

And the Interſections are parallel between themſelves. Proved.

Therefore the Vaniſhing Lines are parallel between themſelves.

Fig. 22. EX. a b c d and c d e f are two Original Planes, whoſe common Section (c d) is parallel to the Picture (A B L M); g h i is a Plane, ſuppoſed to paſs through their common Section, parallel to the Picture.

Now, the Plane a b c d being produced to the Picture, cuts it in L M; and the Plane c d e f, being produced, cuts it in *A B*; wherefore, ſince the Picture and the Plane g h i are parallel, and are both cut by the Planes *A* d c *B* and L a b M, the Interſections (*A B* and L M) are, each, parallel to the common Section, c d; and therefore they are parallel between themſelves.

EX. 2. *L M* is the Vaniſhing Line of the Plane cdef, and LM of the Planes V, U, & W; which are, reſpectively, parallel to the Interſections A B, J F, &c. (by Theo. 4.) and therefore they are parallel, alſo, between themſelves.

3. After what has been ſaid, it is manifeſt that the Directing Lines of Planes, in ſuch Caſe, are alſo parallel.

For, ſince their Interſections with the Picture are parallel, conſequently, if they were produced till they cut the Directing Plane, their Interſections with it would likewiſe be parallel; the Directing Plane being parallel to the Picture.

COR. 1. *The common Interſection of a Plane inclined to the Horizon, with any horizontal Plane whatever, being parallel to the Picture; the Vaniſhing Line, of that inclined Plane, is a Line parallel to the Horizon.*

For, becauſe the common Interſection is parallel to the Picture, the Vaniſhing Lines of both Planes are parallel to each other; by Theorem.

But, one of thoſe Planes is horizontal, therefore its Vaniſhing Line is parallel to the Horizon; by Def. 8.

Conſequently, the other Vaniſhing Line is, alſo, parallel to the Horizon.

COR. 2. *All Original Planes, that have parallel Vaniſhing Lines, have the ſame Vertical Plane; and, conſequently, the ſame Vertical Line; and, alſo, the ſame Parallel of the Eye.*

I K L M is the parallel of the Plane W, cuting the Picture in L M, the Vaniſhing Line of W; and, I K *L M*, is the Parallel, or Vaniſhing Plane of the

the Original Plane X, paſſing through the Eye (at E) and cuting the Picture in *L M*, the Vaniſhing Line of that Plane. Alſo, I K l m being parallel to the Horizon, its Section with the Picture (l m) is the Vaniſhing Line of horizontal Planes; to which, L M and *L M* are parallel, by the Theorem.

But, R S T is the Vertical Plane of all thoſe Original Planes, and of all other parallel to them; i. e. it cuts them all at right Angles, being perpendicular to their common Sections (c d, f g, &c.) and it cuts the Picture in C D the Vertical Line of them all; for the ſame Plane cannot cut the Picture in two Lines.

I K is alſo the Parallel of the Eye of all thoſe Planes; for, it is the common Section of all their Vaniſhing Planes with the Directing Plane.

This Theorem, and the laſt Corollary, may alſo be illuſtrated by Figure 15.

1. B G, the common Interſection of the two Planes A B G H and B F C, is parallel to both Pictures (M N O P or *O P*) their Interſections (*B E* and *B K*) with M N O P, are therefore parallel; alſo their Vaniſhing Lines, R U and O P, on that Picture. The other (M N O *P*) being parallel to the Plane B F C, has but one Vaniſhing Line (*O P*) which is parallel to the Interſections *B G* and MN. Fig. 15.

2. The two Planes, A B G H and B F C, being vertical, a Plane vertical to them, is, conſequently, horizontal; for, no other Plane can cut both, or either of them and the Picture, perpendicularly. (See Fig. 19.)

Wherefore, they have the ſame Vertical Plane; which, in reality, is horizontal; and, conſequently, they have alſo the ſame Vertical Line; which, in this Caſe, is the Vaniſhing Line of horizontal Planes; *viz.* C X or V Y, on either Picture, reſpectively.

N. B. It is the ſame in all Poſitions, whatever, of the Original Planes.

THEOREM VI.

The Planes, which produce the Vaniſhing Lines of two Original Planes, are inclined to each other in the ſame Angle as the Originals; and have their common Interſection, paſſing through the Eye, parallel to the common Interſection of the Original Planes.

DEM. For, firſt, ſince the Planes, producing the Vaniſhing Lines of any two Original Planes, are, reſpectively, parallel to the Originals, they have conſequently the ſame Inclination to each other, as the Originals.

For, if all the Planes are produced (*viz.* the Original Planes and their Parallels) they will cut each other in parallel Lines, and form a Parallelopiped, between their Interſections. - - - - 8. 7. El.

Therefore, they have the ſame Inclination to each other, reſpectively. 13. 7.

EX. U, or W, and X, are Original Planes, and K M, K *M* their Vaniſhing Planes; being parallel to them, reſpectively.

The Angle, which the Original Plane (X) makes with W, is d P N (P d and P N being both perpendicular to their common Section P Q) and it is equal to the Angle L K *L* which their Vaniſhing Planes (K M and K *M*) make with each other Fig. 22.

Becauſe K L is parallel to N P, and K *L* to d P, the Angle L K *L* is equal to d P N ‡.
Alſo, the Angle *L* K L is equal to L d P, which is equal to d P N †.
Therefore, the Planes, extended through thoſe Lines, being reſpectively parallel, are inclined to each other in the ſame Angle; i. e. K *M* has the ſame inclination to K M, as the Plane X has to U or W.

‡ 6. 1. El.
† 4. 1. El.

Plate V. Secondly. Becauſe, in this Caſe, the common Sections (c d, N O, &c.) of the Original Planes are parallel to the Picture, the Interſection (I K) of the Vaniſhing Planes, paſſing through the Eye (E) is alſo parallel to the Picture; and, conſequently, to the common Sections of the Original Planes.
For it is in the Directing Plane, which is parallel to the Picture; by Def. 4.

To illuſtrate this Theorem more familiarly, by moveable Planes.

Fig. 15. No. 2. 1. Raiſe up the Plane of the Picture A O N B, perpendicular, or otherwiſe, to the Horizon. The Directing Plane (G E H) will be parallel to it; and I K L M to the Plane of the Horizon.

No. 3. Then, if the Plane *A D F B* (any Original Plane) whoſe Interſection (*A B*) with the Horizon, or any horizontal Plane, is parallel to the Picture, be raiſed up, on *A B*, making any Angle (P Q R) with the Horizon, and, a Plane, (*I K O N*) paſs through the Eye (at E) parallel to that Original Plane; it is manifeſt, that this Vaniſhing Plane (*I K* O N) makes the ſame Angle with I K L M, as *A D F B* with the Horizon; and *I K* (the parallel of the Eye) the Interſection of the Vaniſhing Planes, is parallel to *A B*, the common Interſection of the Original Planes.

2. The Picture, &c. remaining in the ſame Poſition, perpendicular to the Ground Plane, and to the common Interſection (T U) of the Original Plane *I K L M*; making any Angle with the Horizon, at pleaſure.

No. 2. Then, the Vaniſhing Plane (I K L M) being placed parallel to the Original Plane *I K L M*, it is evident, that it will make the ſame Angle with the Horizon, or with a Plane paſſing through the Eye, parallel to the Horizon, and cuting the Picture in L M (the Vaniſhing Line of horizontal Planes, as the Original Plane (*I K L M*) makes with the Horizon.

Alſo, E C (the Direct Radial) the Interſection of thoſe Vaniſhing Planes, is parallel to T U, the common Interſection of the Original Planes.
For, they are both perpendicular to the Picture.

In Fig. 23. this Theorem is quite general, i. e. the Poſition, the Original Planes have to the Horizon, or to the Picture, is not regarded.

Fig. 23. X and Y are two Original Planes, both inclined to the Horizon, and to the Picture (I T U *N*) which is alſo inclined to the Horizon, towards the Eye (at E.) A B is the common Interſection of thoſe two Planes.

Then, if a Plane (I K L M) be ſuppoſed to paſs through the Eye (E) parallel to the Plane, X; and another Plane (*I K L M*) parallel to A B D, or Y, cuting the Picture in the Vaniſhing Lines, L M and *M L*, of thoſe Planes; they will be inclined to each other in equal Angles, reſpectively; i. e. the Vaniſhing Planes, as the Original Planes; and the common Section (E V) of the Vaniſhing Planes, be parallel to the common Section (A B) of the Original Planes.

COR. 1. *The Radials, producing the Vaniſhing Points of any two Original Lines, make the ſame Angle, at the Eye, as the Original Lines make with each other.*

For, they are reſpectively parallel to their Originals (Def. 22.) and, conſequently, they make equal Angles, reſpectively. - - 5. 7. El.

Fig. 15. EX. E V is the Radial or Parallel of A B, and E Y of C B.
V and Y are, therefore, the Vaniſhing Points of A B and C B; the Radials (E V E Y) being parallel, reſpectively, to the Originals (A B and C B) Def. 22.
Conſequently they make the ſame Angle at the Eye (E) as the Original Lines make with each other; *viz.* V E Y equal A B C. - - 5. 7. El.
But, A B C is a Right Angle; therefore, V E Y is a Right Angle.

This is alſo illuſtrated in Fig. 23, and, by changing Y for M, it is deſcribed in the foregoing; ſave only, that A B C is not a Right Angle, conſequently V E M is not a Right one, being equal to A B C.

SCHOL. If the Original Lines (not being in the ſame Plane) are ſo ſituated, that they croſs, or would croſs, each other, without cuting; then, the Angle, which the Radials of ſuch Lines make with each other, is equal to the Angle that would be made between the Originals, if either of them be moved, parallel to itſelf, in any direction, till it cuts the other.

COR. 2. *The Vaniſhing Point of the common Interſection, of two Original Planes, is the Point in which their Vaniſhing Lines interſect each other.*

For, the Interſection of the parallel Planes, producing the Vaniſhing Lines, paſſes through the Eye, parallel to the common Interſection of the Original Planes (by the Theorem) and conſequently, will cut the Picture in that Point where the Vaniſhing Lines interſect.

But, the Vaniſhing Point of a Line is that Point, in which, a parallel Line, from the Eye, cuts the Picture. - - - - - Def. 22.

Therefore, the Point of Interſection of the Vaniſhing Lines is the Vaniſhing Point of the common Interſection of the Original Planes.

EX. A B is the common Interſection of the Planes X and Y. M L is the Vaniſhing Line of the Plane X, and *M L* of the Plane Y; their interſecting Point (V) is, therefore, the Vaniſhing Point of A B; for E V is parallel to A B. Fig. 23.

The Viſual Rays E A, E B, together with the Line A B, form a plane Triangle, which cuts the Picture in a b (the Repreſentation of A B) whoſe Vaniſhing Point is V; for, it is the Section of a Radial Plane, paſſing through the Original Line and the Eye, with the Picture; which muſt neceſſarily paſs through E V, the Radial of A B †, and cut the Picture in J V. † Ax. 5.

M m is the Vaniſhing Line of the vertical Plane W, and M q of horizontal Planes; both which, cut M L, the Vaniſhing Line of the inclined Plane (X) in M; therefore, M is the Vaniſhing Point of C B, the common Interſection of the Planes W and X, and alſo, of a horizontal Plane (C B D.)

EX. 2. Let the triangular piece (in the Apparatus) be fixed to the direct Picture (of which, it is ſuppoſed to be a continuation) and draw the thread (at W) through the Eye of the Spectator; alſo, bring forward the folding part, into the ſame Plane.

Then, V Y is the Vaniſhing Line of horizontal Planes; R W, of the vertical Planes (B F C and A I L) and V W, of the inclined Plane F G H I. V, Y, and W, are, therefore, the Vaniſhing Points of the common Interſections of thoſe Planes, reſpectively; *viz.* V, of G H, the common Interſection of D G H K and F G H I; and alſo of A B G H, whoſe Vaniſhing Line is O P. Y is the Vaniſhing Point of the common Interſection of horizontal Planes, and of the vertical Planes, B F C and A I L; and, W is the Vaniſhing Point of G F and H I, the common Interſections of thoſe vertical Planes and the inclined Plane F G H I.

SCHOL. This Corollary, being ſo very eſſential in Practice, cannot be too much enforced; for, having found the Vaniſhing Point of a Line (in any Plane whatever) which is either the common Interſection of that Plane with ſome other adjoining Plane, or a Line parallel to it (as V) the Vaniſhing Line of that other Plane muſt, neceſſarily, paſs through that Vaniſhing Point; and, if the Poſition of the other Vaniſhing Line be known, its place on the Picture is determined (as O P) otherwiſe, it is neceſſary to have the Vaniſhing Point of ſome other Line in the ſame Plane (as W, of the Line F G) in order to determine the Vaniſhing Line (V W) of the Plane (F G H I).

Plate IV.

THEOREM VII.

A Right Line drawn from the Center of the Picture, to the Center of a Vaniſhing Line, is perpendicular to that Vaniſhing Line.

DEM. For, becauſe the Direct Radial, producing the Center of the Picture, is perpendicular to the Picture §; and, the Center of every Vaniſhing Line is produced by a Right Line from the Eye, perpendicular to the Vaniſhing Line ‡; conſequently, the Radial of the Vaniſhing Line, producing its Center, and the Direct Radial, together with the Line which joins the two Centers, form a Triangle, and are all in the ſame Plane † which Plane is perpendicular to the Picture. - - - - - P. 9. 7. El.

§ Def. 15.

‡ Def. 19.

† Ax. 8.

For, it paſſes through the Direct Radial, which is perpendicular to the Picture.

And, it is alſo perpendicular to the Vaniſhing Line; becauſe, it is perpendicular to the Picture, and alſo to the Plane which produced the Vaniſhing Line. - - - - - Cor. 1. 9. 7. El.

Wherefore, ſince the Radial of the Vaniſhing Line, and the Right Line joining the Center of the Picture and the Center of the Vaniſhing Line, are both drawn to the ſame Point in a Plane, at which Point the Vaniſhing Line is perpendicular to that Plane; conſequently, the Line, joining the Center of the Picture and the Center of the Vaniſhing Line, is perpendicular to the Vaniſhing Line. Q. E. D. - - - 2. 7. El.

Fig. 18. and 19.

EX. L M is the Vaniſhing Line of the Original Plane Z; *C* is its Center, and C the Center of the Picture. Draw C*C*.

I ſay, that the Line C*C* is perpendicular to the Vaniſhing Line L M.

By Def. 17. E C producing the Center of the Picture is perpendicular to the Picture; wherefore, it is in the Vertical Plane (E *C* D F.)

† Def. 5.

For, it is perpendicular to the Picture (X) and alſo to the Vaniſhing Plane (V) †.

Conſequently, L M, the common Section of the two Planes, V and X, is perpendicular to the Vertical Plane (E *C* D F.) - - 9. 7. El.

Wherefore, E*C*, the Interſection of that Plane and the Vaniſhing Plane (V) is perpendicular to L M. - - - - 2. 7. El.

§ Def. 19.

But, E*C* produced the Center (*C*) of the Vaniſhing Line L M §; and conſequently, *CC*, the Interſection of the Vertical Plane with the Picture, is, alſo, perpendicular to L M; for, E*C* and C*C* are two Lines, cuting each other, in the ſame Plane (E *C* D F) to which the Vaniſhing Line (L M) is perpendicular, at the Point (*C*) of their common Section.

COR. 1. *When the Original Plane is perpendicular to the Picture, the Center of the Picture is the Center of the Vaniſhing Line.*

Fig. 16.

For, a perpendicular (E C) from the Eye to the Vaniſhing Line (M L) producing its Center is the Direct Radial, which is perpendicular to the Picture; and, conſequently, produces the Center of the Picture; which is the Center of every Vaniſhing Line, of Planes perpendicular to the Picture. See Th. 4.

COR. 2. *The Diſtance of every Vaniſhing Line, which does not paſs through the Center of the Picture, is the Hypothenuſe of a Right angled Triangle, whoſe Catheti, or Legs, are the Diſtance of the Picture, and the Diſtance between the Center of the Picture and the Center of the Vaniſhing Line.*

Fig. 18. and 19.

For, E C is the Diſtance of the Picture, C*C* is the Diſtance between the two Centers; and, E*C*, the Hypothenuſe, of the Right angled Triangle E C *C*, is the Diſtance of the Vaniſhing Line L M; being the ſhorteſt Line that can be drawn from the Eye to the Vaniſhing Line. - Cor. 2. 12. 1. El.

To

To ſet this matter in the cleareſt light poſſible I have illuſtrated it by moveable Planes, in Fig. 21.; which, let be raiſed up, in any Angle at pleaſure.

Underneath the Vaniſhing Plane (I K L M) is a triangular Plane (E C *C*) which, being turned down, perpendicular to the Vaniſhing Plane, will repreſent a Part of the Vertical Plane, and determines the Center of the Picture, in a certain angle of inclination, inclined from the Eye, in the Angle E*C*C.

The Triangle E*C*C is right angled, at C; conſequently, E C is perpendicular to the Picture, and C is therefore its Center.† † Def. 17.

It is evident that *C*E and *C*C are both perpendicular to L M, or V M, the Vaniſhing Line of the Original Plane, N G H.

But, *C* is the Center of the Vaniſhing Line, and C is the Center of the Picture; therefore, a Right Line joining the Center of the Picture and the Center of a Vaniſhing Line is perpendicular to that Vaniſhing Line.

Above the Vaniſhing Plane is another Right angled Triangle; which being erected, perpendicular to the Vaniſhing Plane and the Picture, determines its Center, when the Picture is inclined towards the Eye, in the Angle E*C*C.

In which caſe, the Center of the Picture neceſſarily falls above the Vaniſhing Line; as in the former caſe it falls below.

Both theſe Poſitions are repreſented in the 17th Figure, and alſo in the 18th, conſidering either Plane, X or Y, as the Picture, and E or *C* the place of the Eye.

But, when the Picture and its Directing Plane are perpendicular to the Original Plane, as in the 16th, and the 17th Figure alſo ſhews the ſame, the Center of the Vaniſhing Line is the Center of the Picture.

N. B. In this Scheme, and ſome of the following Diagrams, the original Plane (N G H) muſt not be conſidered as neceſſarily being horizontal; for, the whole Theory, here given, is quite general, and applicable to all poſitions whatever, either of the Original Plane or of the Picture.

It may perhaps, to ſome, ſeem unneceſſary to dwell ſo long on this Propoſition, which, at firſt, may not appear of much conſequence; but, I know it to be of the greateſt conſequence, in the moſt difficult part of Perſpective, *viz.* in the practice on inclined Planes; the Theory, of which, being well digeſted and underſtood, the Practice will be found almoſt as eaſy as in the moſt familiar caſes, being founded on the ſame invariable Principles, as will be exemplified in Practice; and therefore, the pains I have been at to enforce it, is by no means to be diſpenſed with.

The Reader may be well aſſured, that every Part of the Theory, given in this Work, is of utility, and neceſſary to be known; and that, the better it is underſtood, the progreſs in practical Perſpective will be thereby facilitated.

THEOREM VIII.

The Perſpective Projection of every original Right Line is a Right Line, in every Poſition or Situation whatever.

DEM. Suppoſe a radial Plane to paſs through the Eye and any original Right Line.

If the Line be ſituated on the other ſide of the Picture, the imaginary Plane muſt, neceſſarily, cut the Picture; and, if it lie on this ſide, the Plane will cut the Picture if produced.

But, the Interſection of two Planes is a Right Line - - Ax. 3.

And, ſince the Eye is in this imaginary Plane, the whole of that Plane appears but a Line. (See Definition of a Plane, Page 41.)

But, the original Line is ſuppoſed to be in that Plane; conſequently, the Original Line and every Line in the radial Plane, have their Repreſentations in the Interſection of that Plane with the Picture; ſeeing, there can be no other Line common to both Planes. Therefore it is a Right Line.† Q. E. D. † Ax. 3.

Plate V. Fig. 15. EX. Let, BFIL be a rectilinear Object, bounded by Planes in various positions. Let MNOP be the Picture, placed direct between the Spectator (at E S) and the Object (BFIL).

FI, FG, or FD, is an Original Line (or, take any other Line in the Original Object) E is the Point of Sight, or place of the Eye; and EI, EF, EG, &c. are Visual Rays; in which, the extreme Points, I, F, G, &c. are seen, forming the Optic Angles FEI, FEG, &c.

Now, since the Original Line (FI, FG, &c.) is on the other Side of the Picture, the Visual Rays (EF, EI, &c.) must necessarily cut the Picture.

But, FEI is a Triangle; for, it is three Right Lines touching or cuting each other, and is, therefore, in a Plane. - - - Axiom 8.

The Ray EF cuts and passes through the Picture in the Point f, and EI in the Point i, &c. the Points f, i, g, &c. are, therefore, the Representations of the Original Points F, I, and G; for, they coincide with their respective Originals.

Wherefore, the Line fi, or fg, joining the Points f and i, or g, is the Intersection of the Plane Triangle FEI, or FEG, with the Picture.

But, the common Intersection of two Planes is a Right Line. - - Ax. 3.

And, fi is the Representation, or perspective Projection of the original Line FI, or fg of FG. (See Projection, Scenography, Page 49.)

For, to the Eye, at E, the Point f coincides with the Original Point, F, i with I, and g with G; consequently, the Line fi coincides with its Original FI, and fg with FG.

THEOREM IX.

The Representation, on the Picture, of a Line parallel to the Picture, is parallel to the Original; and, it has that proportion to the Original, as the Distance of the Picture to the Distance of a Plane, passing through the Original Line, parallel to the Picture.

DEM. The Perspective Projection of every Right Line on the Picture, is a Right Line; and it is produced, in Theory, by the Intersection of a Plane, with the Picture, passing through the Eye and the Original Line. - Theo. 8.

Wherefore, since the Original Line is, in this case, supposed parallel to the Picture, a Plane may pass through that Line, also parallel to the Picture.

But, if two Planes being parallel, are both cut by another Plane, their Intersections are parallel. - - - - - 8. 7. El.

But the Original Line is one of the Sections, and its Projection, on the Picture, is the other (the cuting Plane being supposed to pass through the Original Line and the Eye).

Therefore, the Representation of a Line, which is parallel to the Picture, is parallel to the Original. Q. E. D.

Fig. 22. EX. NO is an Original Line, parallel to the Picture (A L M B) E is the Eye.

Imagine a Plane, passing through the Original Line NO, parallel to the Picture; and, EN, EO, Visual Rays, from the Eye, to each extreme of the Original Line; which, are in the same Plane with NO. - Ax. 8.

Wherefore, NEO is a Radial Plane, passing through the Original Line and the Eye, and cuting the Picture in no, the Representation of NO. - Th. 8.

But, the Plane Y is supposed parallel to the Picture, and NO is in that Plane; consequently, no, its Representation, which is the intersection of the Radial Plane (NEO) with the Picture, is parallel to the Original Line (NO).

For, they are the common Sections of two parallel Planes, by another Plane.

In the same manner, pq, the Representation of PQ, may be proved parallel to PQ.

EX. 2. A H, B G, C D, &c. are Original Lines, parallel to both Pictures (M N O P or O P) E B and E G are Viſual Rays, to the Line B G; which are all in the ſame Radial Plane, cuting the Pictures in b g, and *b g*; which are, therefore, the Repreſentations of B G, on each Picture, reſpectively (by Theo. 8th.) Fig. 15.

But, B G, the Original Line, is perpendicular to the Horizon or Ground Plane, (S M Z) and the Pictures are vertical Planes.

And, the Radial Plane G E B is alſo vertical, ſeeing it paſſes through G B. 9. 7. El.

Conſequently, the common ſections of thoſe Planes are Right Lines, perpendicular to the Ground Plane. - - - Cor. to 9. 7. El.

and, conſequently, they are parallel between themſelves. - 3. 7.

Therefore, b g or *b g*, the Repreſentation of B G, is parallel to B G.

Alſo, c d, or *c d*, is parallel to C D; and a h, or *a h*, to A H.

After the ſame manner, the Repreſentation of any other Line (B C, G D, or G F, &c.) in the Plane B F C, which is parallel to the Picture M N O P, only, may be proved parallel to their reſpective Originals.

DEM. 2. The Repreſentation, n o (of N O) is proved parallel to N O, by the 1ſt Part. Fig. 22. No. 2.

Wherefore, the Triangles N E O, n E o, are ſimilar;

and n o : N O :: E n : E N, or as E o : E O. - - Cor. 3. 2. 6. El.

But, E C ſ is perpendicular to the Picture A B, and conſequently, to the Plane O N S, which is parallel to the Picture, cuting the Picture in C, its Center, and the Plane O N S in ſ. E ſ is therefore the ſhorteſt Line that can be drawn to thoſe Planes, and conſequently meaſures their Diſtances. 12. 1. El.

For, E C is the diſtance of the Picture, and E ſ of the Plane O N S.

But, if two or more Right Lines are cut by parallel Planes, they will be cut proportionally (whether they proceed from one Point or not) - 10. 7. El.

wherefore, E C : E ſ :: E n : E N, or, as E o is to E O.

But, n o : N O :: E n : E N. Th. n o : N O :: E C : E ſ, by equality of Ratios.

That is, the Repreſentation, n o, has that Proportion to N O, its Original, as the Diſtance of the Picture, E C, has to E ſ, the Diſtance of a Plane paſſing through the Original Line parallel to the Picture.

N. B. The ſame Demonſtration holds good if *N O* be conſidered as the Original Line, on this Side of the Picture, and projected to the Picture. (See Projected Perſpective, page 49.)

COR. 1. *From the former part, it is evident, that the Projections of any number of Lines, which are parallel amongſt themſelves and to the Picture, are alſo parallel amongſt themſelves and to the Originals.*

A H, B G, and C D, are parallel amongſt themſelves, and to both Pictures; their Repreſentations, on both, are therefore parallel. Fig. 15.

Alſo, B C and G D are parallel between themſelves and to the Picture M N O P only; their Repreſentations, on that Picture, are therefore parallel between themſelves and to the Originals.

COR. 2. *From the ſecond part of this Theorem, may be clearly deduced; that if an Original Line parallel to the Picture, be any how divided, the Repreſentations of the ſeveral Parts will have the ſame Proportion to each other, and to the whole Repreſentation, as the Parts of the Original Line have to each other, and to the whole Line.*

If from the diviſions, 1 and 2, of the Line C D, the Viſual Rays E 1, E 2 be drawn, they will cut the Repreſentations, c d, on both Pictures, to which the Original Line is parallel, in the ſame Ratio, in the Points y and z.

For, becauſe c d is parallel to C D, the Triangles C E D, c E d are ſimilar.

And, for the ſame reaſon, C E 1, C E 2, are ſimilar to c E y, c E z, &c. C. 3. 2. 6. El.

Wherefore, c y : C 1 :: c z : C 2 : i. e. as c d : C D : - 4. 6. El.

And, conſequently, c y : y z : z d :: C 1 : 1 2 : 2 D; or, as c d to C D.

Plate VI. COR. 3. *Hence it is manifeſt, that if the Eye be moved, ſtill keeping the ſame Diſtance from the Picture*; i. e. *let the Eye be any where in the Directing Plane*; *the whole Repreſentation, and each Segment, will ſtill have the ſame proportion to the Original Line.*

For, by the ſecond part of the Theorem, the repreſentation of a Line parallel to the Picture has the ſame Proportion to the Original, as the Diſtance of the Picture to the Diſtance of a Plane, paſſing through that Line, parallel to the Picture. And the Diſtance of either is not varied, whilſt the Eye is in the Directing Plane; therefore, &c.

COR. 4. *The Angle, which the Repreſentations of any two Original Lines, that are parallel to the Picture, make with each other, is equal to the Angle made by the Original Lines. For the Repreſentations are reſpectively parallel to their Originals.*

Fig. 22. No. 2. In Fig. 22. No. 2. Let the Original Lines, O N and S R, be produced till they interſect (in P) making the Angle O P S.

Becauſe n o is parallel to N O and r s to R S, being produced, they will make the Angle o p s equal to O P S, made by the Original Line. 5. 7. El.

Fig. 15. In Fig. 15. the Angle *g f d*, on the Picture M N *O P*, to which the Plane B F C is parallel, is equal to the Angle G F D; and *f g d* to F G D, &c.

For, *f g* is parallel to F G, *f d* to F D, and *g d* to G D; by the Theorem.

COR. 5. *Original Figures, in Planes which are parallel to the Picture, have their Repreſentations ſimilar to the Originals.*

Fig. 22. No. 2. The Plane NOS is parallel to the Picture, A B; wherefore, the Repreſentation o p is parallel to the Original Line, O P; p s is alſo parallel to P S; and if o s, O S be drawn, o s will be parallel to O S, - by Theo. Part 1ſt.

By Cor. 4. the Angle o p s is equal to O P S; and, by the ſecond Part of the Theorem, o p : O P : : p s : P S; for, each is as E C to E f.

Conſequently, the Triangles p o s, P O S are equi-angular; the Angle o is equal O, and s equal S; and conſequently, o s : O S : : o p : O P, or as p s : P S.

1. 8. El. Therefore, the Repreſentation, p o s, is ſimilar to the Original Triangle, POS.

For, S E O P is a Pyramid, cut by a Plane (A B) parallel to its Baſe (POS).

The Plane B F C, in the Object B F I L, is parallel to the Picture M N *O P*.

Fig. 15. By Cor. 1. *b g* and *c d*, on that Picture, are parallel between themſelves, and to their Originals (B E and C D) and ſo are *g f* and *f d* to their Originals.

And by Cor. 4. the Angles *b g f* and *g f d*, &c. are equal, reſpectively, to the Angles B G F and G F D, &c.

But, it is demonſtrated, that, *b g* : B G : : *g f* : G F; or, *g d* to G D, &c. for, each is to its Original, as the Diſtance of the Picture to the Diſtance of the Plane they are in; by the Theorem, part 2.

Therefore, ſince the Sides are directly proportional, and the Angles contained by correſponding Sides, or Diagonals, equal; the Triangle *g f d* is ſimilar to G F D; and the whole Repreſentation (*b g f d c*) to the Original.

N. B. All Lines, that can be drawn in a Plane, which is parallel to the Picture, are parallel to the Picture; and their Repreſentations have all the ſame Proportion to their reſpective Originals.

THE O-

THEOREM X.

All Right Lines, in any Original Plane, not parallel to the Picture, have their Interſecting Points in the Interſecting Line, and their Directing Points in the Directing Line of that Plane.

Alſo; the Vaniſhing Points, of Original Lines, are all in the Vaniſhing Line of the Planes the Original Lines are in.

DEM. If a Right Line be not parallel to the Picture, it will, if produced, cut the Picture, and the Directing Plane, in its Interſecting and Directing Points. § § Def. 21. and 23.

Now, ſince one part of a Right Line cannot be in a Plane and another part of it out of that Plane; † conſequently, the Original Line muſt cut the Picture, ſomewhere, in the Interſection of the Plane, it is in, with the Picture. † Ax. 12.

And, for the ſame reaſon, it will cut the Directing Plane, in the Interſection of the Plane it is in with the Directing Plane, i. e. in the Directing Line of that Plane. Therefore, all Lines, &c. Q. E. D.

Secondly. All Original Lines, in the ſame Plane, have their Radials in the Vaniſhing Plane of that Plane; their Interſections with the Picture are, conſequently, in the Interſection of the Vaniſhing Plane with the Picture.

But, the Interſection of the Vaniſhing Plane, of any Original Plane with the Picture, is the Vaniſhing Line of that Original Plane. - Def. 8.

Therefore, the Vaniſhing Point, of every Original Line, is in the Vaniſhing Line of the Plane that Original Line is in. Q. E. D.

Examples, of this ſelf evident Theorem, are in the Corollaries.

COR. 1. *Hence, a Right Line, drawn through any two Interſecting Points, is the Interſecting Line; and, a Right Line drawn through two Directing Points is the Directing Line of the Plane, in which the Original Lines are ſituated.*

It is alſo manifeſt, that a Right Line, drawn through any two Vaniſhing Points of Lines in any Original Plane, is the Vaniſhing Line of that Plane.

EX. A B, being produced, cuts the Picture in J, and N A cuts it in *I*; wherefore, a Right Line, drawn through J and *I*, is the Interſection of the Plane X. Fig. 23.

For, J and *I* are the Interſecting Points of Right Lines in that Plane; and if two Points in a Right Line are given, the whole Line is determined. (Cor. 1. Th. 2).

C B, produced, being in the ſame Plane (X) cuts the Picture in d, in the Line J *I*; and ſo, it is manifeſt, would every other Line in that Plane.

A D, produced, cuts the Picture in *N*; J *N* is, therefore, the Interſection of the Plane Y; for A B is alſo in that Plane.

P is the Directing Point of A B, and O of C B; O P is, therefore, the Directing Line of the Plane X; and P Q of the Plane Y*.

The Plane I K L M, paſſing through the Eye (at E) parallel to the Plane X, produces its Vaniſhing Line, M L. - - - Def. 8.

E V is the Radial of A B, and E M of C B; which being parallel, reſpectively to thoſe Lines, are in the Vaniſhing Plane I K L M.

And, the Vaniſhing Points, V and M, are, conſequently, in the Line of its ſection with the Picture, which is the Vaniſhing Line of that Plane; wherefore, if two Vaniſhing Points (V and M) of any two Lines are determined, the Vaniſhing Line (M L) of the Plane they in, is alſo determined.

For, the Vaniſhing Point of every Line, in any Plane, is in its Vaniſhing Line.

* The Point Q ſhould be where A D would cut the Directing Plane, in a Line parallel to J *N*; i. e. where the Interſection, J *N*, and the Vaniſhing Line, *M L* (which repreſent parallel Lines) interſect, which Point is out of the Plate. Therefore, P Q is not the true Directing Line, of the Plane A B D.

Plate VI. COR. 2. *All Right Lines, which are parallel to any Original Plane, have their Vaniſhing Points in the Vaniſhing Line of that Plane.*

Becauſe, a Plane may be drawn through any Right Line, parallel to any Plane to which the Line is parallel; and two or more Planes, being parallel, have the ſame Vaniſhing Line; by Theorem 3d.

COR. 3. *The Interſecting Point of the common Interſection, of two Original Planes, is the Point in which the Interſecting Lines of theſe Planes cut each other.*

For, the common Section of two Planes is a Right Line, in both Planes.

Wherefore, the Interſecting Point, of that Line, is in the Interſections of both Planes; conſequently, it is that Point in which the Interſections cut each other. For, there is no other Point common to both Lines.

Fig. 23. EX. J *I* is the Interſection of the Plane X, and J *N* of the Plane Y. J, the Point of their common Interſection, is, therefore, the Interſecting Point of A B, the common Interſection of the Planes, X and Y.

J *I* is the Interſection of the Plane X; and i d of the Plane W; d is, therefore, the Interſecting Point of C B, the common Interſection of the Planes X and W.

Fig. 15. 2. F D and F G are the Interſections of the vertical Plane B F C, and the inclined Planes, F K and F H; wherefore, *E* and *H* are the interſecting Points of thoſe Lines, on the Picture M N O P; where the vertical Interſection (*B E*) of the Plane B F C, is cut, by the Interſections, *D F* and *F G*, of the inclined Planes.

COR. 4. *The Directing Point of the common Interſection of two Planes, is the Interſection of their Directing Lines.*

Fig. 23. EX. P O is the Directing Line of the Plane X, and P Q of the Plane Y; wherefore, P, the Point of their Interſection, is the Directing Point of A B.

Hence, it is evident, may alſo be deduced the 2nd Corollary to the 6th Theorem, *viz.* The Vaniſhing Point of the common Interſection of two Original Planes, is the Interſection of their Vaniſhing Lines. Which, I adviſe the Student to be quite clear in; for, by that means, he will be able to determine the Vaniſhing Lines of contiguous Planes; and ſo proceed, in the delineation of Objects, from one Plane to another, with the greateſt facility.

This Diagram, though apparently intricate, is the completeſt in the Work, for rendering the knowledge of Vaniſhing Lines and Points, Interſections, &c. general; and I think, it cannot fail of anſwering the end I aimed at, *viz.* to diveſt the Student of that partiality which moſt have to the horizontal Vaniſhing Line; as it is manifeſt, that the ſame Principles extend to all, nor is there any difference in the proceſs; all the difficulty, if any, is in finding and determining them, on the Picture; which, when a clear Idea is inculcated, as by this Diagram, will readily be acquired. It is not poſſible to give, at once, ſo perfect an Idea, in Repreſentation, as in a Model, as is obvious in Figure 15, of the Apparatus; yet, when regularly analized, as in this Theorem, I am perſuaded, that all the intricacy will vaniſh.

THEOREM XI.

The Radial, or Parallel, of an Original Line producing its Vaniſhing Point, makes the ſame Angle with the Parallel of the Eye and Vaniſhing Line, as the Original Line makes with the Interſection and Directing Line, of the Plane that Original Line is in.

DEM. Since the Interſection and Vaniſhing Line, the Parallel of the Eye and Directing Line, of every Original Plane, are parallel amongſt themſelves (Th. 2.) and the Radial, producing the Vaniſhing Point of an Original Line, is parallel to the Original (Def. 22.); it neceſſarily follows, that, whatever inclination the Original Line has to the Interſection and Directing Line (which are all in the ſame Plane) the Radial of that Line has the ſame Inclination to, or makes equal Angles with, the Parallel of the Eye and Vaniſhing Line, which are all in a parallel Plane. - - - 4. of 1. and 7. El.

EX. N O is an Original Line, which, being produced, cuts the Interſection and Directing Line (A B and G H) in equal Angles. - - 4. 1. El. Fig. 20. and 21.

E V is the Radial of N O, producing its Vaniſhing Point (V) in the Vaniſhing Line (M L) of the Plane N G H, making the Angles E V L, with the Vaniſhing Line, and V E I, with the Parallel of the Eye, equal.

And, they are alſo equal to the Angles N *I* A, N *D* H, which the Original Line, (N O) makes with A B and G H, the Intersecting and Directing Lines.

Becauſe, the Original Line (N O) and its Radial (E V) are parallel (Def. 22.) and, becauſe they cut parallel Lines in parallel Planes.

2. It is alſo manifeſt, that, the Angle (V E *C*) made by the Radial (E V) of the Original Line (N O) and E *C*, the Radial of the Vaniſhing Line (V L) producing its Center (*C*) is equal to the Angle which the Original Line, (N *I D*) makes with F D, the Interſection of the Vertical Plane (E *C* D F) with the Plane N G H, in which the Original Line (N O) is ſituated.

For, theſe Angles are the Complements of the Angles made by the Original Line and its Radial, with the Interſection of the Picture, and the Parallel of the Eye or Vaniſhing Line.

3. From the ſame reaſoning, it is evident, that the Angles K E *D*, E *D* H, made by the Director (E *D*) of the Line N O, with the Parallel of the Eye and Directing Line, are equal to the Angles L V *I*, V *I* A, which the indefinite Repreſentation (*I* V) of the Original Line (N O) makes with the Interſection (A B) and the Vaniſhing Line (V L).

For, they are parallel Lines in parallel Planes, and are cut by parallel Lines.

COR. *The Center of every Vaniſhing Line is the Vaniſhing Point of Lines, which are perpendicular to the Interſection of thoſe Planes of which it is the Vaniſhing Line.*

For, by the Theorem, the Radial of a Line makes the ſame Angle with the Vaniſhing Line, as the Original Line makes with the Interſection of the Plane it is in. And in this caſe, the Original Line makes right Angles with the Interſection.

But, the Center of a Vaniſhing Line is produced by a perpendicular from the Eye to the Vaniſhing Line. - - - - Def. 19.

Therefore, it is the Radial of all Lines, which are perpendicular to the Interſection of the Plane they are in, &c.

I would recommend it to young Students, to be very clear in this Theorem, as it is moſt eſſential in Practice. For, from the knowledge it inculcates, the Vaniſhing Points of Lines, in any Plane whatever, may be readily found; knowing the Angle which the Original Line makes with the Interſection of the Plane it is in, with the Picture; or with a Line paſſing through the Station Point, and cuting the Interſection at Right Angles; making an equal Angle, at the Eye, with the Parallel of the Eye; or, with the Radial of the Vaniſhing Line, producing its Center.

Plate VI.

THEOREM XII. Defin. 25th.

The Indefinite Repreſentation, or Projection on the Picture, of an Original Right Line, not parallel to the Picture, is a Line drawn through its Interſecting and Vaniſhing Points.

DEM. For, every Right Line, not parallel to the Picture, will, if produced, cut the Picture, ſomewhere in the Interſecting Line of the Plane it is in.

And the Radial of every Line producing its Vaniſhing Point muſt cut the Picture, in the Vaniſhing Line of the Plane the Original Line is in. Theo. 10.

Wherefore, a Radial Plane, paſſing through the Original Line and the Eye, muſt neceſſarily paſs through the Radial of that Original Line, and conſequently through its Vaniſhing Point. - - - Ax. 5.

For, the Radial of every Line is parallel to the Line. - - Def. 14.

But, the Perſpective Projection of every Right Line, is a Right Line, produced by the Interſection of a Radial Plane with the Picture. Theo. 8.

And, the Radial Plane, paſſes through its interſecting and vaniſhing Points.

Conſequently, the Indefinite Repreſentation of the Original Line, on the Picture, paſſes through its Interſecting and Vaniſhing Point. Q. E. D.

Fig. 24. EX. Let H L be a Right Line, cuting the Picture (A C B) in I, and the Directing Plane in D; let E be the Eye; E V is the Radial of H L, cuting the Picture in its Vaniſhing Point (V) by Def. 22.

Let a Radial Plane (N O P Q) be ſuppoſed to paſs through the Original Line and the Eye (at E) conſequently through the Radial E V (for it is parallel to HL) and alſo through I, the Interſecting Point of H L.

Now, the Points I and V are in this imaginary Plane, and they are alſo in the Picture; therefore, a Right Line, drawn through the Points I and V, is the Interſection of the Plane N O P Q with the Picture (A C B).

But, the Plane (N O P Q) paſſes through the Eye (E) therefore, the Eye is in that Plane; and conſequently, the whole of that Plane, and every Line in it, appears but a Right Line.

Wherefore, the Interſection (g V r) of that Plane with the Picture, is the indefinite Repreſentation of the Original Line (H L) on the Picture; ſeeing that, every Line drawn through E, to any Point (N, M, F, K, &c.) in that Line (except the Point D) is in the Plane N O P Q, and will, if produced, cut the Picture, ſomewhere in I V, produced both ways, to g and r;

But, the Points n, m, f, &c. where the Viſual Rays, or Right Lines, E N, E M, E F, &c. cut the Picture, are the Repreſentations of the Points N, M, F, &c. in the Original Line†; conſequently, the Repreſentation of every Point in the Line H L, except the Point D, where it cuts the Directing Plane, is in the Interſection g V r, produced indefinitely.

† Perſp. Art. 2.

But, the Interſecting Point, I, and V, the Vaniſhing Point of the Original Line, are in that Interſection.

Therefore, the Indefinite Repreſentation of every Right Line is a Right Line, drawn through its Interſecting and Vaniſhing Points.

COR. 1. *The Repreſentation of an indefinite Right Line, ſituate on the other Side the Picture, lies between its Interſecting and Vaniſhing Points.*

If that part of the Original Line (I N) which lies beyond its Interſecting Point (I) was produced infinitely, towards H, its whole Repreſentation, on the Picture, is I V.

For, the Point I is its own Repreſentation; I m is the Repreſentation of I M, and m n of the part M N; and, if the Point H be ſuppoſed at any finite Diſtance, its Repreſentation h, muſt lie between n and V.

For, ſince the Radial, E V, producing the Vaniſhing Point (V) is parallel to the Original Line (I N) the Angle V E H will always be equal to E H D.

And, if the Point H be ſuppoſed at an infinite Diſtance, its Repreſentation, (h) will coincide with V; the Angle V E H will not be ſenſible, and conſequently the Point H will vaniſh, or be loſt to ſight; and, therefore, the whole indefinite Repreſentation of I H, infinitely produced, lies between its Interſecting Point (I) and its Vaniſhing Point (V).

COR. 2. *The Perſpective Repreſentation, of a finite Right Line, is part of a Line drawn from its Interſecting to its Vaniſhing Point.*

For the Viſual Rays (E M, E N, E H, &c.) from the ſeveral Parts of the Original Line (M N, N H, &c.) to the Eye, determine the repreſentations of thoſe Parts; and, ſince the whole infinite Line, from I, its Interſecting Point, is repreſented between its Interſecting and its Vaniſhing Points (I and V) conſequently, the Repreſentation of every finite Part is a part of I V. Fig. 24.

I m, repreſents the Part I M, m n repreſents M N, and, n h repreſents N H; the remaining Part, h V, repreſents all the Line beyond H, infinitely produced.

COR. 3. *The Projective Repreſentation of that Part of an Original Line* (I D) *which lies between the Picture and the Directing Plane, falls on the other ſide of its Interſecting Point; and, its whole Repreſentation is infinite.*

EX. The Point F will be projected to f, where the Viſual Ray E F, produced, cuts the Picture; and the Point G to g. - - - (See Proj. Perſ. P. 49). I f is, therefore, the projective Repreſentation of I F, and f g of F G.

If any Point (K) be taken, near D, its Repreſentation will be projected at a great diſtance from the Interſection I; and, the nearer the Original Point is to D, the farther will it be projected, from I; for, E k will always make an Angle with I g equal to D E K; but, when the Point K coincides with D, it will be projected to an infinite Diſtance.

Therefore, the Part, I D is repreſented by I g, infinitely produced; and conſequently, the Directing Point, D, can have no Repreſentation.

SCHOL. It may be obſerved, that, as the perſpective Part (I V) of the Indefinite Repreſentation, repreſents all that Part beyond its Interſecting Point, to an infinite Diſtance; ſo, the finite part of the Original Line, I D, between the Interſecting and Directing Point, is infinite in its Repreſentation, from I through g.

COR. 4. *All that part of the Original Line, which lies on the other ſide of the Directing Plane, infinitely produced from its Directing Point, is tranſprojected to the Picture, beyond its Vaniſhing Point; and the whole Repreſentation is infinite.*

EX. The repreſentation of the Point Q is at q, and of R at r; and, the nearer any Point (S) is taken to D, the farther will its Repreſentation be from V; but, when S coincides with D, its Repreſentation will be at an infinite Diſtance. (See Tranſprojection, Page 49).

Alſo, the farther any Point (L) is taken beyond Q, the nearer its Repreſentation (C) will approach to V; and, at an infinite Diſtance, it will coincide with V.

SCHOL. Here, it may be obſerved, that an indefinite Original Line is repreſented by an indefinite Line, the Terms being inverted; and that, a Point, on the other Side of D, will be tranſprojected to an infinite Diſtance, on one ſide of the Vaniſhing Point, V. On this ſide, it will be projected to an infinite Diſtance on the other ſide of the Vaniſhing Point; whereas, but one Point (D) lies between; and alſo, that the Repreſentations of two Points, H and L, being conſidered as the extremes of a Right Line, at infinite Diſtances, both ways, from the Interſecting Point (I) coincide at V.

Plate VI. 2. The tranſprojected part of the indefinite Projection, C V, may repreſent a Line in the ſame Plane, above the Eye; from the Point C produced indefinitely, through O; which muſt alſo be infinite, if it be parallel to E V, before the Points o and V coincide.

3. If the Original Line T V paſſes through the Eye, its whole Repreſentation is in its Vaniſhing Point; for, its Interſecting and Vaniſhing Point is the ſame; and, the Point of Sight (E) is its Directing Point.

COR. 5. *The whole indefinite Repreſentation* (I V) *of an Original Line, from its Interſecting Point* (I) *is not varied, the Eye being in any Part of its Radial* (E V).

Suppoſe the Eye, at E, removed to *E*, in the Direction V E.

E V, being parallel to I N, the Original Line, the Vaniſhing Point, V, remains the ſame; and, I, being its Interſecting Point, is invariable; conſequently, the Indefinite Repreſentation (I V) will be the ſame at any Diſtance of the Eye.

But, it muſt be obſerved, that, it is only ſo in reſpect of the whole of I V; for the finite Parts of the Repreſentation are continually varied, while the Eye moves from V to *E*; m being the perſpective Repreſentation of the Original Point M, from the Eye at E; but, removed to *E*, it will appear at m^2; conſequently, I m is the Repreſentation of the finite Part I M, of the Original Line, from the firſt Point of View, and I m^2 from the Point *E*. And ſo, of any other part of the whole Line.

Again, if the Eye move from E, or *E*, in the direction of E M, or *E* M, the Repreſentation m, or *m*, remains the ſame, and the indefinite Repreſentation is varied. For, at *E*, the indefinite Repreſentation is I V; but, at E^2, it is I n; yet I m^2, the Repreſentation of the finite part I M, is the ſame, from both Points of View.

COR. 6. *Hence, it is alſo evident, that, if the Eye be removed to any other Point in* E D (*the Director of the Original Line,* I N) *its whole Indefinite Repreſentation, as well as the finite Parts are varied from every Station.*

For, if the Eye be raiſed to E^3, the Radial is E^3 C; conſequently, I C is the whole indefinite Repreſentation; but if it be depreſſed to E^2, then is E^2 n the Radial, and I n the whole indefinite Repreſentation; and the finite Parts I *m*, *m n*, &c. are raiſed higher, or the Points, m, n, fall nearer to the Interſection, I, as the Eye is raiſed or depreſſed.

COR. 7. *The Repreſentation of the common Interſection of two Original Lines, is the Point, in which the Indefinite Repreſentations interſect each other.*

Fig. 23. For, the Point B, of the Interſection of the two Original Lines, A B and B C, is common to both Lines; wherefore, its Repreſentation is in the Indefinite Repreſentations (J V and d M) of both Lines; and, conſequently, in the Point (b) of their mutual Interſection; for, there can be no other Point common to both.

Alſo, a, the common Interſection of J V and *N V*, the indefinite Repreſentations of A B and A D is the Repreſentation of A, the Interſection of thoſe Lines.

As this Theorem, and what is deducible from it, with the next, contain the whole eſſence of practical, rectilinear Perſpective, reſpecting all Lines which are not parallel to the Picture, I would adviſe the Student to make himſelf particularly well verſed in them before he proceeds to Practice; for if thoſe Theorems be clearly underſtood and retained, he will find the Practice eaſy to be acquired, and, at the ſame time, rationally accounted for.

THEOREM XIII.

The Distance between the Intersecting Point of an Original Line, and the Representation of any Point in that Line, is to the whole Indefinite Representation; as the Distance between the Original Point and the Intersecting Point, is to the Distance between the Original Point and the Directing Point, of that Line.

This Theorem will be best demonstrated in the Example.

M N is an Original Line produced to D; I is its Intersecting, and D its Directing Point; and V is the Vanishing Point of that Line, the Eye being at E; E V is, therefore, its Radial. Fig. 24.

Now, if any Point, M or N, be taken in the Original Line, apart from its Intersecting Point; its Representation, on the Picture, will be, somewhere, between I, the Intersecting, and V, the Vanishing Point of the Original Line.

I say, that, the Distance of m or n, from I, the Intersecting Point, is, in proportion to the Indefinite Representation, I V; as M I or N I, the Distance of the Original Point from the Intersecting Point, is to M D or N D, the Distance of the Original Point from the Directing Point.

Having drawn the Visual Ray, E M or E N; the Representation, m or n, of the Point M or N, is where the Visual Ray cuts the Picture. Persp. Art. 2.

DEM. Now, E V is parallel to N M D †, and, I V is parallel to E D. 8. 7. El. † Def. 22.
Consequently, I V E D is a Parallelogram ‡; and the Triangles N E D, N n I are similar; wherefore, I n : E D :: I N : N D. - - - 4. 6. El. ‡ Def. 33. Geom.
But, I V is equal to E D ‖; therefore, I n : I V :: I N : N D. Q. E. D. ‖ 15. 1. El.

Or, it may be demonstrated thus:

Because E V is parallel to I N, the Triangles I N n, n E V are similar.
Wherefore, I n : n V :: I N : V E; and consequently, - - } 4. 6. El.
I n : I n + n V (= I V) :: I N : I N + V E (= N D) i.e. I n : I V :: I N : N D. }

N. B. This Proportion is invariable, whether the Eye be farther from (as at *E*) or nearer to the Picture; or whether the Eye be raised or depressed, as at E^3 or E^2; for, E^3 D being still parallel and equal to the Indefinite Representation (I C) consequently, I *n* : I C :: I N : N D; and I *m* : I C :: I M : M D.

COR. 1. *From this Theorem, the Indefinite Representation of a Line being given or drawn, and the Distance of any determinate Point, in the Original Line, from the Picture, known, the Representation of that Point, on the Picture, is also determinable.*

For, whatever Plane the Original Line is in, is not material, the Distance of the Point, in question, from the Picture, answers the same purpose, as its Distance from the Intersecting Point of the Line it is in; or, from the Intersection of any Plane with the Picture in which that Line is situated. e. g.

EX. N M is an Original Line cuting the Picture, in the Point I, in the Intersection, (A B) of the Plane (N B C) that Line is in; and the Directing Plane (D E C) in its Directing Point (D). Fig. 25.

I say, that N F (the Distance of the Point N from the Picture, A V B) is to F G, the Distance of the Picture, as N I to I D (the Distance of the Point N

Plate VI. Fig. 25.

from the Intersecting Point, to the Distance between the Intersecting and Directing Points, of the Line MN) or, as NA to AC, its Distance from the Intersection, to the Distance between the Intersection and Directing Line, of the Plane the Original Line is in.

DEM. For (having joined IF and DG) because the Directing Plane is parallel to the Picture, IF is parallel to DG - - - 8. 7. El.
(for the Triangle GND is a Plane cuting them both; Ax. 8.)

Wherefore, the Triangles GND, FNI, are similar; and, for the same reason, GNC, FNA, and also, DNC, INA, are similar;
consequently NF : NG :: NI : ND; or, as NA : NC. - - - 4. 6. El.

Wherefore, the Distance of the Original Point from the Picture, and the Distance of the Picture being known; the Distance of its Representation, from the Intersecting Point, is a fourth Proportional, *viz.* as the Distance of the Original Point from the Picture, added to the Distance of the Picture, i. e. as the Distance NG (of the Original Point, N, from the Directing Plane) is to NF (the Distance of that Point from the Picture) so is the Indefinite Representation, IV (of the Original Line MN) to In; the Distance of n (the Representation of the Point N) from I, the Intersecting Point of the Original Line. Q. E. D. It is, therefore, as NG : NF :: IV : In.

For, (by the Theorem) it is, inversely, as ND : NI :: IV : In.

Or, from the Vanishing Point, it will be, as NG : FG :: IV : nV.

In Numbers, it is thus calculated. The Distance (NF) of the Original Point from the Picture, and the Distance of the Picture (FG) being added (equal NG). The Indefinite Representation, IV, being multiplied by NF, and that Product divided by NG (the Distance of the Original Point from the Directing Plane) gives In; the Distance of n, the Representation of the Original Point, from the Intersecting Point, of the Line it is in. For, NG : NF :: IV : In; as above.

Wherefore, the Rectangle, under NG and In, is equal to the Rectangle under NF and IV; and consequently, NG × In = NF × IV; - 9. 6. El.

It more frequently occurs, in Practice, that the Representation of a certain portion of a Line is required, from some Point already found; the Intersecting Point of the Original Line not being within reach, nor attainable, but only the Vanishing Point. For, the Intersection of every Plane is neither wanted nor can be had, when the Vanishing Line of the Plane is absolutely necessary; it is the same in respect of the Vanishing and Intersecting Points, of Original Lines; as it will be exemplified in the practical part of this Treatise.

COR. 2. *If the whole Indefinite Representation be not given, but only a part, from some determinate Point, in the Original Line; any other portion, of that Line, (from the Point determined) may be perspectively proportioned, geometrically.*

The Representation, m, of the Point M, in the Original Line (MN) being given or found, and the Indefinite Representation (mV) from that Point drawn (V being the Vanishing Point) the Representation, n, of any other Point, N, in the Original Line, will be; as NM : MD :: mn : nV;
or, as NM : ND :: mn : mV.

DEM. For, as NI : ND :: In : IV; and, as MI : MD :: Im : IV; by Theo.
But, - ND : ID :: IV : nV; and, - MD : ID :: IV : mV; 4. 6. El.
for, the Triangles NED, VEn are similar; DE is equal to IV, and ID to VE (Because NDEV is a Radial Plane, passing through the Original Line (NM) and the Eye (E) cuting the Picture and Directing Plane; in IV and ED; and, EV, producing the Vanishing Point (V) is parallel to ND; by Def. 22.)

Now,

Now, N D : I D :: I V : n V; alſo M D : I D :: I V : m V; Fig. 25.
inverſely, - - - - I D : M D :: m V : I V.
Wherefore, by inordinate equality, N D : M D :: m V : n V.
Conſequently, - N D — M D (= N M) : M D :: m V × n V. (= m n) : n V.
that is, - - - - N M : M D :: m n : n V, as it was affirmed.
Wherefore, by addition, - - N M : N D :: m n : m V;
and conſequently, by inverſion, N D : N M :: m V : m n.

Wherefore, the Diſtance of M (whoſe Repreſentation, m, is given) and the Diſtance of any other Point, N, in the Original Line, from the Picture, being known; the difference being N *m*, it will, conſequently, be,
as N G : N *m* :: m V : m n; Q. E. D. For, N G : N *m* :: N D : N M.

Thus may the perſpective Proportion, of the known proportions of any Original Line, from any Point in it, be aſcertained, geometrically; having the Repreſentation of that Point, in the Indefinite Repreſentation given, and the Diſtance of that Point, with any other Point in the Original Line, from the Picture, known; ſeeing, that the Diſtances of the ſeveral Points, from the Picture, are in the ſame Ratio, as their Diſtances from the Interſecting Point; and the Diſtance of thoſe Points, from the Picture, may be had, when, often, their Diſtances from the Interſecting Point of that Line cannot; for ſeveral reaſons.

The Diſtances of the Points M and N, from the Picture, being known, and the Diſtance of any other Point, in that Line (as O) required; it will be, as M N : M O or N O :: *m* N : *m o* or N *o*; (O *o* being ſuppoſed parallel to M *m*).
So that, if *m* F, the Diſtance of the Point M from the Picture, be added, we have the Diſtance of the Point O, from the Picture, equal *o* F.
And the Diſtance of the Picture (F G) being known, the Repreſentation, o, of the Point O, is determinable; for, it will be, as *o m* : *m* G :: m o : o V;
and, by addition, - - - - as *o m* : *o* G :: m o : m V.

By which means, the perſpective Repreſentations of ſeveral Diviſions in a Right Line may be had, geometrically, their Diſtance from each other being known.

Let A B C be an Original Line, and C D E G a Radial Plane, paſſing through the Eye (E) and alſo through the Original Line; cuting the Picture in I V, their common Section. Fig. 26.
E V, parallel to A C, is its Radial; I V is, therefore, the whole Indefinite Repreſentation of A B C; I is its Interſecting, D its Directing, and V its Vaniſhing Point.
Now, if the Repreſentation (a) of any Point (A) in the Original Line, be determined, all other Diviſions (B, C, &c.) in that Line, are alſo determinable, without the Interſection I; the Diſtance of A and B, from the Picture, and the Diſtance of the Picture being known.
For, a b : a V :: A B : B D; a c : a V :: A C : C D; and alſo, b c : b V :: B C : C D.

DEM. For, (having drawn A F and B G, parallel to I V) becauſe a V is parallel to A F, a b : a V :: A *b* : A F; alſo, a c : a V :: A *c* : A F; } Cor. to 6. 6. El.
and, for the ſame reaſon, b c : b V :: *b c* : *b* F; and, as B *C* : B G. }
But, the Triangles G E *C*, *C* C B are ſimilar, for E G is parallel to C B;
wherefore, - - - B *C* : *C* G :: B C : E G (equal B D) 4. 6. El.
Conſequently, by compounding, B *C* : B G :: B C : C D, equal C B + G E.
But, as B *C* : B G :: b c : b V; therefore, b c : b V :: B C : C D.

c is, therefore, the Repreſentation of the Original Point C, and b of B; which are determined from the given Repreſentation of A; the Diſtance of A and any other Original Point from the Picture, being known; no regard being had to the Diſtance of the Point A, from the Interſecting Point (I) or from D, the

Plate VI. the Directing Point of the Original Line; ſeeing that, the Diſtance of the Original Points, from the Picture and Directing Plane, are in the ſame Ratio, as their Diſtances from the Interſecting and Directing Points of the Original Line.

In this Theorem is the Perfection of Practical perſpective. It is, at the ſame time, the moſt mathematical, and the Demonſtrations the moſt perfect, elegant, and convictive, of the whole Theory. By it, we not only know that there is Analogy of Ratios, between the ſeveral Diſtances of Points, in an Indefinite Repreſentation, from the Interſecting Point, and the whole Indefinite Repreſentation; to the ſeveral Diſtances of the Original Points from the Interſecting and Directing Points, or from the Picture and Directing Plane; but, by its means, the Repreſentations of the ſeveral portions of an Original Line, in the Indefinite Repreſentation, are determined, with the greateſt facility, accuracy, and expedition, geometrically; or they may be determined numerically, by a Scale of equal Parts; but that is ſeldom practiced, the method of doing it geometrically, being much readier and more accurate than it is poſſible to calculate, by Numbers.

THEOREM. XIV.

The Perſpective Projection, or Repreſentation, of every Right Line, is parallel to its Director.

Dem. The Radial of an Original Line, producing its Vaniſhing Point, is parallel to the Original Line; — — — — Def. 22.
wherefore, a Radial Plane may paſs through the Original Line and its Radial; conſequently the Eye is in that Radial Plane. — — Ax. 5.

Now, ſince one part of a Right Line cannot be in a Plane, and another part of the Line out of that Plane, the Radial Plane will cut the interſecting Line (of the Plane the Original Line is in) in the interſecting Point, and the Directing Line, in the Directing Point, of the Original Line. — Ax. 1.

But, the Radial Plane paſſes through the Eye; — — Def. 6.
and, conſequently, through the Vaniſhing Point; ſeeing it paſſes through the Radial Line, producing the Vaniſhing Point.

Wherefore, the Section of this Radial Plane, with the Picture, is the Indefinite Repreſentation of the Original Line; ſeeing, it paſſes through its Interſecting and Vaniſhing Points. — — — Theorem 12.

But, it alſo paſſes through the Eye, and Directing Point, of the Original Line; conſequently, it cuts the Directing Plane in the Director of the Original Line. — — — — — Def. 12.

But, the Directing Plane is parallel to the Picture. — Def. 4.

Wherefore, the Sections of the Radial Plane, with the Picture and Directing Plane, are parallel to one another. — — — 8. 7. El.

But, its Section with the Picture is the Indefinite Repreſentation; and, its Section with the Directing Plane is the Director of the Original Line; as above.

Therefore, the Perſpective Projection of an Original Line, being a part of its indefinite Repreſentation, is parallel to its Director. Q. E. D.

Fig. 20. EX. Imagine a Plate (N*D*EV) paſſing through the Eye and the Original Line, NO. This Radial Plane muſt neceſſarily cut the Picture and Directing Plane, in *I*V and E*D*; for EV is parallel to *DI*N, i. e. to NO; wherefore, EV*ID* is a part of the Radial Plane; which, being produced, would paſs through NO, the Original Line.

But, *I* is the Interſecting, and, V is the Vaniſhing Point of NO; Def. 21. and 22.
wherefore, *I*V is the Indefinite Repreſentation of NO; — Def. 25. Th. 12.
and, ſince the Radial Plane paſſes alſo through the Eye and Directing Point,

(E and *D*) its Section with the Directing Plane (E*D*) is the Director of N O ‡. ‡ Def. 12.
Consequently, *I V*, or n o, a part of *I V*, is parallel to E D. - 8. 7. El

In Fig. 21. this is more perfectly illuſtrated, by means of Viſual Rays, from the Extremes (N and O) of the Original Line, N O.

For, NO being produced to the Directing Plane, *D* is its Directing Point; and E *D*, being drawn, is the Director of N O, as before.

But E N *D* is a Triangle, conſequently it is a Plane; - - Ax. 8.
which cuts the Picture in n o, or *I V*, the Indefinite Repreſentation of N O; and, conſequently, it is parallel to E *D*, its Director.

COR. 1. *If two, or more, Right Lines cut the Directing Plane in the ſame Point, they will have parallel Repreſentations.*

For they have the ſame Director, to which they are all parallel, by Theorem.

EX. P Q cuts the Picture in *P*, and the Directing Plane in *D*, the Directing Point of N O; and, becauſe it is perpendicular to A B (the Interſection of the Picture) C is, therefore, its Vaniſhing Point. (E C being perpendicular to L M, is therefore parallel to P Q, being in parallel Planes.)

Conſequently, *P* C is the indefinite Repreſentation of P Q; which is parallel to *I V*, the indefinite Repreſentation of N O; by Theorem, and 4. 7. El.

And alſo, by Theo. 5; for, E *D*, the Director of both (N O and P Q) is the common Interſection of the Radial Planes, *D E V I* and *D E C P*; paſſing through both Lines. Therefore, their Interſections, *I V* and *P* C, are parallel.

N. B. This Corollary is true if the Lines, N O and P Q, are not in the ſame Plane.

COR. 2. *All Lines, not having the ſame Directing Point, but, which cut the ſame Director, indefinitely produced (through the Eye) both ways, have parallel Repreſentations.*

For, if two Original Lines, N O, or P Q, and R S, cut the Directing Plane in the two Points *D* and d, in ſuch wiſe, that a Right Line drawn through the Eye and the Directing Point of one, will alſo paſs through the Directing Point of the other, their Repreſentations, n o and p q, are parallel.

Becauſe they have the ſame Director, *D* E d.

E V is the Radial or Parallel of N O, V is therefore its Vaniſhing Point; and E v is parallel to R S; therefore v is its Vaniſhing Point; and, S v its Indefinite Repreſentation.

COR. 3. *All Right Lines cuting the Directing Plane in* E F, *the Interſection of the Vertical and Directing Planes, will have their Repreſentations parallel to the Vertical Line,* (C D.)

Becauſe, they all have the ſame Director (E F) to which the Vertical Line (C D) is parallel; being produced by the ſection of the Vertical Plane (E C D F) with the Picture and Directing Plane. - - - - 8, 7. El.

As *P* Y, parallel to R S, cuting E F, the Prime Director, in f.
E v is its Radial and *P* v its indefinite Repreſentation.

COR. 4. *All Lines which cut the Parallel of the Eye of any Original Plane, have their Repreſentations parallel to the Vaniſhing Line of that Plane.*

Becauſe, the Parallel of the Eye is the Director of all ſuch Lines, and it is parallel to the Vaniſhing Line; by Theorem 2nd.

T U is an Original Line, cuting I K, the Parallel of the Eye, of the Original Plane N B H, in k; E x is its Radial, x is therefore its Vaniſhing Point, and U x its indefinite Repreſentation; which is parallel to L M, the Vaniſhing Line of the Plane N B H.

† Theo. 2. For, IK is parallel to L M †, and it is the common Section of the Plane, I K L M and E k U x; therefore, U x is parallel to L M, by Theorem 5th.

COR. 5. *The Repreſentation of any Original Line makes equal Angles with the Interſection and Vaniſhing Line, of the Plane it is in, as the Director, of that Line, makes with the Parallel of the Eye and the Directing Line of that Plane.*

Becauſe the Repreſentation is parallel to its Director, by the Theorem; and becauſe they cut parallel Lines in parallel Planes.

COR. 6. *If the Repreſentations of any two Lines are parallel, the Originals are either parallel between themſelves and to the Picture, or they have the ſame Director.*

For, if the Original Lines are parallel between themſelves and to the Picture, their Repreſentations will be parallel (Theo. 9.); but, if they are not parallel to the Picture, they muſt have the ſame Director; ſeeing, there can be but one Line drawn in the Directing Plane, parallel to both Repreſentations.

N. B. Lines, which are parallel to the Picture, have no Directing Point, but the Director of every ſuch Line, is a Line drawn through the Eye parallel to the Original Line.
For, the Repreſentations are parallel to the Original; by Theorem 9th.

This laſt Theorem (and the Corollaries deducible from it) contains the whole Theory of the Directing Plane; as, in it and the thirteen preceding Theorems is contained the whole knowledge of rectilinear Perſpective; or, at leaſt, all that I conceive to be really uſeful, in delineating. It has been my aim, not, merely, to amuſe or to ſhew my knowledge in it, but to give uſeful Inſtruction; and, I dare venture to affirm, that if the whole of this Theory be clearly underſtood, the Student will ſeldom be at a loſs in Practice.

It is a miſtaken notion which many entertain of Perſpective, that, the Theory is unneceſſary to a Practitioner. It is certainly poſſible to practice Perſpective, in all common Caſes, without being able to account, or give a reaſon for any rule that is followed; for, the Rules, being deduced from the Theory, will, undoubtedly, if ſtrictly followed, produce certain effects though we know not how to account for it: as there are many Perſons very acute in Menſuration, Gauging, Surveying, &c. who know nothing of Geometry, the foundation of the whole. The caſe is very different in Perſpective; for I am well convinced that it is of great uſe to underſtand the Theory well, in the firſt place; and that, the Practice will, by that means, be ſooner acquired, and more ſecurely retained. For want of Theory, the Pupil is frequently bewildered, and knows not what he is about; every different Example appearing difficult and ſtrange, though, perhaps, founded on the ſame invariable Principles. In ſhort, the neareſt and moſt certain road, to Perſpective, is to go through the Theory to Practice: and, I will venture to ſtake all my knowledge in it, that when acquired, the loſs of Time (if it be any) will never be regreted.

I ſhall give three more Theorems, on Circles and ſpherical Bodies, and then proceed to Practice. If any Perſon require further knowledge, or a more extenſive Theory, I refer him to the elaborate Work of Mr. Hamilton, which is deſerving of the higheſt encomiums, if it was as uſeful as it is ingenious and learned in the Science; for, he has certainly ſaid all that can be ſaid of it, in Theory: and, I am perſuaded, more than any other Perſon would ever have thought on, and much more than is of real uſe; for, I think I have omited nothing that can be uſeful or neceſſary to be known, by any Practitioner, whatever.

SECTION V.

Of the THEORY of CURVILINEAR PERSPECTIVE.

IN this Section, I ſhall chiefly conſider the Theory of Perſpective relative to circular Objects; which are the moſt common, and moſt uſeful of all curve lined Figures. Other Curves cannot be compriſed in any certain Theory, by which their perſpective Repreſentations can, with certainty, be aſcertained; or if they could, it would anſwer no purpoſe to an Artiſt, ſeeing that, irregular curved Figures or Objects but ſeldom occur, in Practice. I would not be underſtood to mean the curves of the apparent Contours of human or other Figures (endowed with Life or not) which occur in almoſt every Picture, but which, can never be reduced to Rules, for Practice, from an eſtabliſhed Theory, but, irregular curved Figures, in Planes or other Surfaces; winding or ſerpentine Rivers, Rocks, Mountains, Trees, &c. which are bounded by irregular curved Lines and Surfaces, cannot be reduced to Practice, in delineating them, by any Theory in the Science of Perſpective. Notwithſtanding they may be delineated with great accuracy, by any Perſon, who is a little accuſtomed to ſketch by ſight only, by means of an Apparatus, which I ſhall deſcribe in an Appendix to this Work.

To treat, at large, of the various Curves which the Repreſentation of a Circle may take, ſuch as the Parabola or Hyperbola, is foreign to my Deſign; as it ſo rarely aſſumes thoſe forms. Nor is the knowledge thereof of any real uſe in delineating; ſeeing that, the ſmall part of the Repreſentations of ſuch Circles as are or can be repreſented, when they do aſſume either, could not readily be diſtinguiſhed from a portion of an Ellipſis; which Curve, as it is the moſt general and uſeful, ſo it is the eaſieſt to deſcribe, and the only one of real uſe, in Perſpective.

In order to a clear underſtanding of the nature of an Ellipſis and its Properties, it is neceſſary to be acquainted with the Conic Sections; ſince every Repreſentation of a Circle or Sphere, in Perſpective, is either one or other of the Sections of a Cone. But, as a thorough inveſtigation of it is not neceſſary, here, I ſhall refer the Reader, who deſires to be perfectly acquainted with the Conic Sections, to Mr. Steel's, or to a later Work, by Mr. Emerſon.

Nevertheleſs, I find it impoſſible to treat the Theory of the Circle, in Perſpective, without having recourſe to them, in ſome degree; therefore I ſhall, in the firſt place, define what is a Cone, and, the difference between a Right Cone and a ſcalene or oblique Cone; for, without that knowledge, all that can be ſaid of it would be to little or no purpoſe.

The methods of deſcribing an Ellipſis, and all which appertains to it, are treated fully, yet briefly, in ſix Problems in an Appendix to the practical Part (Book 1ſt) of my Treatiſe on Geometry; together with a conciſe Theory, of its moſt eſſential Properties; to which I refer the Reader; I ſhall conſider it, here, only as being the perſpective Repreſentation of a Circle or Sphere.

DEFINITIONS.

A CONE is a geometrical Solid, whoſe Baſe is a Circle, which terminates in a Point, called its VERTEX; and, a Right Line, paſſing through the Vertex and the center of its Baſe, is called its AXE or AXIS*.

* The Axis of any thing is either a real or imaginary Right Line paſſing through its middle, in a certain and determined Poſition.

If the Object be a Plane Figure, its Axe may be either perpendicular to, or in the Plane of the Figure. If the Axe be in the Plane of the Figure, it is divided, by the Axe, into two equal and ſimilar Figures. Any Diameter of a Circle may be its Axe, an Ellipſis has but two Diameters which are Axes, viz. the Tranſverſe and its Conjugate, at Right Angles with each other.

Plate VII. It may be conſidered as a Pyramid whoſe Baſe is a Polygon of an infinite number of Sides; every Section of which, by a Plane parallel to its Baſe, will, conſequently, be a ſimilar Figure; wherefore, the ſection of a Cone, parallel to its Baſe, is a Circle.

2. A RIGHT CONE is that which is formed, or ſuppoſed to be generated, by the revolution of a right angled Triangle, on one of its Legs.

Fig. 27. No. 1. A B C is a right angled Triangle; C is the Right Angle; B C is, therefore, perpendicular to A C, its Baſe.

If you ſuppoſe the Triangle A B C to be revolved quite around, on B C, as an Axis, its Baſe (A C) will deſcribe the Circle A E D F, which is the Baſe of the Cone A B D; and the Hypothenuſe (A B) will have deſcribed the Surface of the Cone; which is every where various, from the Vertex (B) to the Periphery of its Baſe; its Sides are equal every where, as B A, B E, B D, &c. The Perpendicular (B C), which is the Axe of the Cone, remains at reſt; one extreme in B, the Vertex, the other in C, the Center of its Baſe.

No. 2. 3. AN OBLIQUE CONE has, alſo, a Circle for its Baſe, but its Axe is inclined to its Baſe, as *B C* to A D.

As if the Vertex (B) of a Right Cone, was drag'd on one ſide, out of its perpendicular poſition; conſequently, the real Axe, of ſuch a Cone, does not paſs through the center of its Baſe, yet *B C* is called its Axe.

A Right Line biſecting any Angle of a Triangle is called its Axe; wherefore, *B E*, biſecting the Angle A *B* D, is the Axe of the Triangle A *B* D, and conſequently of the Cone; for it paſſes through the middle of ſuch a Solid, whoſe rotundity is elliptical, ſeeing, its Dimenſions, perpendicular to its Axe, are unequal; and, the more the Axe is inclined to the Baſe, the more its Dimenſions differ (its Baſe remaining the ſame) and the real Axe is removed farther from the Center (C) towards A.

If a Section of an oblique Cone be made, by a Plane, perpendicular to the real Axe (*B E*) it is conſidered as an oblique Section of it; which Section is an Ellipſis, and *B E* would paſs through its Center; conſequently, the Solid would revolve regularly on *B E*, its real Axe. Whereas, on *B C* it would revolve very irregularly and unequal; but the Cone, A *B* D, would be equally poiſed on B C, in a horizontal Poſition; wherefore, every Section of a Cone, through the Axe *B C*, biſects the Cone; for the Triangles, A *B* C, C *B* D, are equal, in every Section, through B C; by Prop. 18. 1. El.

THEOREM I.

The Repreſentation, or perſpective Projection, of every Circle, in a Plane to which the Picture is parallel, is a Circle.

The ſcenographic Projection, or perſpective Repreſentation of an Object, is the Section of the Optic Cone, or Pyramid of Rays, by a Plane, paſſing between the Eye and the Object. (See Scenography, P. 47.)

Fig. 28. Let A D G H be a Circle, in any Original Plane (Z) E is the Eye, and E A, E B, &c. Viſual Rays, from every Point in the Circumference, to the Eye, forming an oblique Cone (A E F).

DEM. Now, if the Cone of Rays be cut by a Plane (X) parallel to the Plane Z, in which is the Original Circle; a d g h, the Section of the Rays, by that Plane, is a Circle.

For, it is the ſection of a Cone, by a Plane, parallel to its Baſe; and, whether it be a Right or Oblique Cone, every ſuch Section is a Circle; ſeeing that, the Cone, a E f, cut off by the Plane X, is ſimilar to the larger Cone (A E F).

This is otherwiſe demonſtrable, from Theorem 9, and Corollaries; in which it is demonſtrated, that the Repreſentation of every Plane Figure, parallel to the Picture, is ſimilar to the Original.

Every Diameter in the Original Circle (A F, D H, &c.) being equal, the Repreſentations (a f, d h, &c.) have that proportion to their Originals, as the Diſtance of the Picture to the Diſtance of the Plane of the Original Circle. (Theo. 9.) conſequently they are alſo equal; therefore, the Repreſentation, a d g h, is a Circle. Q. E. D. For, all Circles are ſimilar Figures.

COR. *Hence it is manifeſt, that the Repreſentations, in Perſpective, of various Circles, in the ſame Plane, parallel to the Picture, have all the ſame Ratio amongſt themſelves, and to each other, as the Original Circles.*

THEOREM II.

The Repreſentation of a Circle, in a Plane not parallel to the Picture, is an Ellipſis; except in one certain Point of View, in which, its Repreſentation is, alſo, a Circle.

Let A D G be an Original Circle in the Plane Z; E is the Eye, and E A, E B, &c. Viſual Rays forming (as before) an Oblique Cone. C is the Center of the Circle. Fig. 28.

DEM. If the Cone of Rays be cut by a Plane, (Y.) which is conſidered as the Picture, paſſing through both ſides of the Cone, not parallel to the Baſe, it is an oblique Section; and, conſequently, every ſuch Section, of a Cone, is an Ellipſis; except, as above. Prop. 90. P. 73. Em. Con. Sec.

Wherefore, the Repreſentation, *a d g*, on the Plane Y, (which is an oblique Section of the Cone of Rays E A, E B, E D, &c.) is an Ellipſis. Q. E. D.

Every Circle, which is viſible, appears an Ellipſis, except when the Axe of the Eye is perpendicular to the Plane of the Circle, and paſſes through its Center; for, in that Caſe, only, the Viſual Rays, from the Eye to every Point in the Circumference, are equal, and conſequently they generate a Right Cone, the Axe of the Eye being the Axe of the Cone. In every other poſition, whatever, they muſt neceſſarily form an Oblique Cone, ſeeing that, the Axe of the Eye (which is always the Axe of the Cone) muſt be inclined to the Plane of the Circle, if it be not perpendicular; and conſequently, the Viſual Rays, forming the ſurface of the Cone, have various Inclinations to the Plane of the Circle, and therefore they are unequal.

Wherefore, ſince the ſection of an Oblique Cone, perpendicular to its central Axe, is an Ellipſis, and the more the Axe is inclined to the Baſe, the more excentric is the Section; ſeeing that, if the Eye be nearly in the Plane of the Circle, the Section approaches nearly to a Right Line; and, the Section, made by a Plane in that poſition, is the only true Appearance of the Circle; or rather, by a ſpherical Surface, perpendicular to the Axe, which truly meaſures the Optic Angle (the Radius being equal to the Diſtance) under which, the ſolid dimenſions of the Cone, every way, are ſeen.

COR. *If the height of the Eye, or its diſtance from the Directing Line, be taken a mean Proportional, between the diſtance of the neareſt convex part of the Circumference and the diſtance of the fartheſt concave part, from the Directing Line, the Repreſentation will then be a Circle.*

Plate VII. Fig. 29. Let A B be a Diameter of the Circle, A D G, perpendicular to the Directing Line (K L) B S is the diſtance of the Circle from it.

Make S E : S A : : S B : S E; i. e. let S B : S E : : S E : S A, by Pr. 30. Geo.

At E, as the Point of View, the Repreſentation of the Circle A D G, on any Plane (X) parallel to the Plane K E L, will be a Circle, and in no other Point whatever, on that Plane, and at the diſtance B S.

DEM. Draw B I parallel to S E; and, ſuppoſe A E B a ſection, by a Plane paſſing through C E, the Axe of the Cone, and the Diameter A B, of its Baſe; and alſo through S.

Then, becauſe B I is parallel to S E, the Triangles A I B, A E S are ſimilar;

Wherefore, - - - - A B : B I : : A S : S E.
But, by Conſtruction, - - A S : S E : : S E : S B.
Wherefore, - - - - - S E : S B : : A B : B I;

+ 5. 6. El. ‖ 4. 1. and therefore, the Triangles A I B, B E S are ſimilar†, for the Angle A B I is equal to E S B‖; E B S is equal A I B, and I A B equal B E S; ſeeing that, the Sides which ſubtend thoſe Angles are proportional.

‡ 4. 1. El. But, the Angle B E S is equal E B I‡, for they are alternate; wherefore, E B I is equal I A B (of the Cone A E B) and conſequently, the leſſer Cone (a E b) cut off by the Plane X (parallel to B I, and the Plane K E L) is ſimilar to the Cone B E A; for, the Angle E (at the Vertex) is common to both; the

§ C. 5. 10. 1. Angle E b a (equal E B I) is equal E A B, conſequently, E a b is equal E B A§, and therefore, the Cone (A E B) is cut ſub-contrary, by the Plane X.

But, if an Oblique Cone be cut ſub-contrary, the Section is a Circle.

Therefore, the Repreſentation, a h b g, on the Plane X, of the Original Circle A D B F, is a Circle. Q. E. D. Prop. 89. P. 73. Em. Con. Sec.

N. B. The Center (s) of the Repreſentation, is not the Repreſentation of the Center (C) of the Original Circle; for it is at c, where the Viſual Ray E C cuts a b, the Repreſentation of the Diameter A B. Wherefore, C E is not the real Axe; ſeeing that, it cannot be the Axe of the Cone A E B, and alſo, of the leſſer Cone (a E b) which is cut ſub-contrary, and is therefore ſimilar to A E B; conſequently, the true Axe is common to both Cones.

So likewiſe, d f, the Repreſentation of the Diameter D F, in the Original Circle, is not a Diameter of the Repreſentation; but, g h, the Repreſentation of the Chord G H, (which biſects a b) is its Diameter perpendicular to a b.

Wherefore, the Repreſentation of the Segment G B H is the lower Semicircle, g b h; and, g a h, the upper Semicircle, repreſents the large Segment (G A H).

It is evident, that, if the Eye be raiſed, as at E', the Diameter a b, in the Repreſentation, will be lengthened; ſeeing, that the Viſual Rays, E' A, E' B, cut the Picture more oblique; whilſt the other Diameter, g h, remains of the ſame

§ Cor. 3. Theo. 9. length §; and conſequently, the Repreſentation of the Circle, from that Point of View, will be an Ellipſis, and a b its Tranſverſe Diameter, or Axe.

But, if the Eye be lowered, to E³, the Diameter a b (now c e) will be ſhorter than the Diameter g h (which, being parallel to the Interſection of the Picture, will have the ſame length, ſeen from any Point in the Directing Plane, as above) and it will then be the Tranſverſe Axe; and c e (the Conjugate to it) is the ſhorteſt Diameter of the Ellipſis.

If the Eye be removed on either Side of the Point E, the Repreſentation will be an Ellipſis; for, the Section will not, then, be ſub-contrary (the Picture remaining as before). The Repreſentation (a b) of the Diameter A B, will ſtill be a Diameter of the Ellipſis, becauſe it will paſs through its Center; but it will be neither the tranſverſe nor the conjugate Axe, for they are always at right angles with each other.

Hence, the Point of View, from which the Repreſentation of a given Circle, on any Picture, however ſituated, will be a Circle, is eaſily determined.

The Original Plane (K A L) in which the Circle is ſituated, being produced if neceſſary, cuts the Picture in M N, its Interſection.

Having fixed on the Diſtance, or Station Point, S (at pleaſure) in the Diameter A B, produced, which is perpendicular to the Interſection of the Picture; let K L be drawn, through S, parallel to M N; K L is the Directing Line. - Def. 10.

Draw S E perpendicular to K L, and parallel to the Picture.

Make S E a Mean Propprtional, between S A and B S; - - Pr. 30. Geo.

E is the place of the Eye, or the Point required; from which, a Circle, a g b h, (on the Plane X) whoſe Center is c, and Diameter a b, will truly repreſent the Original Circle A D B F, on the Plane A K L.

N. B. The diſtance of the Picture, in this Caſe, is not material; for, the Point E being fixed (as above) every Section of the Cone of Rays E A, E F, E B, &c. parallel to the Plane X, or to the Directing Plane, K E L, being cut ſub-contrary, will, conſequently, be a Circle.

From what has been advanced, it is evident, that the Repreſentation of every Circle, in Perſpective, is ſome one or other of the Conic Sections; which, being a diſtinct Science, would not be proper to enter on here; but, from that conſideration and the preceding Theorems, the two following Corollaries may be deduced.

COR. 1. *If the Circumference of the Original Circle touches the Directing Line, in a Point only, and is not cut by it, the Repreſentation of that Circle will be a Parabola.*

For, whether the Original Circle (A D B F) touches the Directing Line, in S, (the Station Point) or in any other Point (B) it is the ſame; ſince that Point, and alſo the Eye (E) which is the Vertex of the optic Cone of Rays, are both in the Directing Plane (Y) it is evident, that the Directing Plane touches the Cone in the Right Line E B, from its Vertex (E) to its Baſe (at B). Fig. 30.

Wherefore, ſince the Picture (X) is always parallel to the Directing Plane ‡, the Cone, A E B, is cut by a Plane parallel to its Side (E B) the Curve produced by every ſuch Section is a Parabola. Prop. 76. P. 223. Em. Con. Sec. ‡ Def. 4.

That part of the Original Circle (M G A N) which lies beyond the Picture, is repreſented perſpectively, and falls above the Interſection M N (as M a N); the remainder of the Circle, lying between the Picture and the Directing Line, is projected below the Interſection; as l is the repreſentation of the Original Point L. Every other Point (as K) is projected further from the Interſection as it lies nearer to the Directing Line (B S) and, except the Point B, in which it touches the Directing Line, may be ſuppoſed to have a Repreſentation, on the Picture, though at an immenſe diſtance; but, the Point B can have none; becauſe, the Line (E B) which ſhould produce it, is parallel to the Picture, ſeeing, it lies in the Directing Plane, and therefore can never cut the Picture. The Repreſentation of which Point, only, is wanting to compleat the Figure and form an Ellipſis; but, for want of that Point, in the Repreſentation, it is kept open, and falls off in Right Lines, nearly, at a Diſtance, to all ſenſe, infinite.

Hence it is plain, that, on account of the Diſtance, but a ſmall portion of that part of the Curve which falls below the Interſection, can be repreſented, in a Picture; and the part which can, together with the part which lies beyond the Picture, differs ſo little, in its Repreſentation, from an elliptic Curve, that, in delineating, it would be needleſs to deviate from it; and, I queſtion that ever a diſtinction was made, in Practice. Nor can it ever be of uſe, but in delineating the inſide of a large Rotunda, Circus, or circular Area; when the Spectator may be ſuppoſed to ſtand on the hither part of the Circumference, or in a Line which is a Tangent to it, and parallel to the Picture; in which caſe, only the farther, concave part of the Curve can, properly, be repreſented.

COR.

Plate VII. Fig. 31.

COR. 2. *If the Directing Line* (KL) *cuts the Original Circle in two Parts; that part* (KFAHL) *which is on the ſame ſide with the Picture, will, in its Repreſentation, form an Hyperbola, below the Vaniſhing Line* (OP) *of the Plane of the Circle; the other part* (KBL) *of the Circle (if it be repreſented on the ſame Picture) will be tranſprojected, and form an oppoſite Hyperbola, above the Vaniſhing Line, which is equal and ſimilar to the other.*

For, if the Directing Line paſſes through the Center of the Original Circle, and the Eye, which is the Vertex of the Cone, is in the Directing Plane; in which Caſe, the Section, made by the Picture, is parallel to the Axe of the Cone, and the oppoſite Hyperbolas are equal, and equally diſtant from the Vaniſhing Line (OP).

But, when the Circle is cut unequally, by the Directing Line (KL) the Cone (BEAF) is alſo unequally cut by the Directing Plane (KEL) the Section of which, with the Cone, is the Triangle KEL; the Picture (NOPM) being parallel to the Directing Plane, cuts the larger Segment of the Circle, in MN, and the Cone, in the Curve f g a N, which is an Hyperbola, Prop. 104. P. 160. Em. C. Sec.

If the Sides (AE, FE, BE, &c.) of the Cone BEA, are produced through the Vertex (E) forming an oppoſite Cone (a E b d) the Picture being produced, will alſo cut that Cone, produced; its Section, with it, is the oppoſite Hyperbola, or tranſprojected Repreſentation of that part of the Circumference of the Circle, (KBL) which lies on the other Side of the Directing Line, by means of the Rays BEb, QEq, &c. produced to the Picture; which, notwithſtanding it is the Repreſentation of the leſſer Segment of the Circle, is generated by a ſimilar Cone, (a E b) and, although the Section is made at a greater diſtance from their common Vertex; it is equal and ſimilar to the perſpective and projective Repreſentation (f M g a N d) of the larger Segment.

This Curve, like the Parabola, can never be generated, or of uſe, but when the Area of the Circle is ſo large, that the Spectator is ſuppoſed to ſtand within it; and that Segment (M G A H N) which lies beyond the Picture, only, is required; and which differs ſo little from an elliptic Curve, in its Repreſentation, (M g a h N) that the diſtinction is not very obvious, and ſeldom, if ever, regarded.

Theſe are all the variety of Curves which a Circle in Perſpective can aſſume, however ſituated in reſpect of the Picture, or of the Eye; the chief of which is the Ellipſis. Every Circle, which the Eye is capable of taking, in at one View, it is manifeſt, has the Appearance of an Ellipſis; except, when the Axe of the Eye is perpendicular to the Plane of the Circle, and paſſes through its Center; in which poſition, only, it can appear, to the Eye, a true Circle; although all Repreſentations of Circles, in Planes parallel to the Picture, are Circles†; but, being ſeen oblique, when the Eye is in the true Point of View, they have the Appearance of Ellipſes.

† Theo. 1.

THEOREM III.

The Repreſentation, in Perſpective, of a Globe, or Sphere, is an Ellipſis*; except when the Center of the Sphere coincides with the Center of the Picture; in which Caſe, only, it is a Circle.

The Cone of Rays from the Eye, as its Vertex, to the apparent Circumference of a Sphere, is always a Right Cone. For, a Sphere can have but one Poſition, either to a Plane, Line or Point; and conſequently, the Diameter, every way, always preſents itſelf to the Eye, and always forms a Right Cone.

* I have found it very difficult, nay almoſt impoſſible, to convince ſome Perſons of this plain and well known truth; becauſe, as they truly obſerve, the Diameters of a Sphere always appear equal; which is as much as to ſay, that the Viſual Rays, under which a Globe is ſuppoſed to be ſeen, always form a Right Cone, i. e. whoſe Axe is perpendicular to its Baſe; which is not ſo with Circles, but in one poſition only. But, ſuch Perſons ſeem to forget, that the Repreſentation is on a Plane, and that Plane is conſidered as the Plane of the Section; which muſt, conſequently, cut every Cone oblique, but that which has its Axe perpendicular to the Picture.

Now,

DEM. It is evident, that, when a Sphere is ſo ſituated, in reſpect of the Picture and the Eye, that, the Direct Radial coincides with the Axe of the Cone, (i. e. when it paſſes through the Center of the Sphere) the Section of the Cone, made by the Picture, is parallel to every apparent Diameter of the Sphere, which is the Baſe of the Cone; and, being equal every way, it is conſequently a Circle.

But, the Direct Radial is the Axe of the Cone, which paſſes through the Center of its Baſe, and alſo through the Center of the Picture.

Conſequently, the Center of the Picture coincides with the Center of the Sphere, in this Section; and therefore, the Repreſentation is a Circle. Theo. 1ſt.

For it is the Section of a Cone, parallel to its Baſe.

2. But, if a Sphere be ſo ſituated that the Direct Radial does not paſs through its Center, the Axe of the Cone muſt be inclined to the Picture; which, being the Plane of the Section, the Cone of Rays are cut obliquely by the Picture.

But the oblique Section of a Cone, through both its Sides, is an Ellipſis. Pr. 90 Em.

Therefore, the Repreſentation of a Sphere, whoſe Center is not the Center of the Picture, is an Ellipſis. Q. E. D. For it is an oblique Section of a Cone.

EX. Let A B be a Sphere and C its Center (ſuppoſed to be ſeen). Let E be ſuppoſed the Eye of a Spectator, and E A, E B, Viſual Rays, from the Eye, to the apparent Diameter, A B *. A E B may, therefore, be ſuppoſed a Section of a Right Cone, through its Axe, E C, which is an Iſoſceles Triangle; the Sides, E A, E B, &c. of a Right Cone being equal ‡. Fig. 32. ‡ Def. 2.

Now, if F G be the Picture (which is a Plane) parallel to A H B I (the Baſe of the Cone) the Line a b, in which the Plane A E B cuts that Plane, is a Diameter of that Section; which Section, being every way equal, is, conſequently, a Circle.

But, E C, the Axe of the Cone, being perpendicular to its Baſe, is perpendicular to the Picture §, and to its Section, F G, with the Triangle A E B; the Point, c, where the Picture is cut by the Axe of the Cone, E C, is the Repreſentation of C, the Center of the Sphere. § 6. 7. El.

And, becauſe A B is a Diameter of the Baſe of the Cone, and C its Center, A B is biſected in C; conſequently, a b is alſo biſected in c; for, the Triangles C E A, C E B are congruous ||; and, becauſe a b is parallel to A B, the Triangles a E c, A E C, and c E b, C E B, are all ſimilar ‡. || 7. 1. El. ‡ 2. 6. El.

But, E is ſuppoſed the Eye of a Spectator, and, E c, being perpendicular to F G (the Picture) is the Direct Radial ¶; and the Point c, where it cuts the Picture, is the Center of the Picture §, which is alſo the Center of the Repreſentation of the Sphere, on that Picture; therefore it is a Circle. Theo. 1. ¶ Def. 15. § Def. 17.

Now, if the Cone A E B be ſuppoſed to be cut by any other Plane, paſſing through the Diameter h i, and conſequently through c, the Center of the Picture and of the Repreſentation; that Section will be an Ellipſis.

Let S D be a Section of another Plane or Picture, with the Triangle A E B, cuting the Rays E A, E B, in e and d; then is e d, in that Section, the repreſentation of the apparent Diameter A B, of the Sphere, and c is the repreſentation of its Center, as before, in the Section a b; but, e d is not biſected in c.

DEM. Let a c e be an Iſoſceles Triangle; a c = e c; and, a c = c b; th. e c = c b: Ax. 3. El.

But, becauſe b c E, in the Triangle b E c, is a Right Angle, c b E is acute †, and, the Angle c b d, in the Triangle d c b, is, conſequently, obtuſe. Wherefore, c d, ſubtending the obtuſe Angle, is greater than c b; And conſequently, e c + c d, equal e d, is greater than a c + c b, equal a b. † C. 3d 10. 1. El. C. 2. 1. 12. 1.

But, the Section H E I, which is vertical to A E B, cuts both Pictures in the ſame Line, h i, equal to a b; as above.

* See the Note to Art. 4. Page 14, on Direct Viſion.

Plate VII. Therefore, the Section, e h d i, is an Ellipſis; for, the Diameter e d is larger than any other, in that Section.

But, E S, perpendicular to S D, is the Direct Radial; wherefore, S is the Center of that Picture ‖; and it does not coincide with the Center of the Sphere (the Eye being at E) conſequently, the Repreſentation of a Sphere cannot be a Circle, except when the Center of the Picture coincides with the Center of the Sphere, in the Center of its Repreſentation.

‖ Def. 17.

Fig. 33. EX. 2. To illuſtrate this further. Let X, Y, and Z be three Globes, whoſe Centers are all in the ſame Right Line, parallel to a h, the Interſection of the Picture; whoſe Center is C, and E C its Diſtance.

Draw the Tangents E A, E B, E D, &c. to the three Globes, the Chords, A B, D F, and G H, of thoſe Tangents, are the apparent Diameters of each; which, it may be obſerved, are ſtill turned towards the Eye; and are conſidered as the Diameters of the Baſes of the three Cones, A E B, D E F, and G E H, which are all Right Cones.

Now, ſince they are all cut by the ſame Plane (of which a h is a Section) each Cone, except the firſt, A E B (whoſe Axe, E X, coincides with the Direct Radial, E C) is cut oblique; and, conſequently, one Diameter, of each Section, is larger, as the Globe is farther from the perpendicular E C; as d f, and g h, the Repreſentations of a Diameter of each Globe (Y, and Z) d f being larger than a b, and g h ſtill larger than d f.

§ Cor. 3. 1 h. 9. But, the other Diameters, perpendicular to thoſe, are equal, in all §, for they are ſuppoſed parallel to the Picture, and equally diſtant from it; conſequently, the Repreſentation of the Globe X, only, which is in the middle of the Picture, or, the Center, C, of its Repreſentation, in the Center of the Picture, is a Circle; becauſe, the Diameters perpendicular to each other are equal: all others (as of Y and Z) are Ellipſes, becauſe they are oblique Sections of Cones, whoſe Diameters are proved to be unequal.

N. B. It is the ſame however the Globes are ſituated; whether above, below, or ſideways of the Center of the Picture. If they are equally removed from the Perpendicular E C, and equally diſtant from the Picture, the Globes being equal, their Repreſentations are equal and perfectly ſimilar, though differently ſituated; and the farther they are remote, from the Perpendicular, the more excentric is the Ellipſis.

It is obvious and demonſtrable, that a b, the repreſentation of the Diameter A B, is biſected in C, the Center of the Picture, and the repreſentation of the Center of the Globe; for, the Triangle a E b is Iſoſceles. But, d f, and g h are not biſected, in i and k, the repreſentations of the Centers of the Globes Y and Z.

For, becauſe the Right Lines E Y and E Z (from the Eye to the Centers of the Globes) are perpendicular to the Chords, D F and G H, the Angles D E F, G E H are biſected by thoſe Lines; becauſe the Chords are biſected. C. 9. 1. El.

But, in the Triangles d E f and g E h, becauſe the Angles, at E, are biſected, by the Right Lines E i, E k; d f and g h are cut, by thoſe Lines, in the Ratio of the other ſides of the Triangles; - - - 3. 6. El.

and conſequently, d i : i f : : E d : E f; and g k : k h : : E g : E h.

† 12. El. But E g is leſs than E h †; becauſe, the Angle g h E is acute, and E g h is obtuſe; wherefore, g k is leſs than k h; and alſo, d i than i f.

Therefore, d f and g h are not biſected, by the Lines E Y and E Z. And conſequently, ſince they repreſent Diameters of the Spheres, Y and Z, their Repreſentations are not Circles; conſequently they are Ellipſes.

SECTION

SECTION VI.

Containing a full refutation of ſeveral Errors and abſurd Opinions, which many Artiſts entertain of Perſpective; and, therefore, look on it as an imperfect and fallacious Science.

I Shall, in this Section, in the firſt place, explain the reaſon why the Repreſentations of the Diameters of Columns, on a Picture which is parallel, or nearly ſo, to the Columns, are continually larger the farther they are removed from the Center of the Picture, and conſequently from the Eye.

As this is a particular circumſtance, which many Perſons ſeem inclined to diſpute, or, if it be admited, they look on it as an imperfection in Perſpective, I ſhall endeavour, and doubt not, to make it appear conſonant to reaſon and Perſpective, to their entire ſatisfaction. It has ſo near affinity to what has been ſaid, in reſpect of a Sphere, that the ſame Diagram might have done for both; but, in order to avoid miſtakes, and to keep the Ideas diſtinct and ſeparate, I have given another.

Let V, X, Y, and Z be the Sections of four Columns, by a horizontal Plane, in which is the Eye, at E. Let L M be a Section of the Picture, parallel to the Columns, and E I, E K, E A, E B, &c. Viſual Rays, from the Eye to the apparent Diameters of the Columns, which are ſtill turned towards the Eye, as Globes. Fig. 34.

It is evident, that the Viſual Rays cut the Picture more oblique, the farther the Columns are from the Perpendicular E C; and, notwithſtanding the optic Angles D E F, G E H are leſs, as the Columns are continued, their repreſentative Diameters, d f, and g h, intercepted between the Viſual Rays, continually increaſe; and would, if the Columns were continued, till the Interval between them was loſt; the Repreſentations of their Diameters ſtill increaſing till they touch and cut each other. For, the Space from Center to Center, of the Columns, are equal, and are, conſequently, repreſented ſo on a parallel Picture, if continued infinitely; what, then, can become of the Space between the Columns, if it be not added to their Diameters, in the Repreſentations on the Picture?

I preſume, no Perſon will ſay that there is any imperfection in Perſpective, in this Caſe; I do affirm there is none in Perſpective; the buſineſs, of which, is to repreſent Objects, truly, on a Plane; according to their Magnitudes, Diſtances, and Situations in reſpect of each other, of the Eye, and of the Picture; where, then, is the imperfection in this?

If a Perſon, not knowing how to chooſe a proper Diſtance, take, into the Picture, more than the Eye is capable of taking in at one View; or if, through ignorance, the Picture be abſurdly ſituated, in reſpect of the Object, is the fault in Perſpective, or in his Judgment? In Perſpective there is not, nor can be, on the Principles here laid down, any, the leaſt error, if the Elements of Euclid are to be depended on, upon which the whole Fabric is erected; if one falls, the other falls with it.

The Art of Perſpective is to repreſent Objects on a Plane, by geometrical Rules, according to their Situations, &c. (See Perſpective, Page 47.)

It is well known (or ought to be) that no Perſpective Repreſentation can appear perfect, i. e. it cannot truly repreſent the Original Object, though ever ſo accurately delineated, but when the Eye is in the true Point of View.

Suppoſe, then, E to be that Point; and A E B, D E F, &c. the Optic Angles under which the Columns X, Y, Z (being equal) are ſeen.

It is alſo, I preſume, allowed, that Objects appear to have the ſame proportion to each other, reſpectively, as the Angles under which they are ſeen. Th. 1. S. 3. D. Viſion.

But,

Plate VII. But, the Angle DEF is leſs than the Angle AEB; becauſe their Subtenſes are equal, AB=DF (ſuppoſing the full Diameter of a Column to be ſeen) and the Viſual Rays EB, ED, EF, &c. the Sides of thoſe Angles, ſtill longer, the farther they are from the Perpendicular, EC†. How comes it then, that the Diameter of that Column (on the Picture) is the largeſt, which is ſeen under the leaſt Angle? The reaſon is obvious; becauſe, the Picture, LM, cuts thoſe Rays moſt oblique, where the Angle is the leaſt; in the Points g and h, &c.

†C. 14. 1 El.

DEM. If EF and EH are produced till they cut a Right Line drawn through the Centers of the Columns (which is parallel to the Picture) in *F* and *H*; ED and EG cuts that Line in *D* and *G*; all which are beyond the Circumferences of the Circles; wherefore, *DF* and *GH* are, each larger than a Diameter; and, if it was not ſufficiently obvious, it would be eaſy to prove that *GH* is larger than *DF*. But, AB is leſs than a Diameter; conſequently, ab is leſs than df, and df than gh; for ah is parallel to *AH*. - - - 4. 6. El.

Now, if the Eye (at E) be turned towards the Column V; or, if the ſituation of the Picture be changed to NM, then is S, where ES cuts NM, the Center of that Picture; on which, it is evident, that, *ik*, the repreſentative Diameter of the Column V, being neareſt the Center, is the leaſt; which, on the Picture LM is equal to that of Y; for, they are equally diſtant from the Center of that Picture; and *g*h, on the Picture NM, the repreſentation of the Diameter of the Column Z, is conſiderably larger than gh on the other. Yet, to the Eye, at E, both theſe Pictures truly repreſent the Diameters of the four Columns V, X, Y, and Z; V and Y appear equal on both, they being equally diſtant from the Eye; X, the neareſt, will appear the largeſt, and Z, the fartheſt, from the Eye, will appear the leaſt.

DEM. With any radius, as ES, on E as a Center, deſcribe an Ark of a Circle, cuting all the Viſual Rays EI, EK, &c. from the four Columns.

The parts, *ab*, *df*, &c. intercepted between the Rays, are the true proportions of the apparent Diameters of the Columns, and conſequently, of their Repreſentations, on both Pictures.

But the Ark *ab* is the greateſt, *ik* is equal to *df*, and *g h* is the leaſt.

Wherefore, the Angle aEb is greater than dEf (equal IEK) and gEh is the leaſt; and conſequently, the apparent Magnitudes of the Columns, X, Y, and Z, are in the ſame Ratio. - - - Theo. 1. Direct Viſion.

It is unneceſſary to enforce this, by dwelling longer on it; as it is certain, if either of theſe Pictures be viewed from any other Station, they could not repreſent the four Columns V, X, Y, and Z, in the Poſition and Situation they are in.

Suppoſe the Eye removed to E^2 (the Point of View ought always to be oppoſite to the middle of the Picture) and, from that Station, to view the Picture NM.

Draw the Viſual Rays E^2a, E^2b, &c. and on E^2 deſcribe an Ark of a Circle, cuting them, in 1, 2, 3, &c. The ſlighteſt glance of the diſproportion of their Appearance, from that Station, is ſufficient conviction, that *ab* (which appears the largeſt from the true Point of View) whoſe apparent Magnitude is the Ark 3 4, does not appear half ſo large as *df*; and *g*h appears larger than *df* (which ought to appear the leaſt) as the Arks 3 4, 5 6, and 7 8, ſufficiently evince.

That the true, apparent magnitudes of the Columns V, X, Y, and Z, from the Point of View, E, are the portions *ab*, *df*, &c. of the Ark *idh*, is manifeſt, when we conſider, that the Picture, on which the Columns are repreſented, is a Plane; but that, their true Appearances can only be repreſented on a ſpherical Surface; i. e. on the Surface of a Sphere, the Repreſentation and the Appearance are the ſame, the Eye being in its Center.

Does any one imagine that there is a real Arch in the Heavens, which has that Appearance? in which, the Stars, &c. appear, equally diſtant from the Eye in its Center. The Celeſtial Globe, for inſtance, is a Picture of the Heavens, Planets,

Stars, &c. each Repreſentation of a Star, on its Surface, if they are truly depictured, would, to an Eye in the Center of the Sphere, exactly coincide, and be in the ſame Right Line with its Original in the Heavens; and, their apparent Diſtances, from each other, are meaſured by an Ark of the Sphere; whereas, their real Diſtances from the Eye and from each other, reſpectively, have not the leaſt affinity to their Diſtances, as repreſented on the Surface of the Globe or from its Center. Wherefore, to an Eye at E, the true Point of View for either Picture (L M or M N) each Diameter, being ſeen under its true Angle, and the ſame as its Original, will appear leſs and leſs, the farther they are diſtant from the Eye, or from the Center of the Picture; although their Repreſentations are continually larger and larger, on thoſe Pictures, as the original Columns recede.

And this will ever be the caſe, on a Picture parallel to the Columns, in ſome degree, at any diſtance of the Picture; but, at a proper Diſtance, for taking in the whole, the difference is ſo little, and that ſtill leſs as the Diſtance is increaſed, that, it is and ought to be deſpenſed with, by making them equal: but, I muſt obſerve, it is not, then, true Perſpective.

2. Methinks I hear ſome carping Critic ſay, I muſt allow, then, that Perſpective is ſomewhat defective; by no means; I have not yet given up the Point in debate. I ſay, that, although, at any Diſtance, there muſt and will be a difference, though ſcarce perceptible at a proper Diſtance, yet I would never adviſe a Perſon, who would repreſent a row of Columns, in full Front, to make the leaſt difference in their Diameters; for, ſince they ſupport an Entablature, which is repreſented perfectly horizontal, their Pedeſtals or Baſes the ſame, conſequently parallel, it would be very improper to make the Columns differ in width when they are equal in height, for this reaſon; becauſe it is impoſſible to confine the Eye to the true Point of View, always; from which if you deviate, ever ſo little, the whole Repreſentation is diſtorted and imperfect.

But, if it were poſſible to confine the Eye, I would not ſtep the leaſt aſide from Perſpective, on any account; let the Repreſentation be ever ſo diſtorted and prepoſterous, it will, and muſt, if truly repreſented (by the Rules hereafter preſcribed) appear to the Eye, in the Point of View, as the Original. If we may be allowed to take liberties, in any Caſe, where ſhall we draw the line between the perfect and imperfect Repreſentation? for the whole is more or leſs diſtorted; conſequently, the Rules of Perſpective are not to be depended on, at all. To what, then, muſt we have recourſe? the Eye is not, in many Caſes, a competent judge; we ſhould, if we follow its dictates, implicitly, have as many Points of View, in a Picture, as Objects; becauſe, if every Object be repreſented exactly as it appears to the Eye, on the ſame Plane or Picture, there can be no point of View for the whole; and conſequently, in a long, connected, and continued Object, compoſed of Planes and Right Lines (as in Buildings of any kind, of regular Architecture) it would be beyond the power of Art to connect the ſeveral detached pieces or projectures ſo, together, as to compoſe one entire and uniform Picture of the whole.

Notwithſtanding an ingenious Author has treated it ludicrouſly, in a ſuppoſed Dialogue between a Lady and an Artiſt, who was determined to abide by the Rules of Perſpective; his Argument has not the leaſt weight, and muſt be imputed to him, as not having a right notion of Perſpective. The Eye is conſidered as a Point; therefore, whether we ſuppoſe it confined to a Pin-hole or not 'tis the ſame; for he muſt allow (or he was very unfit to write on Perſpective) that there can be but one Point of View for a Picture, in which it can be perfectly ſeen, as intended by the Artiſt; conſequently, when the Eye is not in that Point, the Objects muſt neceſſarily appear more or leſs diſtorted, according as they are ſituated nearer to or farther from the Center of the Picture, or that Point which is oppoſite to the Eye.

3. I ſhall, however, for the ſake of the Argument, allow each Object to be truly repreſented, as they appear to the Eye, on a Plane Picture of a tolerable length, as a h: Globes are the fitteſt Subject to expatiate on, becauſe they are every way the ſame. Fig. 33.

Plate VII. Let C be the Center of the Picture, the Eye being at E; and therefore, the Picture may be ſuppoſed to be extended equally towards a as to h.

Now it is certain, that the three Globes appear round, let the Eye be ſituated where it may; but they cannot, or ought not to be ſo repreſented, on the Plane a h, to be viewed by the Eye at E; for if they were, they would not appear round, but gibbous, or Egg like, ſtanding erect, all but the Globe X, in the Center; and the more ſo, as they are farther removed from it; the Eye muſt be removed oppoſite to each, and conſequently, there would be as many Points of View as there are Objects, which is an abſurd Hypotheſis, in one Picture.

Suppoſe, from the Point of View E, I would repreſent the three Globes, X, Y, and Z, as they appear; that is, the outline of each to be a Circle; the Eye, and its Axe, E C, muſt be turned towards Y and Z, as E Y, E Z; and conſequently, the Picture is turned with it, into the Poſition b l, and l m. For, the Picture muſt be perpendicular to the Axe of the Eye, if the Object be repreſented as it appears; in which Caſe, there are three diſtinct Pictures, *viz.* a b, b l, and l m; each having a diſtinct Center, and the ſame Diſtance, E C or E c; on which Pictures, each Globe is repreſented by a Circle, and the fartheſt from the Eye (Z) is the leaſt in its Repreſentation, as they really appear.

It muſt be obvious, if the Globes were repreſented ſo on one Plane (of which a h is ſuppoſed a Section, and C its Center) that they would be under a worſe predicament than thoſe repreſented ſtrictly perſpectively: ſuch a Picture could not appear true in any Point of View, whatever, every Object having a diſtinct and ſeparate one.

Suppoſe them repreſented on the Plane a h, as they appear to the Eye, at E, i. e. round; the Repreſentation of the Globe Y leſs than that of X, and, of Z, leſs than Y, as they are repreſented on b l and l m; would they appear in the proportion they are repreſented, at E, or in any other Point? No, certainly; for, at E, the Repreſentations would appear leſs than the Original Objects, and not round (except X, only) Y would appear more round than Z, and Z would alſo appear rounder than any other, more remote from C; but they would, all, except X, appear eliptical Spheroids, and not Globes (which is obvious, to any Perſon tolerably acquainted with Optics or Direct Viſion) the fartheſt, from X, ſtill more ſo than the laſt.

Now, let the Eye be removed to *E*, or oppoſite to Z; Y, or Z, would then appear round, but not of the ſame proportion, as from the Point of View E; but, at *E*, neither X nor Z would appear round; and, although equally diſtant from the Eye, X would appear much the largeſt; where, then, in this Caſe, muſt the Eye be placed, to ſee thoſe Repreſentations ſuch as the real Objects appear? There cannot be a Point determined, for each Repreſentation has a ſeparate Point of View. Can, then, the Picture a h, in this Caſe, be a true perſpective Repreſentation of the three Globes, X, Y, and Z, as they appear to the Eye? certainly no, but each Repreſentation, on the Plane a h, is as much a diſtinct and ſeparate Picture, as the three Pictures, a b, b l, and l m; the difference is only, that the three Pictures have but one Point of View, and the Picture a h has three, equally diſtant from it; by ſuppoſing the three Pictures placed in a Right Line, a h.

4. If what I have here advanced be not ſufficient, to divert thoſe Artiſts of their abſurd notions of Perſpective, I ſhall give them one obſervation more, which they have not, perhaps, conſidered with that attention it requires; and then leave them to purſue their own way, if it appears to them more eligible and reaſonable, and will produce a better and more agreeable Repreſentation, of any Object whatever.

They quarrel and find fault with Perſpective, but without reaſon; becauſe, it is an infallible and moſt perfect Science. They would have all Objects repreſented, in Perſpective, exactly as they appear to the Eye; there is no ſuch thing to be done; 'tis not in the power either of Art or Science to repreſent, on a Plane, any ſingle Object, except a Sphere, or a plane Circle, having the Eye oppoſite to its Center, and the Picture parallel to it, as it appears; and yet, Perſpective will give a true and juſt Repreſentation of every regular Object.

The

The real, and the only cauſe of all their errors, and falſe notions of Perſpective, is their not rightly diſtinguiſhing between the Repreſentation of an Object, on a Plane, and the true Appearance of it; two diſtinct things, which can never be united, on a Plane Surface or Picture.

The perſpective Repreſentation of any Object is the ſection of the Cone or Pyramid of Rays, by a Plane; and the Appearance of an Object is the ſection of the Rays by the ſurface of a Sphere, only; to which, every Viſual Ray, from the Eye to the Object, muſt be Perpendicular; conſequently, the portions of the Arks, intercepted between the Viſual Rays, meaſure the Optic Angles, under which, every part of the Repreſentation is ſeen (the Ark a, or *a*, *d b*, Fig. 33 and 34, may be ſuppoſed a ſection of a Sphere, cuting the Viſual Rays E A, E B, &c). It is therefore manifeſt, that the true Repreſentation, and, at the ſame time, the true Appearance, can only be repreſented on a ſpherical Surface, the Eye being in its Center; but that is not a perſpective Repreſentation.

5. I expect it will, again, by ſome Perſons, be alledged, here, that Perſpective, then, is not ſufficient, to repreſent Objects as they appear to the Eye. I affirm that it is. Let us, therefore, once more enquire, candidly, what is meant by Perſpective, and what effects it is expected to produce.

Perſpective is a repreſentation, on a Plane, of an Object, or Objects, in a fixed and determined Poſition and Point of View. (See Perſpective, Page 47.)

I have already ſhewn the bad effect of viewing a perſpective Picture, out of the true Point of View; from which, if we deviate, we cannot expect that the ſeveral parts of a Picture can vary their Bearings and Proportions, to each other, as the real Objects; no, certainly, that muſt be a reality, not a Perſpective; which is but a Deception, a Repreſentation of a real Object, on a Plane; and, which, can never repreſent the Object, truly, from any other Point of View, but that for which it was delineated.

I have alſo ſhewn, to ocular conviction (by the Apparatus) that there may be as many various Repreſentations of the ſame Object, from the ſame Station, or Point of View, as there can be Poſitions of the Picture; all which, are true Perſpective, and will affect the Eye alike, in the true Point of View; which, is the Vertex of the Pyramid of Rays; ſeeing that, every correſponding Line, on each Picture, is ſeen under the ſame plane Angle as the Original, and every Surface, as well as the whole Object, is ſeen under the ſame ſolid Angle, or Pyramid of Rays, as the correſponding original Surfaces, or as the original Object.

What, then, is it we are caviling about? would ye have a real, ſolid Object on a Plane, or a Repreſentation of it only, in a certain Poſition? Certainly then, if the Eye be not in the true Point of View, the Picture does not truly exhibit an appearance of the intended Object, at the fixed Station; and, although it may be a juſt, perſpective Repreſentation, it may, nevertheleſs, be a very diſtorted and diſagreeable Picture; not owing to any fault, or imperfection in Perſpective, but to the choice of the Situation of the Picture, or the Diſtance and Poſition of the Object and the Picture.

Does not almoſt every Object, except a Sphere, appear different from every different Point of View? and can any Perſon be ſo unreaſonable as to expect, that a Repreſentation of a ſolid Object, on a Plane Surface, can appear truly to repreſent the Original in any other Point of View, but in the Vertex of the Pyramid of Viſual Rays, under which the Object itſelf is ſuppoſed to be ſeen? the very ſuppoſition is abſurd to the laſt degree; becauſe, no two parts of the Repreſentation can, at the ſame time, be ſeen under the true Optic Angle, in any other Point; conſequently, the Repreſentation, muſt appear erroneous.

6. Now, although the true Repreſentation, and alſo the true Appearance are depictured on a ſpherical Surface, yet, I affirm that ſuch a Picture is ſubject to much greater imperfection than a true Perſpective, on a Plane Surface; becauſe, if the Eye be the leaſt removed out of the Center, the whole Appearance and Effect is deſtroyed,

Plate VII. deſtroyed, and exhibits a much worſe Image of the Object, than a perſpective Repreſentation, on a plane Picture, can poſſibly exhibit, in any Point of View; which is ſo very obvious, that it is needleſs to point out the reaſon. For, ſuppoſe the Eye, at *E*, viewing the ſpherical Picture, a *d b*; the Viſual Rays, *E* a, *E d*, &c. ſhew, at firſt ſight, how much worſe ſuch a Picture muſt appear, than that on the Plane, a h, from the ſame, though falſe Point of View; not one of them will appear round, and the Appearance, of all, is prepoſterous.

From the circumſtance I have mentioned, in reſpect of the true Repreſentation and Appearance being depictured, at once, on a ſpherical Surface, ſome Artiſts imagine, that the Repreſentation on a Plane ought to be ſo delineated; it cannot be; 'tis impoſſible, in the nature of things. Suppoſe a true Repreſentation of a long Building, in full front, delineated on a ſpherical Surface, and it were poſſible, afterwards, to reduce the ſpherical Surface to a Plane; is any Perſon ſo weak as to ſuppoſe, that ſuch a Repreſentation would appear like the Original, in any Point of View? he muſt be weak, indeed, and have ſtrange miſtaken notions of Perſpective, who can; and yet I have heard this Point ſtrenuouſly ſupported, or rather argued for, ſupported it could not be; for, any thinking Perſon (who can think with propriety about it) muſt be ſenſible, that, what ſhould repreſent Right Lines will be curved, and, the whole, will give the Idea of a Rotunda, or externally round Building; ſeeing that, the extremes would fall off, not in Right Lines but curved, and they would appear leſs than the real Object; to ſay nothing of the almoſt impoſſibility of producing or delineating ſuch a Picture, at all, or by any means; I ſhould be glad to be informed, how, or by what Rules.

7. There may be alledged another circumſtance, which, I think, muſt convince the moſt obſtinate, in Perſpective. I am perſuaded, that, if the Eye be fixed in a Point, at a proper Diſtance from a tranſparent Plane, placed between the Eye and an Object, whilſt the Hand traced, accurately, every Line of the Object, as it appeared on the tranſparent Plane; ſuch a delineation, all muſt allow, would be a true one. Let thoſe, who are not otherwiſe to be convinced, try the Experiment. I will ſtake all my knowledge in Perſpective, that every Repreſentation of a Right Line is a Right Line, on the Plane; that Columns or Cylinders of equal magnitude, and parallel to the Plane, will be larger as they are more remote from that Point, on the Plane, to which the Eye is oppoſite; that the Repreſentation of a Circle or Sphere, ſeen oblique, is an Ellipſis; that Objects of equal magnitude, and equally diſtant from the Picture parallel to them, however othetwiſe ſituated or elevated will be repreſented equal; with various other circumſtances; all which may be fully proved to ocular conviction, which will not admit of the leaſt doubt. Surely then, if by the rules of Perſpective the ſame thing be effected, which certainly will, in every reſpect, it muſt exhibit true Repreſentations of Objects.

It has been obſerved, that the buſineſs of Perſpective is to produce the Figure of a ſection of the Cone or Pyramid of Rays, by a Plane, in any determined Poſition; which, if the Rules it preſcribes be truly followed, it will moſt certainly effect, without any ſenſible error. For, wherever any Point, or Angle of an Object, appears on a Plane, or other Surface, between the Eye aud the Object, there the Viſual Ray would cut and paſs through the Plane; but when the diſtance of the Eye is ſuch, that the Viſual Rays cut the Plane very oblique, or in Angles, nearly, or perhaps leſs than, half Right ones, the Repreſentation will conſequently be diſtorted and prepoſterous, and, in other Points of View, will have a diſagreeable and unnatural Appearance.

Here, then, lies the miſtake, which, through ignorance or inadvertency, is attributed to Perſpective, and ſuppoſed to be a deficiency or imperfection in it, 'Tis, generally, in the Point of View, the Situation, &c. of the Picture or Object; which, by being too near the Eye, occaſions that Diſtortion and prepoſterous Repreſentation we perceive, in ſeveral Pictures; for, if the Optic Angle, under which the whole Picture is ſeen, exceeds 50, or at the moſt, 60 Degrees, the Diſtance is not ſufficient; as, the Viſual Rays will cut the Picture very oblique, near its extremes,

and occaſion a diſagreeable diſtortion of the Objects on the extreme parts of it. Yet, as I have obſerved, at any Diſtance, the repreſentations of the Diameters of Columns, or other cylindrical Objects, on a Picture parallel to them, will, in true Perſpective, ever be the leaſt which are neareſt to the Center.

8. I ſhall now take notice of another great difficulty, which ſeems to be a ſtumbling Block to many Artiſts; who, one would imagine, would not heſitate one moment, to determine about it with propriety; which is, to repreſent, on a vertical Picture, the appearance of a direct Deſcent; which, Mr. Kirby has affirmed impoſſible, in the nature of things, to be done; that it is a ſtrong inſtance of the inſufficiency of Perſpective, and that, we muſt have recourſe to Experience, only, in ſuch Caſes; intimating, that it is not poſſible, by the rules of Perſpective, to give the Repreſentation of a deſcending Plane; which is ſo ridiculous an aſſertion, that, any Perſon, who underſtands Geometry tolerably, will eaſily be convinced of the contrary.

For, firſt, we are to conſider whether the Diſcent (which I ſhall ſuppoſe a Plane) is perceivable or not. If this deſcending Plane can be ſeen, at all, from any fixed Station, it may, undoubtedly, be repreſented on the Picture, from that Station, by the ſtrict Rules of Perſpective, or there is no truth in Perſpective; either it is a perfect and infallible Rule, or it is no Rule at all. If the Plane can be ſeen, it is a ſubject of Perſpective; if it cannot be ſeen, it is no ſubject for a Picture; which needs no Demonſtration. To tell us, what deſcends, and we actually know to go down-hill in Nature, will, if ever ſo correctly drawn, appear to riſe, upwards, on the Picture, is ſaying nothing to the purpoſe, the expreſſion is vague and nugatory; for, if the Plane deſcended ſo much as not to appear to riſe on the Picture, it could have no place or repreſentation thereon; but if it can be ſeen at all, it muſt, neceſſarily and unavoidably, appear to riſe; or rather, it muſt, really, riſe on the Picture, for, the Appearance is to deſcend.

And yet, he ſays we muſt have recourſe to Experience. What kind of Experience can teach us to do, what is impoſſible (as he ſays) in the nature of things, to be done? Experience will teach us all that can be done; Experience, in Perſpective, will teach us to delineate regular Objects which we never ſaw, yea, which never exiſted; but no Experience whatever, can make us perform impoſſibilities; to repreſent Objects which are loſt to ſight in the Originals, from the determined Station. But, how can any Perſon be ſo unreaſonable, as to require that to be done, in a Picture, which is ſo liable to falſe conſtructions, or determinations, in Nature; for, a level, or horizontal Plane will appear either to aſcend or deſcend, by the affinity and concurence of Lines and Objects connected with them; as for Example. When we are going down a regular deſcent, towards a level Plane, of Land or Water, the greater the deſcent, the more the level before us will appear to riſe; yea, ſo ſtrong is the Deception, that we are not eaſily convinced of the contrary; except it be Water, before us, which, we are certain, cannot aſcend. Likewiſe, when we are going up a gentle aſcent, towards a level Plane, which is in ſight, it will appear to deſcend; more eſpecially when we ſee the boundary Lines of both, as in regular Walks, in a Garden, &c. Nor is there any thing ſurpriſing in it, if we conſider, that, in the former Caſe, it is owing to the much greater breadth which we ſee, of the level Plane, and having the ſame appearance as an Aſcent, before us, when we ſtand or move on a level. In the other Caſe, the contrary effect, in appearance, is the cauſe of the Deception.

To draw two parallel and horizontal Lines acroſs the Picture, and to give an Idea, that the ſpace, between them, repreſents a deſcending Plane (of a certain length) without ſhape, bounds or limits, ſideways, or any Object ſituated on the inclination, to bias the judgment, is indeed impoſſible; but that is giving too great latitude to the meaning of the Expreſſion. I ſay, it is abſolutely impoſſible, in this Caſe, to give, by Art, an Idea of a deſcending Plane, ſimply as ſuch. For, although a proper gradation of Light and Colour may make a ſmall ſpace, on the Picture, appear to repreſent a great length, yet, 'tis not poſſible to ſay whether it deſcends,

Plate VII. or repreſents a level Surface; becauſe, the ſame ſpace, on the Picture, may repreſent an equal length, on a declining Plane, to a Spectator ſtanding, as is requiſite to repreſent a level Ground, to a Perſon laid along, or ſiting; conſequently, it is not poſſible to ſay, with abſolute certainty, whether it repreſents one or the other.

And yet, I queſtion if a ſkilful and ingenious Painter, in aerial Perſpective (who had critically obſerved and copied Nature) might not, by the effect of Colour, ſimply, even in this Caſe, deceive the Eye, and give the appearance of a Deſcent. But, if there are Objects ſituated on the inclined Plane, or, if the ſhape or figure of the Plane, itſelf, is to be deſcribed; whatever can be ſeen of ſuch Objects (whether Tops or Bottoms it matters not) they may and can be repreſented, truly and exactly, by the infallible Rules of Perſpective; and that, on the ſame invariable Principles, as the moſt common and ordinary Caſes, whatever, are ſubject to.

To illuſtrate what I have advanced, by a ſimple geometrical Scheme, will not be very difficult; which may be conſidered as a vertical Section through the whole. Each Plane is, therefore, repreſented by a Right Line, making the true Angle of the Inclination of the Planes (as I ſhall call them) with each other.

Fig. 35. Let A F repreſent the Plane of the Horizon, and A B a Deſcent, as a ſlope Bank, or any other Declivity; making the Angle A B G with the Horizon. Let C D be a Section of the Picture, E the place of the Eye, and E F its Altitude above the Horizon, at the Diſtance E C from the Picture, whoſe Center is at C.

Now, if the Eye (at E) be ſo ſituated, that a continuation of the inclined Plane would paſs through the Eye (as B A E) it is evident that the Plane (A B) from that Station, is totally loſt to ſight; its whole Repreſentation being the Line of its Interſection with the Picture†. Conſequently, if the Eye be moved further back (as at *E*) its appearance, or place on the Picture, would deſcend; which cannot be, unleſs the Plane be conſidered ſimply, a Plane, without ſubſtance, and ſeen on the under ſide; its apparent width from that Station is a D; but if the Eye be removed, nearer to the Picture, in any direction above B A E) the Repreſentation of the Plane riſes on the Picture. At E^2, its width on the Picture is a b, as the Rays, E^2 A, E^2 B, evince.

† Cor. 2. Theo. 2.

If the Eye be moved back again, in the direction B E^2, to E^3, it is plain that, b, the apparent height of B, remains the ſame, wherever the Eye is ſituated in that Line; but the repreſentative width, a b, of A B, will be increaſed, towards D, as it recedes, if the Picture be at any diſtance from the Deſcent. Suppoſe the Eye brought forward again to E^4; the width of A B from that Station is *a b*; and, if the Plane was continued infinitely, a V is its whole apparent breadth, E^4 V being parallel to A B.

9. I preſume, this is intelligible and clear, hitherto. I will, next, ſhew that 'tis impoſſible for the Eye to judge, merely from the width of its Repreſentation on the Picture, whether the Plane be deſcending or aſcending, or perfectly on a level.

From the ſituation of the Eye at E^4, the repreſentative width, on the Picture, of the Slope, A B, is *a b*; whereas, if the Plane was more inclined to the Horizon, i. e. if the Deſcent was more gradual, as A I, it would repreſent a leſs ſpace; if it was horizontal, the ſame *a b* repreſents the length A H, only; but, if the Plane aſcended from the Picture, as A K, it ſtill repreſents a ſhorter length (A K) on that Plane (the Arks I i, H h, &c. ſhew how much each is ſhorter than the other) and A L, making right Angles, B L A, A L E^4, with the Viſual Ray E^4 B is the ſhorteſt length, from the Point A, that can be repreſented, by *a b*, from that Station, ſituation, and poſition of the Picture.

Now, if any Object (as X) be ſituated on the Inclination A B; it is evident, if a continuation of the Plane of its Top (M N) paſſes above or through the Eye, at E^4, it cannot be ſeen from that Point; whereas, if the ſame, or any equal and ſimilar Object be ſituated on the Horizontal Plane A H, the top, O P, may be ſeen from that Station, notwithſtanding the Object is more elevated; becauſe, a continuation of the Plane of the Top, O P Q., falls below the Eye, at E^4.

The figure of the inclined Plane, itſelf, or the figure of any Object ſituated upon it, is deſcribed, perſpectively, in the ſame manner, and on the ſame Principles as on a horizontal Plane; which is exemplified in the practical part of this Work.

10. In the laſt place, I ſhall demonſtrate, that the repreſentations of Objects, which are elevated perpendicularly, above the Horizon, have the ſame proportion*, on a vertical Picture, as thoſe of the ſame Magnitude, ſituated on or near the Horizon; the Object being parallel to the Picture.

It is a miſtaken notion which ſeveral Perſons entertain, that the parts of a Building, which are elevated high above the Horizon, appear to diminiſh in a greater ratio than thoſe which are extended horizontally; ſuch an opinion may be eaſily refuted. If the ſecond Part of the 9th Theorem be well conſidered, it is ſufficient refutation.

Let A G be ſuppoſed a high Obeliſk, and A B, B D, &c. ſeveral equal Diviſions thereon. Let E S be a Spectator, E the Eye, and a g, or *a g*, a Section of the Picture; which, being vertical, is parallel to the Object A G. Fig. 36.

Now, if the Viſual Rays E A, E B, &c. be drawn, they will cut the Picture in a, b, d, &c. then is f g, the repreſentation of F G, equal to a b, the repreſentation of A B, or to b d, the repreſentation of B D†. † 4. 6. El.

For, ſince the parts A B, B D, &c. of the Original Object, A G, are equal, the repreſentations of them, on the Picture, being parallel to the Object, are alſo equal. But, A B is an Object, direct before the Eye, at E, on the Horizon A S; and, F G is one of equal length, elevated greatly above it; therefore, the Repreſentations of equal Objects at an equal Diſtance from the Picture, and parallel to the Picture, are equal. Q. E. D.

The ſame Object may be conſidered as a Building, extended horizontally, and A E G a horizontal Plane, in which is the Eye, and alſo the Viſual Rays E A, E B, &c. a g, or *a g*, is a Section of the Picture, as before. The interſections of the Viſual Rays with the Picture, it is evident, are the ſame.

11. There is yet another point of controverſy I have ſome times been entertained with; which is, that, when we ſtand, oppoſite the middle, near a long Building, or range of Buildings in a Right Line, the horizontal Lines, in the Cornice, &c. appear to decline towards either End, yet make no Angle; and therefore, they imagine that the repreſentation of thoſe Lines, in ſuch Caſe, will be curved. What a poor Idea muſt a Perſon have of Perſpective, who advances and is really prepoſſeſſed of ſuch an Opinion.

I ſhall ſay very little on this Point, becauſe, the eighth Theorem is full and perfect Demonſtration, that the Repreſentation (on the Picture) of every Right Line, is a Right Line; and, it is ſo if extended infinitely. Becauſe, a Plane may be ſuppoſed to paſs through any Right Line and the Eye; and, the Interſection of this imaginary Plane, with the Picture, is the indefinite Repreſentation of every Line it paſſes through; for, the Eye being in a continuation of it, the whole Plane is loſt to ſight †. Therefore, the Repreſentation of every Right Line is a Right Line. † Art. 3. of a Plane.

The truth, of which, any Perſon may ſoon be convinced of, by applying a perfectly ſtreight Ruler, before his Eye, parallel, or otherwiſe, to a Right Line, in an Object, of any length, and imagine the edge of the Ruler to cut the Picture, which is a Plane; then certainly, if the Ruler coincides with the Original Line, from one end to the other (which it undoubtedly will) the Right Line, in which it is ſuppoſed to cut the Picture, is the Repreſentation of the Original Line. It is the ſame as if a tranſparent Plane was placed between the Eye and the Object, and any Right Line, in the Object, traced on it, exactly, whilſt the Eye is fixed in a Point; the Line, ſo deſcribed, will be a Right Line.

Thoſe Perſons never conſider (but 'tis plain they are not furniſhed with the means) that the Picture, being placed in the true Point of View (conſequently at its proper Diſtance) will appear the ſame as the Original; for the ratio of the Parts is always in proportion to the Diſtance. Conſequently, every part of the Picture, being ſeen under the ſame Angle as the Original, will have the ſame Appearance, in every reſpect; and conſequently, the Right Lines, on it, will appear to decline either way, or both ways, the ſame as in the Object.

* What I would ſignify by the proportion of Objects, in this place, is, ſimply, length and breadth.

12. Reſpecting the appearance of parallel Right Lines direct before the Eye, there is ſomething paradoxical, which is not eaſily reconciled to reaſon; for it is manifeſt, from the foregoing, and the eighth Theorem, that every Right Line appears a Right Line, when the Eye is directed to it; and yet it is certain, that parallel Right Lines, however ſituated, appear to converge, and conſequently to approach each other.

Suppoſe two, parallel, horizontal Lines, direct before the Eye. It is manifeſt, that their Repreſentations, on a Picture parallel to them, are alſo parallel Right Lines. Notwithſtanding which, it is certain, that the neareſt Diſtance ſubtends a greater Angle, at the Eye, than any other; and that, the Angle ſubtended, by their perpendicular Diſtance, is continually leſs, the farther they are extended on either ſide. How, then, can they appear Right Lines? ſeeing that, they appear wider aſunder, direct before the Eye, than towards their extremes; nevertheleſs, ſince a Plane may paſs through each Line and the Eye, they muſt neceſſarily appear Right Lines, and conſequently parallel; ſeeing they ſubtend equal Angles, at equal Diſtances from thoſe parts which are neareſt to the Eye.

This very extraordinary Phænominon may be better conceived, by imagining the Eye in the Center of a concave Sphere; and, imagine two great Circles, which are Meridians, or longitudinal Circles, making an Angle of 15 or 20 Degrees, or more. Now, the Eye being in the common Center of both Circles, each appears a Right Line, which way ſoever the Eye be turned, towards the Circumference. But, the Meridians interſect and croſs each other, in the Poles; which, being diametrically oppoſite, cannot both be ſeen at the ſame time; but, in looking towards either, the Circumferences of the Circles will appear like two Right Lines, converging to a Point; the Pole, in which the Circumferences interſect.

Now, ſuppoſe the Eye directed to the Equator of thoſe Meridians; and imagine two equal Chord Lines drawn, one in each Circle, at equal Diſtances from the Eye, and parallel between themſelves. Thoſe Chord Lines, it is manifeſt, will appear to coincide with the Circumferences of the Circles, the Eye being in their common Center; and would, if they were continued beyond the Circles, infinitely. But, it is certain, that the Circumferences, being meridian Circles, croſs each other twice; wherefore, the parallel Lines, one in each Plane of the Circles, will appear to converge to a Point towards each extreme, which are their Vaniſhing Points; and, which, muſt be infinite, before they can appear to coincide with the Poles of the Circles. Thus, it is evident, that Right Lines do always appear Right Lines, and are repreſented by Right Lines; notwithſtanding which, parallel Right Lines appear, when direct before the Eye, to converge to a Point, towards each extreme.

I have now, I hope, fully refuted thoſe truly ridiculous and abſurd Opinions, of Perſpective, which many have imbibed, and are not eaſily diveſted of; they, rather, obſtinately perſiſt in them, without being able to give a ſolid reaſon for their Opinions, and are determined not to give up their Prejudices, at any rate, right or wrong. What ſtrange infatuation muſt poſſeſs that Perſon, who, having no argument of weight, to ſupport his falſe notions of Things, has recourſe only to Sophiſtry; and, becauſe he cannot come at ſterling Truth, himſelf, imagines there is no ſuch thing to be found. In points of natural Philoſophy, &c. where no certain Criterion can be obtained, to fix our aſſent, it is no wonder that we meet with ſo many, widely different, Opinions; and, though few of the arguments advanced have the leaſt foundation in Reaſon, 'tis amazing with what eagerneſs and warmth each Aſſailant attacks his Opponent. But, in Sciences, purely mathematical, all muſt agree; when Truth appears, there is no reſiſtance can be made; we cannot with-hold our aſſent ſo prevalent is her influence.

To ſuch as are open to Conviction, and are deſirous of coming at Truth, I think I have ſaid enough for their conviction, on the Points debated; but if they are not, all that can be ſaid is to no purpoſe, 'tis waſte of Time and Words. I ſhall, therefore, leave them to enjoy their Opinions, and pleaſe themſelves with their great Sagacity; and proceed to lay down certain and infallible Rules for the Practice of Perſpective, deduced from a perfect and well founded Theory; which, if truly followed, and proper attention be given to the Leſſons, contained in the Introduction, will moſt certainly produce Harmony and true Effect, of any regular Object, ſo far as comes within the province of linear Perſpective.

BOOK III.

Of the Practice of PERSPECTIVE.

SECTION I.

An INTRODUCTORY PREFACE.

I Come, now, to the practical, and, in that, the uſeful part of Perſpective; to which, the foregoing Book is an Introduction, only, but a very neceſſary Introduction; inſomuch, that, without the knowledge inculcated by it, we ſhould proceed in ignorance and uncertainty. Nevertheleſs, thoſe Perſons who have not ſtudied Geometry, and have not, now, perhaps, either leiſure or inclination to ſtudy it (though, in my Treatiſe of Geometry, they will find it neither ſo abſtruſe, tedious, or dry a Study, as many look on it to be, having treated that moſt uſeful branch of Science in a more familiar and intelligible manner, than has been done heretofore) I ſay, that, without a ſufficient fund of Geometry, to perceive and be clear in the Demonſtrations, if they will but treaſure up, in memory, the Theorems and Corollaries, and take all for granted (as they may depend on the Truths contained in them), they will find the great advantage of it, in Practice.

Every branch of Science is in two parts, *viz.* theoretic and practical. Theory teaches the knowledge of all that is neceſſary for Practice, in Speculation; the other applies that Knowledge to real uſe. It is neceſſary, firſt, to know how, before we can perform any thing; but notwithſtanding, many Perſons may be ſaid to know Perſpective (as a mathematical Science) yet know not how to apply it to Practice, with ſucceſs. So, every Art, dependant on Science, may be acquired without the Theory of it, by cuſtom or habit; as in mechanic Trades, derived from Geometry and Mechanics; all which, require time and application, to become familiar to us. So likewiſe, Perſpective may be practiced, without being ſaid, properly, that we underſtand it; ſeeing, we may not be able to give a ſufficient or ſatisfactory reaſon for the effects of it, in any inſtance. And, being well verſed in the Theory, we ſhall nevertheleſs find, that it will require time and aſſiduity to apply it, in all Caſes that may occur, in Practice, although founded on the ſame invariable Principles, without familiar Leſſons, in every Caſe, being given. But we ſhall, certainly, with the aſſiſtance of Theory, be able to comprehend, and to underſtand the various methods of applying the Rules, better than without it; as very little reflection, when we are at a loſs, will ſet us right again, and enforce the univerſality of its Principles.

It is not, to be wondered at, that the beſt of Treatiſes on Perſpective has been the leaſt underſtood; *viz.* that, by Dr. Brook Taylor; becauſe it is very obſcure. I call it the beſt, on account of his general Principles, not from its real utility to Artiſts, in reſpect of practicing from its Rules. It can ſcarce be ſaid, with propriety, that Perſpective exiſted before; it certainly was not properly underſtood, as a Science. How inſipid and imperfect are all the Books on the Subject before his, except Graveſande's and Ditton's; both which ſeem to touch on the ſame Principles, but are far ſhort of a perfect, general Syſtem.

There are, in that ſmall Treatiſe, Rules ſufficient, in almoſt all Caſes, for Plane Objects; but, various Examples requiring various ways of applying them, he has not made it ſo uſeful as it would have been, had he, inſtead of refering to former Problems, ſhewn how to apply it there. As, in Example 3rd, Page 24, Book the firſt, Fig. 21. We are told to make CB repreſent a Line, equal to that which is

repreſented by CA (by Prop. 15.) Now, I queſtion that one Perſon in fifty (who underſtands the whole) ever ſaw the leaſt Affinity between the two Examples; the different ſituations of the Lines makes it a very different Operation, though built on the ſame Propoſition of Euclid.

Reſpecting plane Objects, only, Perſpective is ſoon acquired; knowing how to delineate Figures in horizontal Planes, it is the ſame in any other; having found their Vaniſhing Lines, with their Centers and Diſtances; which I have firſt ſhewn, in Practice (as in Theory) how to find, in all uſeful Caſes whatever. But, without the embelliſhments of Mouldings and other architectural Ornaments, in Buildings, &c. all the reſt would be to little purpoſe. That Treatiſe comprehends only Perſpective in Plano; but I have, in this Book, ſhewn how to apply it in every neceſſary Caſe that can be deviſed; and doubt not, to render it, by that means, the moſt uſeful work, of the kind, yet publiſhed in any Language, that we know of.

Brook Taylor has, indeed, to his immortal Fame (in Perſpective) furniſhed us with new and extenſive Principles; but his Work, at beſt, is imperfect, and greatly deficient. His Theory is too conciſe, and is not regularly digeſted. It may perhaps, by ſome, be objected, that I have made too much of the Theory. I could have ſaid much more, but know not where to curtail it; the Examples, given for Illuſtration rather than Demonſtration, I am perſuaded, will not be found unneceſſary, to ſome, or trifling to any. One Example, in each Caſe, to ſome Perſons, would be ſufficient; to others there can ſcarce be too many, ſo they are various, and not a repetition of the former. I ſhall be guilty of the ſame fault (if it be a fault) in Practice; I had rather ſay too much than not ſay enough, yet I would not be tedious; becauſe, all that can be ſaid, to ſome, will be too little, or rather too much, ſeeing it will be all to no purpoſe. To ſteer the middle courſe is a difficulty not eaſily obviated; but, it is my fixed deſign to aim at it; others muſt determine how I have ſucceeded.

It is the opinion of many Artiſts, that the whole of uſeful Perſpective may be compriſed in a little Compaſs; that nothing is a greater diſcouragement to the ſtudy of it, than to ſee a voluminous Work on the Subject. 'Tis certain, that the Principles, on which the Theory of it is founded, are contained in a very ſmall Compaſs; and, I would recommend Brook Taylor's Epitome for that very reaſon, which contains *multum in parvo*. Yet, notwithſtanding that valuable Treatiſe has been ſo long publiſhed, it is, at this time, but little known, and leſs underſtood; which is a ſufficient reaſon, with me, to ſuppoſe that he has not ſaid enough on the Subject. The elaborate Work of Mr. Hamilton is ſpun out to an immoderate length; yet to as little uſe as the other. 'Tis my Deſign, to compriſe the whole of uſeful Perſpective in this Book. Neither of theſe Authors, I am perſuaded, had either taught or practiſed the Art of delineating; and conſequently, they were not qualified for treating it in an eaſy and familiar manner; one great requiſite in a Work of this kind. I have had experience in both, and am well convinced, that, to make it uſeful, it cannot be compriſed in ſo little compaſs as many imagine; that it requires frequent repetitions of the ſame Leſſon, ſomewhat diverſified, to familiarize the Rules, in various Caſes; without which, not one in twenty will ever be a Proficient. Dr. Taylor truly ſays, it is much better for the Student to deviſe Examples, himſelf, in particular Caſes, than to go through thoſe of others; but how few are capable of doing ſo? nay, I find, many are not able to comprehend them at any rate, nor by any means; and therefore, to make ſuch a Work really uſeful, variety of Caſes and Examples muſt be deviſed, for Practice and Experience.

In Practice, our Author has given ſome Problems, containing the moſt elegant and general Rules that can be, notwithſtanding they are but ſeldom practiced; becauſe he has not ſhewn, properly, how to apply them: that ſhall be my care to do, where they can be applied uſefully. His Diagrams are, in general, very imperfect, and badly deviſed; 'tis evident he was no practitioner or delineator, himſelf, even in Plane Objects (for he has given us no other). But, he has departed from his

own Principles, in Example V. Fig. 19. Part II. having projected the Dodecahedron, in Perſpective, by means of the Ichnography and Orthography (as by the old Authors) inſtead of Vaniſhing Lines and Vaniſhing Points; which is much more maſterly, elegant, and perfect; and is what the difference chiefly conſiſts in. Such Subjects are indeed of little uſe, except to familiarize us to find Vaniſhing Lines and Points in all poſitions of Planes and Lines, to the Picture, and ſituations of the Object, or Picture.

Some Perſons are ſo bigoted to the old Authors, that they cannot be reconciled to the new Terms, by Brook Taylor; nor indeed to his new Principles, till they find their Excellence by experience. It is not to be wondered at. 'Tis not eaſy to diveſt any Perſon of old habits and methods of practice, though ever ſo abſurd; becauſe it is impoſſible that they can ſee the difference, at firſt, and conſequently cannot judge of it; but it is ſurprizing that they are not to be prevailed on to try; and if they do, it is with ſeeming reluctance, and with a fixed reſolution to prefer, and perſevere in their old Prejudices. I am as much againſt capricious innovations in Science as any Perſon; but, if there be an appearance of any acquiſition to it, we ought, candidly and unprejudiced, to make a fair trial of their merits; without which we cannot judge of their Excellence.

In reſpect of the new Terms given us by Dr. Brook Taylor, ſuch as Interſection of the Picture, Center of the Picture, Vaniſhing Lines and Points, &c. (together with Directing Plane and Line, which are moſt eſſential, in Theory, though but of little uſe, in Practice) I am of opinion that no better Terms could poſſibly be deviſed; nor any other ſo expreſſive of what is meant by them. How narrow, how limited is the Baſe Line and Horizontal Line (the only Vaniſhing Line known to the old writers on Perſpective) when compared with them! What difference is there, either in Theory or Practice, between the horizontal Vaniſhing Line and any other, of Planes perpendicular to the Picture? None at all, ſeeing, they have the ſame Center and Diſtance (Th. 4.); nor indeed in any other, except in finding them; the Practice, in all, is the ſame, in every reſpect.

It was impoſſible for him to make the Principles of Perſpective general, but by general Terms; which does not regard any Poſition either of the Picture or of the Original Plane, ſince all Planes (ſimply as Planes) are the ſame. The Interſection of the Picture includes every Interſection whatever, as well as the Baſe Line, and they are all of the ſame uſe. Vaniſhing Line is not only general, but is, at the ſame time, ſo ſimple and expreſſive, that it conveys its utility at once to the Mind.

In the ſecond Theorem of the firſt Book, it is proved, that parallel Right Lines, however ſituated, appear to approach towards each other; and, conſequently, if they are produced, infinitely, they will appear to meet, and vaniſh, in a Point at an infinite Diſtance. So likewiſe, parallel Planes appear to meet each other, and to vaniſh in a Right Line, (ſuppoſed to be infinite) or, properly, in a Point.

Now, if this Theorem be well conſidered and underſtood (together with the foregoing) it will be found to be the foundation of the new Principles of Perſpective. For if a Plane be ſuppoſed to paſs through the Eye, parallel to any Plane, whatever, or any number of parallel Planes, and being produced, or continued till they are loſt to ſight, they will all appear to unite, and to meet the Plane paſſing through the Eye, at an infinite Diſtance.

But, if the Eye be in the continuation of a Plane, the whole of that Plane is loſt to ſight, and appears but a Right Line (Art. 3. of a Plane, P. 41,) And, the Interſection of two Planes is a Right Line, (Ax. 3.) Wherefore, if a Plane (which may be conſidered as the Picture) be ſituated any how; the Line, in which this imaginary Plane would cut the Picture, is that in which the parallel Planes unite, and vaniſh; conſequently it repreſents an infinite Diſtance; and conſequently, the Line, ſo produced, is their Vaniſhing Line; for they cannot, if continued infinitely, appear to go beyond it.

Hence,

Hence, thoſe Planes are, very aptly, ſaid to vaniſh, they being loſt to ſight. Therefore, all Parallel Planes have the ſame Vaniſhing Line. (Theo. 3rd.)

Alſo, if a Right Line be ſuppoſed to paſs through the Eye parallel to any number of Lines, they will appear to converge towards that Line, and to meet it in one Point, at an infinite Diſtance (Theo. II. Direct Viſion.)

Wherefore, if this Right Line from the Eye cuts a Plane, any how ſituated, it will cut the Plane in a Point only; which repreſents a Point at any Diſtance whatever, in that direction, and conſequently, it repreſents the Point in which the Lines, parallel to it, converge; which is infinite. Therefore, it is their Vaniſhing Point; for, they are loſt to ſight before they appear to reach it, ſeeing it is infinite.

And, ſince the Line, producing that Point, paſſes through the Eye, the whole Line is loſt to ſight, ſeeing that one Extreme is at the Eye; and the Extremes of Lines are Points. Therefore, the Point, in which it cuts the Picture, is its whole Repreſentation; and conſequently, all Lines, parallel to it, tend to that Point.

Now I muſt own, that I cannot conceive any Term ſo fit to expreſs that Line, or Point, in which parallel Planes, or Lines, meet each other, as Vaniſhing Line, and Vaniſhing Point; becauſe they are truly ſaid to vaniſh in them. For the ſame reaſon, perhaps, Mr. Noble, the laſt writer on Perſpective, (except Fergusſon) has made uſe of the Terms Entering Line, and Entering Point; ſeeing that, the Plane, or Line, begins at the one and vaniſhes in the other. Had this Author been the inventor of thoſe Principles, I ſhould not have found fault with the Names he had given them; but, ſince there were Names already given, by the Author, which are more ſignificant, I muſt blame even an attempt to alter them; becauſe, a multiplicity of Names, for the ſame Thing, occaſions a confuſion of Ideas, in the Mind, of their ſignification and uſe; and cannot poſſibly be of advantage to the Science.

In the proceſs of this Book, after ſome neceſſary obſervations, on the proportion of the Picture, the Height and Diſtance of the Eye, &c. in the third Section, I have, firſt, ſhewn how to determine on the Poſition of the Picture, in reſpect of the Object and the Eye, the Station being previouſly determined; then, how to prepare the Picture, for Practice, according to the Principles contained in the foregoing Theory.

The Picture being prepared, the Situation and Diſtance of the Object, and the Poſition of the Picture being determined, the following Problems, in that Section, ſhew how to find the Interſections and Vaniſhing Lines of Planes, in all Poſitions to the Picture, if they are not parallel to it (for all ſuch have no Vaniſhing Line. Theo. 1ſt.) with their Centers and Diſtances. Then, to fix the Vaniſhing Points of certain Lines in thoſe Planes, and determine their Diſtances; by which, the Lines, vaniſhing in them, are proportioned.

The Poſition, Situation, and Diſtance of the Picture (in reſpect of the Object and the Eye) being determined, the Original Lines, in the Objects, are produced (if neceſſary, and not parallel to the Picture) to their Interſecting Points; which are always found in the Interſection of the Plane they are in (Theo. 10th) or being much inclined to the Picture they do not, perhaps, fall within the compaſs of the Picture; then, other expedients are uſed, to find the repreſentation of ſome principal Point in the Line, from which Point, the indefinite Repreſentation is drawn (Theo. 12th.) And laſtly, the finite Parts, which repreſent certain portions of Lines in the Original Object, are determined, by Theorem 13th; by which means, the Object is completed; proceeding from one Plane, or Face of the Object, to another; drawing all the Figures, in each, by means of their reſpective Vaniſhing Lines. Each two adjoining Faces, having one Line common to both, the Vaniſhing Point, of that Line, is in both Vaniſhing Lines (Theo. 10th) conſequently, it is in their common Interſection (Cor. 2nd. Theo. 7th.) by the help of which, the Vaniſhing Lines of contiguous Faces are determined.

In Section 4th, I have briefly illuſtrated all the remaining, practical, Problems in Brook Taylor's Eſſay, reſpecting the proportioning of Right Lines, perſpectively, and ſhewn their great and extenſive utility; each of which, founded on the moſt ſolid and permanent Principles, is of immenſe value. For, without knowing the whole of Perſpective, or practicing by its Rules, rigidly, an Artiſt, who is accuſtomed to ſketch, by ſight, whatever he ſees before him, with ſeeming accuracy, may, by theſe Problems, rectify any errors, in right lined, or circular, Plane Objects, from the known proportion of one part to another; the affinity of the Planes and Lines to each other, being known, and the ratio of one Part to another; which, may frequently be obtained, when the true meaſures of thoſe Parts cannot.

Theſe Problems contain all the Rules neceſſary for Practice. They may be compared to the five fundamental Rules in Arithmetic, by which all others are worked; and, a Perſon might, with as much propriety, imagine that he had given Arithmetic enough, for every Occaſion, in thoſe Rules, as Brook Taylor had of Perſpective, in his firſt Eſſay; whereas, the Rules, he has there given, are no more than the Elements of practical Perſpective.

This Section contains, alſo, various Expedients; *viz.* for determining Vaniſhing Points, when they fall beyond the limits of the Picture, geometrically and arithmetically, i. e. to determine their Diſtance from the Center of the Vaniſhing Line and from the Eye, both which are neceſſary to be known; how to draw Lines to a Vaniſhing Point which is beyond the bounds of the Picture, &c.

In Section 5th, thoſe Rules are applied to real uſe, in delineating all kinds of Plane Figures; and, on Planes in various Poſitions. Firſt, by means of the Figure being geometrically drawn, in the Original Plane. Secondly, without it, by their known Proportions, their poſition to the Picture, and the properties of the Figure, being regular.

In the 6th Section, from Plane Figures, I have proceeded to Solids, compoſed of Planes, of various Figures; and in various Poſitions.

In the 7th, I have applied them to ſtreight Mouldings, compoſed of Planes and cylindrical Surfaces, in Cornices, Entablatures, &c.

Section the 8th treats of curve lined Objects, in general.

The 9th ſhews how to apply the whole to compound Objects, in regular pieces of Architecture, and Buildings of various kinds.

The 10th is for internal Views, and horizontal Pictures, in Ceiling Pieces, &c.

The 11th is adapted for the particular Profeſſions of Cabinet-makers, Coach-makers, &c. and for Machines, in general.

The 12th, and laſt, is on inclined Pictures and Planes, in general; and, applied to Fortification, or military Architecture.

N. B. A Scale of equal Parts is always adapted, or determined on, of the Proportion we intend to delineate the Objects, on the Picture.

*** The methods of making Scales, for various purpoſes, are given in the Appendix to my Treatiſe of Geometry.

Plate V.

SECTION II.

A preparatory and elementary INTRODUCTION, to the PRACTICE of PERSPECTIVE.

AS this Book is intended for a compleat practical Treatiſe, I have treated it in ſuch a manner (in this Section) as if no Theory, or Elements, had been given. For which reaſon, I have defined a few more Terms, which are ſuited to Practice only; as there are ſeveral Terms in the theoretic Liſt which may be omited here; and, there are alſo, in Practice, ſeveral which are not uſeful, in Theory. I have deduced from the Definitions ſuch uſeful Leſſons, which, if carefully attended to, will contain all the neceſſary Theory for a Practitioner.

PERSPECTIVE, is the Art of delineating the true Repreſentations of Objects, on a Plane Surface, by geometrical Rules; according to the Poſition, and Diſtance of the Objects, in reſpect of the Picture and of the Eye. (See the Apparatus.)

Figure 5.

The PERSPECTIVE REPRESENTATION of an OBJECT, is the Section of the Pyramid of Rays, AEI, by a Plane, in any Poſition; which, is the Subject of this Third Book. For, the Picture of every Object, truly delineated in Perſpective, is ſuppoſed to be ſo ſituated, in reſpect of the Object and the Eye, that if Viſual Rays, or Threads, i. e. if Right Lines were drawn from each Angle, or other Point in the Object, to the Eye, they would paſs through the correſponding Points in the Repreſentation, on the Picture.

As EA, EB, EF, &c. cut the Pictures in *a*, *b*, *f*; a, b, f, &c. which are, therefore, the Perſpective Repreſentations, of the Points, A, B, F, &c. on each Picture.

It is evident, that the Perſpective Repreſentation of an Object is nothing more than the Figure projected on a Plane, by its Interſection with the Viſual Rays from the Eye to the Object; wherefore, the whole buſineſs, of practical Perſpective, is to find the true Figure of the Section of the Rays, in all Poſitions, whatever, of the Picture (which is conſidered as the interſecting Plane) and in every ſituation of the Object, or of the Eye.

In order to which, it is neceſſary to reconſider, well, the conſtruction of the elementary Planes; as in Plate IV; or, to make them more familiar, I have given another, which is better ſuited for Practice; as they are the foundation of the whole, and the origin of all the Lines and Points uſed in Practice. Alſo the general Introduction to this Work ſhould be attentively peruſed, and tolerably well underſtood, by every one who would be a Proficient in the Practice of Perſpective.

Let BFIL be an Original Object, repreſenting a Building, and, ES a Spectator, viewing that Object, from the Point of View E (the Eye.)

It is manifeſt, that whilſt the Eye remains fixed, in that Point, the Object cannot vary in its Appearance. But, it is alſo obvious and demonſtrable, that every different Section of the Pyramid of Rays, by a Plane in different Poſitions, will exhibit a different Picture; ſo very different, indeed, they may be, that they can ſcarcely be ſuppoſed to be Repreſentations of the ſame Object, much leſs from the ſame Point of View and Poſition of the Object, when viewed direct.

Compare the Pictures, MNOP and *OP*; how different are the Images of the ſame Object, to each other. The one, on MN*OP*, has the Repreſentation, *bgfdc*, of the end of the Building, BFC, ſimilar to the Original, in every reſpect; the Repreſentation, *ahgb*, of the Front, AHGB, is diſtorted, and drag'd out to a prepoſterous length, when the Eye is oppoſite to it; whilſt the other Picture exhibits a pleaſing and natural Appearance, in almoſt any Point of View, but moſt ſo in the true one. Yet, both Pictures affect the Eye alike, in the true Point of View, and appear the ſame as the Original Object, itſelf; each Line, Plane,

or Figure (it is evident) being ſeen under the ſame Optic Angle as in the Object. Conſequently, the Picture being a true Perſpective Repreſentation of the Original, if it had likewiſe the ſame degree of Colour, Light and Shade, it would not be poſſible for the Eye, at E, to diſtinguiſh whether the Image (*a i f c*, or a i f c) delineated on a Plane, or the Object itſelf (A I F C) was preſented to view.

Having, thus clearly, explained what Perſpective is, I ſhall next define all the practical Terms, by means of which, the whole proceſs is performed, and the Repreſentations effected,

Let the Plane A I K B (which may be ſuppoſed the Picture) be raiſed up into a vertical Poſition, i. e. perpendicular to the Plane it is fixed to; which is the moſt natural, moſt general, and convenient Poſition. Pl. VIII. Fig. 37. No. 1.

Alſo, raiſe up the Plane G K I H, parallel to the Picture. This Plane is called, the Directing Plane (Def. 4.) and is very uſeful, in Theory. In it, the Eye is always ſuppoſed to be, as at E; and if the Plane N*I*K L be turned down, parallel to A B G H, the Line *I K* meeting I K, in the Directing Plane; then, E C is the Diſtance of the Eye from the Picture (equal F D); and E F, (equal C D) is its height above the Plane A B G H, on which, the Spectator (E F) is ſuppoſed to ſtand. Therefore, at whatever Diſtance the Eye is from the Picture (as E C) a Plane G K I H is ſuppoſed to paſs through the Eye, parallel to the Picture.

The Plane V being turned up, perpendicular to the Picture, is conſidered as a part of the prime or chief Vertical Plane; and, N*I*K L is called the Horizontal Plane; both which are ſuppoſed to paſs through the Eye, at E.

N. B. This Conſtruction of the elementary Planes, ſo eſſential in Theory, is, alſo, very neceſſary to be well conſidered by every Practitioner, who may not have an inclination to go through the Theory; he will, moſt certainly, find his account in it.

DEFINITIONS.

DEF. A. The GEOMETRICAL, or GROUND PLANE is that Plane on which an Original Object, intended to be delineated, is ſeated.

As G H Z Z; on which the Plans, A C D F and X Y Z (ſuppoſed to be the Seats of ſome Objects) are geometrically drawn, which are to be repreſented on the Picture, A I K B.

On this Plane is always drawn, or ſuppoſed to be drawn, the Figures of the Seats or Plans of Objects ſituated on it, in their true geometrical Proportion, according as they are ſituated, in reſpect of each other and of the Picture; and they are (in common Perſpective) always underſtood to be beyond the Picture.

In the Apparatus, the Plane Z S Z is the Ground Plane, repreſenting the real Ground, on which the Object, B F I L, ſtands. The Rectangle A B C L is its Plan, or Seat on the Ground Plane.

Note. If the Object ſtands on a Floor or Table, &c. it is conſidered as the Ground Plane.

DEF. B. The GROUND LINE, or BASE LINE, is a Right Line on the Picture, drawn parallel to its Top and Bottom, i. e. parallel to the Horizon.

As A B, ~~Fig. 37. No. 1 and 3.~~ ~~(See Interſection. Def. 7. in the Theory.)~~

The Plane A I K B, being erected, repreſents the Picture, and A B Z Z, being horizontal, is conſidered as the Ground Plane. Therefore, the Line A B is the Interſection of the Picture with the Ground Plane. And, ſince all Objects which are to be repreſented, on the Picture, are ſuppoſed to be beyond it, they muſt neceſſarily appear above the Line of Interſection; for which reaſon, it is, very properly, called the Ground Line.

Note. On the Ground Line is applied all the meaſures, geometrically, of all Figures (as Plans, &c.) of Objects on the Ground Plane, to be delineated; and frequently, for other horizontal Planes, whoſe Interſection with the Picture is not given or drawn.

N. B. Of the ſame uſe is the Interſection of any other Original Plane with the Picture, *viz.* for proportioning Lines and Figures drawn on that Plane.

DEF. C. The HORIZONTAL LINE, is a Right Line drawn parallel to and above the Ground Line; as M L. (See Vaniſhing Line Def. 8; or G.)

The Horizontal Line is called ſo by way of eminence, or for diſtinction from all other horizontal Lines, being the firſt, or principal Vaniſhing Line; from its fixed and determined Poſition and Diſtance, all other Vaniſhing Lines, whatever, are determined; the Vertical Line is determinable without it.

N. B.

Pl. VIII.

Fig. 37.

N. B. Its Diſtance from the Ground Line is always equal to the height of the Eye from the Ground, being conſidered as a Plane, on which the Object, to be delineated, is fixed. So that, whether we be ſitting, ſtanding, or elevated, the height of the Eye, from the Ground Plane, being determined, and AE, or DC, being made equal to it (by the Scale of Proportion) NL, drawn through the Point E, or C, parallel to AB, is the Horizontal Line; or Vaniſhing Line of horizontal Planes.

Note. It is ſuppoſed (in Theory) to be produced by a horizontal Plane (N*I*KL) paſſing through the Eye and cuting the Picture, in NL, their mutual Interſection.

DEF. D. The VERTICAL LINE, is a Right Line drawn at right angles with the Horizontal and Ground Line; cuting the Picture into two equal Parts, perpendicularly. As ED. (See Def. 11.)

DEF. E. CENTER of the PICTURE (or POINT of VIEW) is the Point C, in which the Horizontal and Vertical Lines cut each other. (See Def. 16 and 17.)

N. B. The real Point of View is the place of the Eye, where it ought to be fixed when viewing a perſpective Picture. The Point C, on the Picture, oppoſite to it, where a Perpendicular from the Eye would cut the Picture, is, therefore, its Center; and is, generally, underſtood to be the Point of Sight; i. e. to which the Eye muſt be oppoſite.

Hence, if the Center of the Picture, C, be firſt determined (as it frequently is) then a Right Line drawn through C, parallel to the Horizon, as LM, is the Horizontal Line; and, another Line, drawn through C, perpendicular to it, as ED, is the Vertical Line.

DEF. F. PARALLEL of the EYE. If EC be taken, in the Vertical Line, equal to the Diſtance of the Picture; and, through the Point E (which is conſidered as the Eye) if a Right Line be drawn, parallel to the Horizontal Line, as IK, it is called the Parallel of the Eye, of horizontal Planes. (See Def. 9.)

DEF. G. VANISHING LINE, of any Original Plane, is a Right Line on the Picture, ſuppoſed to be produced by an imaginary Plane paſſing through the Eye, or Point of View, parallel to the Original Plane; the Line, in which ſuch a Plane would cut the Picture, is the Vaniſhing Line of that Original Plane.

COR. Hence, the Horizontal Line is the Vaniſhing Line of the Ground Plane, and all other horizontal Planes.

For, it is produced by a horizontal Plane (N*I*KL) paſſing through the Eye, and cuting the Picture.

COR. 2. And hence, all Planes, which are parallel between themſelves, (but, not to the Picture) have the ſame Vaniſhing Line †.

† Theo. 3.

For, there can be but one Plane paſſing through the Eye parallel to them all; and ſince it can produce but one Line on the Picture, by its interſection with it, that Line is, conſequently, the Vaniſhing Line of them all.

N. B. The Vaniſhing Line, and the Parallel of the Eye, of any Original Plane, are parallel to the Interſection of that Plane with the Picture ‡.

‡ Theo. 2.

Therefore, LM, the Horizontal Line, and IK, the Parallel of the Eye, of horizontal Planes, are both parallel to AB, the Ground Line; which is the Interſection of the Ground Plane, with the Picture.

§ Theo. 1.

N. B. 2. Original Planes which are parallel to the Picture have no Vaniſhing Line, nor Interſection §.

DEF. H. VISUAL RAY is a Right Line ſuppoſed to be drawn from the Eye to any Point in an Original Object.

Let the Picture, AIKB, be turned up into a vertical Poſition; and, let the Plane W be turned over, till *C* coincides with the Center of the Picture; then will *E* coincide with the Eye, or Point of View (E) if the Directing Plane, GEH † be turned up parallel to the Picture.

† Def. 4.

Let the Plane AFDC, on the other Side of the Picture, be alſo turned over, till it falls into the ſame Plane with W. Then, the Right Lines EA, EB, EC, are Viſual Rays from the Eye (E) to the Original Points, A, B, and C, on the other ſide of the Picture; which, by their Interſections with the Picture, give the Repreſentations a, b, and c, of the Original Points, A, B, and C, on the Picture.

See the Apparatus; in which, the Threads EA, EB, EF, EG, &c. are Viſual Rays; producing, by their Interſections with the Picture, the Repreſentations, a, b, f, g, and *a*, *b*, *f*, *g*, &c. of the Original Points A, B, F, G, &c. in the Original Object.

PM and *P*M are the Ground Lines of thoſe Pictures; VY, and CX are the Horizontal Lines; and, CL, and *O*P are the Vertical Lines. C, their Interſection, is the Center, or Point of View of each Picture; for, the Eye, conſidered as a Point, is oppoſite to either.

DEF. I. DISTANCE of the PICTURE is the length of the Perpendicular, or the ſhorteſt Line that can be drawn from the Eye of a Spectator, in the true Point of View, to the Picture; the Diſtance between the Eye and the Center of the Picture. As EC. (See Def. 15 and 18.)

DEF. K. INTERSECTING POINT, of an Original Line, is that, in which any Original Line (being produced) would cut the Picture.

B, P, and S, are the interſecting Points of XY, ZY, and ZX of the Three Sides of the Triangle XYZ.

In the Apparatus, *F*, *G*, and *H*, are the Interſecting Points of the Original Lines, IF, HG, and FG.

N. B. The Interſecting Point, of every Line, is in the Interſection of the Plane the Line is in, with the Picture (Theo. 10.) Therefore, *FGH*, drawn through the Interſecting Points *F*, *G*, and *H*, is the Interſection of the Plane FGHI. Alſo, *BG*, drawn through the interſecting Points, *B* and *G*, of the Lines AB and HG, is the Interſection of the Plane ABGH.

DEF. L. VANISHING POINT, of an Original Line, is that Point, on the Picture, in which, a Right Line, from the Eye or Point of View, parallel to any Original Line, would cut the Picture.

N, O, and L, are the Vaniſhing Points of the three ſides of the Triangle XYZ. C is the Vaniſhing Point of the Lines AC and FD; and, M and L of the Diagonals AE, BF, &c.

N. B. The Vaniſhing Point, of every Line, is in the Vaniſhing Line of the Plane it is in. (Th. 10.)

Therefore, a Right Line drawn through V and W, the Vaniſhing Points of IF and GF, is the Vaniſhing Line of the Plane FGHI. Fig. 15.

Alſo, RW, drawn through the Vaniſhing Points, Y and W, is the Vaniſhing Line of the Plane BFC; in which, the Lines BC, or GD, and GF, are ſituated.

COR. All Lines which are parallel amongſt themſelves have the ſame Vaniſhing Point‡; except they are parallel to the Picture; in which Caſe they have no Vaniſhing Point (Cor. to Theo. 1.)

‡ Cor. 1. Theo. 3.

For, EV, producing the Vaniſhing Points, C and V, on both Pictures, being parallel to AB, is conſequently parallel to GH, FI, &c§. and, ſince it can produce but one Point on each Picture, that Point is, conſequently, the Vaniſhing Point of them all, by the Definition. § 4. 7. El.

Alſo, EW, being parallel to FG and HI, produces their Vaniſhing Point, W.

In the Apparatus, V is the Vaniſhing Point of all the Lines AB, GH, and FI: Y is the Vaniſhing Point of BC, and GD, &c. and W, of the Lines GF and HI; on the Picture MNOP.

The Vaniſhing Point of FD is out of the Picture, below the Ground Line.

In the Picture MN*OP*, C, its Center, is the only Vaniſhing Point; *viz.* of the Lines AB, GH, &c. to which EC is parallel. For they are perpendicular to that Picture.

All the other Lines in the Original Object, in the Plane BFC, are parallel to that Picture; and therefore, they have no Vaniſhing Point.

Alſo, BG, AH, &c. being parallel to both Pictures, have no Vaniſhing Point, on either.

COR. 2. Hence, the Center of the Picture is the Vaniſhing Point of all Lines which are perpendicular to the Picture.

For EC, the Direct Radial, produces the Center of the Picture (Def. 17.) and, becauſe it is perpendicular to the Picture, it is parallel to all Lines that are perpendicular to the Picture, and conſequently it produces their Vaniſhing Point; by this Definition.

DEF. M. STATION POINT is at the foot of the Spectator; or, it is that Point, in which a perpendicular from the Eye would cut the Ground Plane.

If the Directing Plane, GHIK, be turned up, on GH the Directing Line, perpendicular to the Ground Plane; E being the Eye, EF perpendicular to GH, cuts it in F, the Station Point; and EF is the height of the Eye above the Ground Plane. Fig. 37.

DEF. N. STATION LINE is a Right Line drawn from the Station Point, in the Ground Plane, perpendicular to the Ground Line; or, it is the Interſection of the Vertical Plane†, with the Ground Plane. As FD, produced.

† Def. 5.

SECTION III.

Containing ſome preliminary obſervations, concerning the Proportion and Poſition of the Picture; of the Height and Diſtance of the Eye, &c. Secondly, How to prepare the Picture, for Practice. Thirdly, How to find Vaniſhing Lines, in all common Caſes; and to fix the Vaniſhing Points, of Lines in all Poſitions, in reſpect of each other and of the Picture.

IT may appear ſomewhat ſtrange, and unaccountable to many, of what uſe are all the imaginary and elementary Planes in delineating Objects; or how ſo many Planes, cuting each other, can poſſibly be applied, in Practice; ſeeing that, the Plane or Picture, on which the Object is delineated, is the only real Plane made uſe of. It may alſo be a matter of wonder, how a Viſual Ray can be drawn from an Object, which is beyond the Picture, to the Eye, on this Side; by means of which, it is very obvious (from the Apparatus) the Repreſentations of the ſeveral Angles, &c. in the Object, are projected on the Picture; but how this can be effected in delineating, muſt appear ſtrange to a Novice in Perſpective. All which, are, I preſume, in the following Pages, accounted for, very ſatisfactorily.

The Art of drawing in Perſpective has this advantage of all other. It is no a random ſketch, depending on the Hand and Eye; but, every Line (Right Line at leaſt) may be drawn to the utmoſt exactneſs; and the Points, where the Viſual Rays would paſs through the Picture, are determined, mathematically true; yet may be done by a Perſon entirely unacquainted with Mathematics, who ſhall adhere to the Rules contained in the following Sections.

It is evident, ſeeing that Viſion is conveyed (from the Object to the Eye) in Right Lines, that every Section of the Pyramid of Rays will appear the ſame, to an Eye in the Vertex. Conſequently, whether the Section be made nearer to the Eye or to the Object, whether by a Plane or other Surface, whether it be direct or an oblique Section, the effect, to the Eye, is the ſame; but it is obvious, that it can be ſo only in that Point of View; for, every different Section has a different Repreſentation; but, all parallel Sections are ſimilar Repreſentations. So that, whether the Picture be drawn by a larger or a ſmaller Scale of Proportion, the Repreſentation is the very ſame, except in Dimenſions; and ſuppoſes the Section to be made farther from, or nearer to the Eye, the Vertex of the Pyramid of Rays.

It may alſo be obſerved, that, if the Viſual Rays were produced or continued, beyond the Model, they would diverge, ſo, as to take in a real Building, ſituated exactly as the Model, of any Dimenſions. Which being premiſed, it is manifeſt, that a Picture, delineated, truly, by a ſcale of equal Parts, of any Proportion, from a real or imaginary Model, of a Building, &c. will as truly repreſent the Original, as if the full meaſures of it were applied; which could not poſſibly be done.

This conſideration may account for the Meaſures applied in Practice; which muſt always be in the ſame Ratio or Proportion, to each other, as the real Meaſures of the correſponding Parts in the Original Object.

PRELIMINARY OBSERVATIONS.

IN reſpect of the Shape and dimenſions of the Picture, no Rules can be preſcribed, it is always at the diſcretion of the Artiſt; unleſs it be proportioned to ſome particular Place which determines its Figure and Dimenſions. The oblong Rectangle is, in general, a more agreeable and convenient ſhape than a Square; about the Proportion of 3 to 2, i. e. if the length be three feet, the width may be two, or thereabout; as conveniency, for taking in the Objects, may require. Some Objects requiring it upright; others, and more generally, length-ways.

2. Neither

2. Neither can a certain and invariable Rule be given, for fixing the height of the Eye, and, conſequently, of the Horizontal Line. To fix it to half or a third part of the height, abſolutely, would be ridiculous; it muſt ever be at Diſcretion, in proportion to the Scale of the Drawing; a Landſcape View, from an Eminence, may raiſe the Horizon to the middle of the Picture, or higher, yet may be very natural. In general, five feet, or five feet ſix inches, the natural height of the Eye, is the moſt agreeable, being moſt accuſtomed to ſee Objects at that height; altho' it may not be, perhaps, above one fourth or fifth part of the height of the Picture. Too low, in a general View, is not agreeable, becauſe the recedings of the parts of Objects, on the Ground Plane, are not ſo diſtinguiſhable, as, they approach nearly to Right Lines, with each other.

3. Reſpecting the Diſtance, ſomething may be aſcertained. The Diſtance of the Picture is a material Circumſtance which ought to be well attended to; otherwiſe the whole performance may be a diſagreeable Diſtortion, inſtead of a pleaſing and natural Repreſentation. The Diſtance ought always to be conſiderably more than the height of the Eye; although, Brook Taylor and others, in their Diagrams, have made it much leſs, which produces a very bad effect; the Repreſentations of the receding parts of the Object, on the Ground Plane, are, by that means, drag'd out to an immoderate length. To illuſtrate it.

EX. Let AFEB be ſuppoſed the prime Vertical Plane, in which the Eye is ſituated, at E. Let EC, be its Diſtance from the Picture; of which, CD may be ſuppoſed a Section. Fig. 37. No. 2.

This, I preſume, is eaſy to conceive; for, ſuppoſe two Planes cuting each other at right angles, one of which is conſidered as the Picture; the other, a Plane paſſing through the Eye, perpendicular to it. The Right Line CD being conſidered as their common Section; and the Planes turned around on it, as an Axis (ſee Fig. 8. Pl. 3. in the general Introduction) in which revolution, either Plane will become direct before the Eye, and the other paſs through the Eye, cuting the former in CD.

Now, E is ſuppoſed the Eye of a Spectator, and, C the Center of the Picture; EC is, therefore, its Diſtance. Let an Object be ſuppoſed beyond the Picture, whoſe Seat on the Ground is AD.

If a Right Line, EA, be drawn, cuting CD, the common Section of the Picture and the Vertical Plane, in a; it is obvious that, to the Eye at E, the repreſentation of the Point A will be at a, on the Picture; and the length of Ground, between the Picture and the Point A (equal DA) is repreſented by Da. For, EA is a Viſual Ray, from the Eye to the Point A; in which Direction, that Point is ſeen; and conſequently, a is the Point in which it would paſs through the Picture.

As it may be obſerved in the Apparatus, by placing the Eye in a continuation of either Picture and the Ground Plane; i. e. in the Line of their common Section, PM.

Then (conſidering the Eye of the Spectator as the Point E; and, A, or any other Point, on the Ground Plane, anſwering to A, in the Figure) EA repreſents the Viſual Ray, and a, the Point where it paſſes through the Picture; which is repreſented by the Right Line CD.

Now, Da repreſents the length of Ground between the Picture and the Point A, the Eye being at E; but, being moved to *E*, the Appearance of it is at *a*; and, the Point a repreſents, at that Station, the length DG only, which is prepoſterous.

For, ſince DC is the whole indefinite Repreſentation of DA, produced (Theo. 12.) and Da, a*a* repreſent the finite, ſmall Portions DG, GA, only; conſequently, the remainder, *a*C, of the indefinite Repreſentation, DC, repreſents the whole of DA, produced infinitely beyond A.

It is evident, that the Point, a or *a*, from either Station, is mathematically determined; agreeable to Theorem 13th. For, DC being the whole indefinite Repreſentation of DA (Th. 12.) the repreſentation of any Point, A, is found; by making Da (or D*a*) to DC, as AD to AB (or AD added to CE) i. e. Da : DC :: AD : AB, as it is demonſtrated in thar Theorem.

The Triangles AaD, AEB, and CEa being ſimilar; Da : aC :: AD : CE; conſequently, Da : Da+aC (i. e. DC, equal BE) :: AD : AD+CE, equal DB (i. e. AB.)

N. B. While the Eye moves in the Direction EC, the Indefinite Repreſentation DC remains the ſame, and the finite Parts (Da or D*a*) of AD is continually varied †.

Alſo, whilſt it moves in the Direction EA the finite part Da remains the ſame, and the indefinite Repreſentation is varied. As, at e, the whole indefinite Repreſentation is D*a*, and the finite part is ſtill *D*a.

† Cor. 5. Theo. 12.

Thus, it is evident, that the Height of the Eye is productive of as great Diſtortion as the Diſtance; but, to determine, abſolutely, in what proportion one ſhall be to the other, is not poſſible, as various circumſtances may render all ſuch Rules exceptionable. In general, the Diſtance ought not to be leſs than twice the Height of the Eye (as at E) or, at the leaſt, as three to two (as at E²) but there may be a neceſſity, in ſome Caſes, to make it equal, or perhaps higher, for the conveniency of ſhewing ſome particular parts of the Object.

Pl. VIII. Mr. Kirby makes it a general Rule, for the Diſtance not to be greater than the Perpendicular of an equilateral Triangle, whoſe Side is the length of the Picture; nor leſs than half the Diagonal; which (when the Center of the Picture is in the middle) if rightly underſtood and applied, will not produce great Diſtortion, in the Repreſentations of Objects thereon.

No. 3. For, if S be the Center, or Point of View, of the Picture A N O B, the Diſtance S N, or S O, may be ſufficient, if there be no Objects near the Extremes, at A or B; but if there are, they will be diſtorted; becauſe, it is evident that the Optic Angle, under which they are ſeen, is a Right Angle, or 90 Degrees.

But, this Rule, for fixing the Diſtance, is ſometimes injudiciouſly adhered to; when the Center, or Point of View, is near either Extreme, as at M or L; in which Caſe, eſpecially if the View be internal, or have Objects ſituated near the other Extremes A or B, they will be greatly diſtorted.

Let A S or S N be taken for Radius, and, on M, deſcribe an Ark of a Circle, from E to F. It is manifeſt, that all the part beyond that Circle, towards L, being ſeen under an obtuſe Angle (which the Eye cannot poſſibly take in) will be prepoſterous.

Now, this is ſo very obvious, to a Perſon of any diſcernment, or a ſmall ſhare of knowledge in Geometry, that 'tis needleſs to expatiate longer on it; yet, this is one principal reaſon, why many Artiſts quarrel with Perſpective, and pronounce it deficient. For, not being acquainted with the Theory, they would have, on every part of the Picture, Objects repreſented as they appear; which cannot poſſibly be, if they are remote from the Center; for the reaſon given above.

Wherefore, if the Center of the Picture be judiciouſly fixed (as at C) in the middle of the Picture (reſpecting its length, only, not height) as it ought always, except in particular Caſes, to be, and C N or C O be taken for the Diſtance, there can no great inconvenience accrue, eſpecially in external Views; as there will, very probably, be nothing but Clouds repreſented, at N, or O.

The reaſon of all this is evident. For, wherever the Center of the Picture is fixed, every part of the Picture, ought, at the moſt, to come within a Circle, whoſe Radius, or Semi-diameter, is its Diſtance.

I ſhall however give one general Rule; which is, to make the Diſtance, at leaſt, equal to the length of the Picture, *E F*, the Center, C, being in the middle; but, if it be on either Side, as at M, or L, it muſt be equal twice M*F*, or *E* L, inclusive; ſeeing that, the Eye is always ſuppoſed to take into the Optic Angle, as much on one ſide of the Center as the other, every way.

4. The Poſition of the Picture, in reſpect of the Object and the Eye, is another eſſential Point to be well conſidered, and determined on. Without due regard to that, the other Preliminaries are to little purpoſe; as all the imagined caution, in the Height and Diſtance, may be rendered abortive, by that means.

The Diſtance of the Picture ought always to be regulated and governed by the real Diſtance of the Object, if it be a ſingle one; or, if there are a multiplicity of Objects, it muſt be calculated from the neareſt, intended to be repreſented.

Fig. 38. Let A B C, be the Seat or Plan of a Building, which is intended to be delineated from the Station E; the Diſtance from the neareſt part is E B.

Let D F be the Interſection of the Picture with the Ground Plane, or Horizontal Plane; and E the Station Point, or the Eye; then is E S the Diſtance of the Picture, applied cloſe to the neareſt part of the Object, A C, which is ſeen under the Optic Angle A E C.

The Diſtance E S (equal D F) is ſufficient for that Picture, i. e. for the Object A C. But, if there are more Objects, as X and Z, extended to the full length of the Picture, on either ſide, it is little enough; they being ſeen under the Angle D E F, which the whole length of the Picture ſubtends, viz. 55 Degrees, on the Ark *df*; and is the leaſt Diſtance I would ever uſe, in ſuch Caſe, when the Objects extend the full width of the Picture; although it is more than Mr. Kirby's greateſt.

ES is therefore the Diſtance of the Picture DF, and alſo of the Objects, or of a Plane paſſing through the neareſt Planes of X and Z; in which Caſe, it is evident that the Picture muſt be as large as the Objects themſelves, being applied cloſe to them.

If they are ſuppoſed real Buildings, and the Scale of proportion be determined on, *viz.* a 10th or 12th part, or any other; take Eſ (one third part of ES) for the Diſtance, and draw df parallel to DF; then is Eſ equal df; for Eſ: df :: ES: DF; i. e. they are equal, and conſequently, the Objects x, y, and z, being reduced to the ſame Proportion will all ſubtend the ſame Angles, reſpectively, as the Originals. By which means the Picture may be delineated of any Proportion.

Now, if the Station (E) be determined, from which the Object AC is to be drawn, the Poſition of the Picture is alſo determined. For, ES, the Station Line, biſecting the Optic Angle, AEC or DEF, ought always to be perpendicular to the Picture; conſequently, the Poſition of the Picture, DF or df, is determined; i. e. it muſt be perpendicular to ES.

For, when the Station, or Point of View, is fixed, from which we are determined to delineate the Object, is it not moſt rational to ſuppoſe the Picture, on which the Object is to be repreſented, placed direct before the Eye? (as MNOP in the Apparatus) or is it more eligible to place it parallel to either Plane, ABGH or BFC (as MNOP) i. e. to AB or BC, as DC or BG, whoſe Center, or Point of View, is at C or G. The very ſuppoſition of it is abſurd to the laſt degree; and yet, this abſurdity is commited by every Artiſt who places the Point of View at either Extreme, or perhaps entirely off the Picture, as is frequent. Fig. 15. Fig. 38.

The difference muſt be obvious to every Perſon who conſiders it. For, the Picture being placed direct, according to DF, the Optic Angle is no more than AEC, or *a*E*c*, about 23 or 4 Degrees; but, being placed in the Poſition DC, the Optic Angle is, *a*E*g*, 102 or 3 Degrees; for, EC, perpendicular to DC, biſects the Optic Angle, on that Picture; as ES biſects the Angle AEC, on the Picture DF; notwithſtanding, the Object, ABC, does not occupy one fourth part of the Angle *a*E*g*.

In the Apparatus, S*P*, on the Ground Plane, biſects the Optic Angle, pSt, under which the Object is ſeen, according to the Poſition of the Picture MNO*P*; for, the Angle, at the Station Point (S) is the ſame as in the Horizontal Plane, at the Eye.

How to prepare the PICTURE for PRACTICE.

Let AIKB (when raiſed perpendicular to the Ground Plane) be ſuppoſed the Picture; alſo, let the Directing Plane, GHIK, and Horizontal Plane, *I*KLM, be placed parallel to the Picture, and the Ground Plane. Fig. 37. No. 1.

Suppoſe the Figures AD, and XYZ, on the other ſide, are to be repreſented on the Picture; and the Points a, b, c, &c. where the Viſual Rays EA, EB, &c. would paſs through the Picture, to be geometrically determined thereon.

In order to which, ſeveral preparatory Lines are neceſſary to be drawn on the Picture; the Center and Diſtance muſt always be known or determined, together with the poſition or ſituation of the Object, in reſpect of the Picture and of the Eye, and alſo its height, above the Ground Plane.

Now, E is the place of the Eye, EC is the Diſtance of the Picture; and EF is the height of the Eye; the Point C, where the Perpendicular EC cuts the Picture, is its Center. ML, the Line in which the Horizontal Plane cuts the Picture, is the Horizontal Line (Def. C) or the Vaniſhing Line of horizontal Planes; and AB, in which the Picture cuts the Ground Plane, is the Ground Line (Def. B.) conſequently parallel to the Horizontal Line. Theſe two are the firſt and moſt uſeful Lines; the Center of the Picture (C) in this Caſe, is in the Horizontal Line, the Picture being vertical.

Pl. VIII. The next Line to be conſidered, and of uſe, is the Vertical Line, ED (Def. D.) If the Plane V be turned up, perpendicular to the Picture, it will paſs through the Eye, at E, and conſequently through EC, cuting the Picture in EC, the Vertical Line; which, being produced will paſs through D.

This Line always paſſes through the Center of the Picture, and is of the ſame uſe for all vertical Planes, which are perpendicular to the Picture, as the Horizontal Line for horizontal Planes; i. e. it is their Vaniſhing Line, which has, conſequently, the ſame Center (C) and Diſtance (EC).

The conſtruction of theſe five Planes ſhould be well attended to, and the Lines they generate, by their Interſections; three of which, *viz.* the Ground Line, the Horizontal Line, and the Vertical Line, are already produced on the Picture; and the Parallel of the Eye, IK (whoſe real place is out of the Picture) is tranſpoſed to the Picture, by turning up the Horizontal Plane on ML, its Interſection, till it coincides with, or falls into the Picture; and with it the Direct Radial, or Diſtance, (EC) together with the Eye (E) which falls into the Vertical Line, at E.

When the perpendicular poſition of the Planes has been conſidered attentively, let them be puſhed either from or towards you, keeping the Directing and Horizontal Planes, joined in IK; the Vertical Plane (V) may ſtill be ſuppoſed to cut them all at Right Angles, and generates ſtill the ſame Line on each. In all Poſitions, i. e. let the Angles they make with each other be what it may, the Planes are ſtill parallel to each other reſpectively; and their Interſections are ſtill parallel amongſt themſelves. (Theo. 2nd.)

Now, let the Picture be turned down, on AB, its Interſection with the Ground Plane, till they coincide; and let the Horizontal Plane, with the Eye (E) and the Parallel of the Eye (*IK*) be alſo turned down into the Picture (as it is repreſented on the Picture); alſo, let the Vertical Plane (V) be turned down on either ſide, into the Picture, and with it the Direct Radial (EC) i. e. the Diſtance of the Picture, falling into the Horizontal Line, with the Eye at *E*. And, laſtly, let that part of the Ground Plane, which lies beyond the Picture (on which the Seats of Objects are geometrically drawn out) be ſuppoſed to be turned, on AB (its Interſection with the Picture) quite over to the other ſide; and imagine it, ſimply, a Plane, without thickneſs; ſo that, all the Figures, deſcribed on it, are ſeen on the other Side, inverted; as *ABEF*, and XYZ; in which Caſe, it is obvious, that they have the ſame Poſition, or Situation to the Picture, as before; and the Lines (which are not parallel to the Picture) being produced, cut the Picture, in the Interſection AB, in the ſame Points, as before.

Thus, is the Picture prepared for Practice, in common Perſpective; and all the elementary Planes are reduced to one Plane, viz. the Picture*.

All the ſame Lines and Points, may be ſeen in No 3. which is diveſted of that apparent intricacy of Planes on Planes, conſequently it is more ſimple and intelligible; having only the preparatory Lines, anſwering to the Interſections of the elementary Planes with the Picture, on it. The Figures below AB, the Ground Line, are Plans of Objects, on the Ground Plane, intended to be delineated.

Fig. 37. No. 3. To prepare which, let the Ground Line (AB) be firſt drawn, at ſuch a convenient Diſtance from the bottom of the Picture, and parallel to it, as to allow room, below it, for drawing the Plans of Objects (as X, Y, Z) intended to be delineated, in their true geometrical Proportion, Place, and determined Poſition to the Picture, if neceſſary. The ſpace, below AB (as AFB) is not conſidered as a part of the Picture, but of the Ground Plane, whoſe real place is beyond the Picture.

* The Directing Plane (GHIK) not being of uſe in common Perſpective is ſuppoſed to be turned down, or removed out of the way.

Through

Through the middle of the Picture, draw ED, perpendicular to the Ground Line, dividing the Picture into two equal Parts. This Line is the Prime vertical Vaniſhing Line, in which, the Center of the Picture, or Point of View, is always (though not always in the Horizontal Vaniſhing Line, but when the Picture is vertical, as it is now ſuppoſed to be); make CD equal to the determined height of the Eye, C is the Center; through which, draw ML, the Horizontal Line, parallel to AB, the Ground Line.

Then, with the Radius EC (the determined Diſtance) deſcribe a Semicircle, cuting the Horizontal and Vertical Lines in *E*, E, and *F*; i. e. make C*E*, &c. equal to the Diſtance of the Picture, which are conſidered as the Eye, tranſpoſed to the Picture * (generally underſtood by the Points of Diſtance) and, through E, draw IK, the Parallel of the Eye, parallel to the Horizontal Line; then is the Picture prepared, having all the fixed Lines and Points determined thereon. It only remains, now, to find other Interſections, Vaniſhing Lines, and Vaniſhing Points, neceſſary for delineating the Objects intended.

The Ground Line, or Interſection of the Picture with the Ground Plane, is the firſt and principal Interſection; and the only one, in general, made uſe of as ſuch. But, the Interſections of other Planes are often wanted, and particularly vertical Interſections; on which, the meaſures of the heights of Objects are applied; although they are frequently made uſe of, yet very few conſider them as Interſections; and conſequently, they do not ſee the generality of the Principles and Rules by which the Work is performed; as they look on the operation in Planes which are vertical or inclined, in a different light from ſuch as are horizontal; whereas, if it be well conſidered, they will find it is the very ſame; for, wherever the geometrical meaſures are applied, in proportioning any Line, it is conſidered as an Interſection of ſome Plane, in which that Line is or may be ſituated. And, ſince the Interſection of any Plane determines the Poſition of its Vaniſhing Line, I have ſhewn how to find the Interſections of Original Planes, in all neceſſary and generally uſeful Caſes.

Inclined Planes, in general, I have reſerved for the laſt Section; but, as various Planes, which are perpendicular to the Picture, are inclined to the Horizon, and vertical Planes are frequently inclined to the Picture, I ſhall not conſider them, in either Caſe, as inclined Planes, but ſuch only as are inclined to both.

As the Horizontal and Vertical Lines both paſs through the Center of the Picture, they are, therefore, the Vaniſhing Lines of Planes perpendicular to the Picture † (being vertical) which, from their fixed and invariable Poſition, determine all other Vaniſhing Lines whatever. † Theo. 4.

Therefore, in the following Problems, I have, in the firſt place, ſhewn how, by them, to determine the Vaniſhing Lines of all Planes that are perpendicular to the Picture, either with or without the Interſection; which are ſubject to one general Rule.

Secondly, how to find the Vaniſhing Lines of vertical Planes, in all Poſitions to the Picture (except parallel) and alſo, of certain Planes inclined to the Horizon, which are ſubject to the ſame invariable Rule, as vertical Planes.

The fifth Problem ſhews how to find the Vaniſhing Lines of Planes any how inclined, both to the Horizon and to the Picture (being vertical) from their known inclination to the Horizon, &c. as ſpecified in the Problem. All which, are frequently neceſſary in common Practice.

* The Eye, or Point of Diſtance, may be any where in the Circumference of a Circle, whoſe Radius is EC, as occaſion requires; or, according to the Poſition of original Planes, which are perpendicular to the Picture, and their Vaniſhing Lines; all which, have the ſame Center and Diſtance (Theo. 4th.)

Plate IX.

PROBLEM I.

The Intersecting Point of any Line, in a Plane which is perpendicular to the Picture, being given, together with the Angle of its Inclination to the Horizon; to determine its Intersection and Vanishing Line; the Center of the Picture being given.

Fig. 39. Let I be the Intersecting Point of some Line, given; C is the Center of the Picture; and X is the Angle of Inclination of the Plane to the Horizon.

Through C, draw AB, the Horizontal Line, parallel to the Bottom of the Picture; and, through I, draw ID parallel to AB.

Make the Angle DIF equal to the given Angle X; and, IF is the Intersection required.

Through C, the Center, draw GH parallel to IF; GH is the Vanishing Line.

Note. The Vanishing Line may be determined, by making the Angle ACG or BCH equal to the Inclination known; without the Intersection.

DEM. Let ID be supposed the Ground Line, and I the Intersecting Point of the common Intersection of the Ground Plane and the other Plane; which, seeing both Planes are perpendicular to the Picture, is

† Cor. 1. 9. 7. El. also perpendicular to the Picture †.

‡ Th. 10. And, since I is the Intersecting Point, of a Line in the Plane, the Intersection of the Plane must necessarily pass through that Point ‡; and consequently, it will make the same Angle with the Ground

‖ 4. 1. El. Line and Horizontal Line, as the Planes make with each other (DIF, equal BLF ‖) equal to the given Angle X. (See Article 4th, of Planes and their Positions. Page 42.)

Therefore, IF is the Intersecting Line of that Plane.

§ Theo. 4. But, the Vanishing Lines of all Planes, which are perpendicular to the Picture, pass through the Center of the Picture §. And, the Vanishing Line of every Plane is parallel to its Intersection. (Theo. 2.)

Therefore, GH is the Vanishing Line of a Plane perpendicular to the Picture, and inclined to the Horizon in the Angle HCB equal FID, equal X.

N. B. If F be the Intersecting Point of any other Line in that Plane, and EF be drawn parallel to the Horizontal Line; the Angle EFI, equal FID (equal X) produces the same Intersection.

Also, if K be the Intersecting Point of any Line in another Plane, parallel to the former; then, AK, parallel to IF, is its Intersection.

For, all parallel Planes have parallel Intersections. But they have the same Vanishing Line (GH).

If *E*C, or EC, be equal to the Distance of the Picture; AK, or EF, may be considered as the parallel of the Eye; and *E*C, or EC, (perpendicular to GH, or AB) is the Vertical Line.

SCHOL. If the Angle of Inclination be greater or less, the process is the same. But, if the Angle be increased or enlarged to a Right one, the Plane is no longer inclined, but vertical; and, if the Intersecting Point, J, be in a Line, passing through C, perpendicular to AB, the Intersection and Vanishing Line are the same, *viz.* the Vertical Line EJ; which is the Vanishing Line of all vertical Planes that are perpendicular to the Picture. See this Problem illustrated by moveable Planes; Fig. 15. No. 2.

Fig. 15. No. 2. *IKLM* is the Original Plane, TU is its Intersection with the Ground Plane, or other horizontal Plane; *I* is its Intersecting Point, and *IN* is the Intersection of that Plane with the Picture; making the Angle *NI*B equal to the Angle PQR, of the Original Plane with the Ground Plane.

And, if IKLM (the Vanishing Plane of all Planes which are perpendicular to the Picture) be placed parallel to the Original Plane, *IKLM*, it will cut the Picture in ON, the Vanishing Line of *IKLM*, in that Position.

For, it passes through the Eye (at E) and the Direct Radial (EC); consequently

† 9. 7. El. it is perpendicular to the Picture †, and passes through C, its Center.

Hence; all Vanishing Lines, which pass through the Center of the Picture, have the same Center and Distance, *viz.* the Center and Distance of the Picture.

Note. If the Plane inclined on the other Side of its Intersection TU, the Angle of its Inclination is made on the other side of the Vertical Line; to which, particular regard must always be had.

PROBLEM II.

The Vaniſhing Line of a Plane being given, with its Center and Diſtance; and the Angle of inclination which any Original Line, in that Plane, makes with the Interſection, of the Plane it is in, with the Picture; to find the Vaniſhing Point of that Line, and to determine its Diſtance.

AB is the given Vaniſhing Line, and C is its Center. Fig. 40.

Draw CE perpendicular to AB, and equal to the Diſtance of the Vaniſhing Line given or found.

Through E draw GH (the Parallel of the Eye) parallel to the Vaniſhing Line. Make the Agle GEA equal to the Angle which the Original Line makes with the Interſection, cuting the Vaniſhing Line in A, the Vaniſhing Point ſought; and EA is its Diſtance, E being conſidered as the Eye.

Note. Regard muſt always be had, on which ſide the Original Line inclines, and the Angle GEA, or HEB muſt be made accordingly.

DEM. Imagine the Plane AGHB turned over, on AB, till E coincides with the Eye; then is GH in its true Place (ſtill parallel to the Picture); and EA will be parallel to the Original Line, producing its Vaniſhing Point†; and making the ſame Angle with the Vaniſhing Line (AB) and Parallel of the Eye, as the Original Line makes with the Interſection and Directing Line, of the Plane it is in‖. † Def. 22. ‖ Theo. 11.

EX. The Picture, AIKB, being erected, turn over the Vaniſhing Plane, *IK*NL, parallel to the Original Plane. Fig. 37. No. 1.

Then, the Radials EN, EO, EL, &c. are reſpectively parallel to their Originals, XY, YZ, and XZ, in the Triangle XYZ; and conſequently, they make equal Angles, reſpectively, with NL, and *IK*, as the Originals make with AB, the Interſection of the Plane they are in.

PROBLEM III.

The Interſecting Point of any Line, in a vertical Plane, being given, and the Angle of Inclination of the Plane to the Picture, to find its Interſection and Vaniſhing Line; the Center and Diſtance of the Picture being given, and the Picture ſuppoſed vertical.

Let I be the interſecting Point given, and C the Center of the Picture. Fig. 40.

Through C, draw AB, the Horizontal Line, and ED the Vertical Line at right angles; and through I, the Interſecting Point given, draw IH, parallel to ED; which is the Interſection required.

Make CE equal to the Diſtance of the Picture; and make the Angle CEA equal to the Complement of the given Angle of Inclination; cuting AB in A.

Or (having drawn EG parallel to AB) make GEA equal to the given Angle; and through A, draw FG parallel to IH, which is the Vaniſhing Line required.

DEM. Becauſe the Picture and the Original Plane are both vertical, their Interſection is perpendicular to the Horizon‡; and conſequently, ſince I is the Interſecting Point of ſome Line in that Plane, IH perpendicular to FI, the Ground Line, is the Interſection; by Theorem 10th. ‡ Cor. to 9. 7. El.

And, becauſe EAC (equal AEG) is equal to the Angle of Inclination, EA is the Radial, or Parallel of the common Interſection of the vertical Plane with horizontal Planes, producing its Vaniſhing Point. (By Prob. 2.) Therefore, FG, drawn through A, parallel to IH, or ED (i. e. perpendicular to FI) is the Vaniſhing Line (Theo. 2nd).

Plate IX.

For, imagine the Picture FGHI turned up, vertical, and the Triangle AEC vertical to it, i. e. horizontal; repreſenting a part of the Horizontal Plane; E is the Eye.

Then, a vertical Plane, paſſing through the Eye and the Line EA, will cut the Picture in FG, the Vaniſhing Line, which is parallel to IH, the Interſection; by Theorem 2nd.

For, they are the Sections of parallel Planes with the Picture (Def. 8).

N. B. If the Interſecting Point of any other Line, in the Plane, had been given, as B or H, the Interſection IH would be the ſame.

And conſequently, if the Vaniſhing Point of that Line, or any other, as F, or G, in any vertical Plane, be given or found, the Vaniſhing Line is determined.

For, it paſſes through that Point (Th. 10.) perpendicular to the Ground Line, from its Poſition.

EXAMPLE, by the Apparatus.

Fig. 15.

If the Planes ABGH and BFC were produced, they would cut the Picture MNOP, in *BK* and *BE*, the Interſections of thoſe Planes; which are parallel between themſelves, becauſe their common Section, BG, is parallel to the Picture †; and, becauſe BG is vertical, they are, alſo conſequently, vertical, being parallel to BG reſpectively ‡.

† Theo. 5.

‡ 4. 7. El.

And, if a Plane (RSTU) be ſuppoſed to paſs through E (the Eye of the Spectator) parallel to BFC, cuting the Picture MNOP, produced, in RU, it is the Vaniſhing Line of that Plane, parallel to its Interſection, *BE*.

Alſo, a Plane (STOP) parallel to ABGH, cuts both Pictures, in OP and OP, the Vaniſhing Lines of that Plane, on both; parallel to RU or W, i. e. to *BK*.

When the Original Plane is inclined both to the Horizon and the Picture, having its common Section with horizontal Planes, parallel to the Picture; it is a ſimilar Caſe to this; and by turning the Picture ſideways, on IH, (Fig. 40) 'tis the very ſame in every reſpect; as it is fully illuſtrated in Fig. 15. No. 3.

No. 3.

EX. *ADFB* is the Original Plane; *AB*, its Interſection with the Ground Plane, is parallel to the Picture; i. e. to AB. Let it be raiſed up, making the Angle PQR, with the Horizon. AONB is the Picture, which being raiſed up vertical, the Plane KONL, paſſing through the Eye, at E, will be parallel to the original Plane, *ADFB*; and cut the Picture in ON, its Vaniſhing Line, parallel to the Horizontal Line, LM.

And, if the Original Plane was produced, it would alſo cut the Picture in a Line parallel to LM, or AB (i. e. to ON) below AB, the Ground Line.

But, if the Original Plane inclined towards the Picture, on this ſide of *AB*; then, its Vaniſhing Line would fall below the Horizontal Line, and the Interſection above it.

SCHOL. If the Interſection of an Original Plane, in any poſition whatever, be given, and the Inclination of that Plane to the Picture known, its Vaniſhing Line is determined as by this Problem; ſeeing that, the Vaniſhing Line of a Plane is always parallel to its Interſection with the Picture. Conſequently, if the Interſection, be parallel either to the Horizontal or Vertical Line, the Vaniſhing Line ſought is alſo parallel to them.

But, when the Interſection of the Original Plane is not given; and which, by reaſon of the great Diſtance of the Plane, from the Picture, or Inclination to it, cannot be had, nor its Poſition aſcertained; then other Expedients are uſed, to find the Vaniſhing Lines of ſuch Planes; *viz.* by finding the Vaniſhing Points of two Lines in the Original Plane. (See Prob. 5.)

N. B. The Center of every vertical Vaniſhing Line is the Point in which it is cut by the Horizontal Line; and the Center of every Vaniſhing Line, which is horizontal, is the Point in which it is cut by the Vertical Line (by the 7th Theorem) the Picture being vertical.

Fig. 40.

The Diſtance of every Vaniſhing Line which does not paſs through the Center of the Picture, is the Hypothenuſe of a Right angled Triangle (as AEC) whoſe Baſe and Perpendicular are the Diſtance of the Picture, and the Diſtance, AC or EC, between the Center of the Picture and the Vaniſhing Line.

For, in reſpect of the Vaniſhing Line, FG, of vertical Planes, if CE be the Diſtance of the Picture, EA is the Diſtance of the Vaniſhing Line, and A is its Center.

But, if GH be a Vaniſhing Line of a Plane inclined to the Horizon, in the Angle GEA, and to the Picture, in AEC, having the ſame Diſtance, AE, E is its Center, and AC is the Diſtance of the Picture.

PROBLEM IV.

The Vaniſhing Line of a Plane, its Center and Diſtance, being given, and the Vaniſhing Point of ſome Line in that Plane; to find the Vaniſhing Point of other Lines, making a given Angle with that Line, whoſe Vaniſhing Point is given.

A B is the Vaniſhing Line, C is its Center, and A the given Vaniſhing Point. Fig. 41.

Draw C E perpendicular to A B, and equal to the Diſtance of the Vaniſhing Line, given or found.

Join A E; and make A E B, or A E D, equal to the Angle, which the Original Lines make with each other, cuting the Vaniſhing Line in B or D, the Vaniſhing Point ſought.

DEM. Imagine the Triangle A E B turned up, on A B, perpendicular to the Picture (if A B be conſidered as the Vaniſhing Line of a Plane perpendicular to the Picture) or, making the ſame Angle with the Picture, as the Plane, of which, A B is the Vaniſhing Line.

Then is A E B the Parallel of whatever Plane the Original Lines are in, producing its Vaniſhing Line A B‡; E coincides with the Eye, and E A, E B, &c. are reſpectively parallel to the Original Lines; ſeeing they paſs through the Eye and the Vaniſhing Points of thoſe Lines (Def. 22.) ‡ Def. 8.

But, the Radials of two Lines producing their Vaniſhing Points, make the ſame Angle, at the Eye, as the Original Lines make with each other†. Therefore, B, or D is the Vaniſhing Point required. † Theo. 11.

N. B. If the Angle A E B be obtuſe, it is not the Angle of Inclination of the Lines. In which Caſe, regard ought particularly to be had to the Poſition of the Original Lines, in reſpect of each other, and of the Picture.

For, ſuppoſe F G to be the Repreſentation of a Line, whoſe Vaniſhing Point is A; then, if F B be drawn, A F B repreſents an Angle equal to A E B; conſequently, the obtuſe Angle is towards the Picture. In which Caſe, let the Angle of Inclination be made on the other Side; that is, produce A E to I, and make I E B equal to the given Angle; for if the Angle A E B (i. e. A E D) was made equal to the Angle of Inclination, the Point D (in that Caſe) would not be the Vaniſhing Point required.

PROBLEM V.

The Angle of the Inclination of a Plane to the Horizon, together with the Angle which its Interſection, with any horizontal Plane, makes with the Picture, being given, to find its Vaniſhing Line; the Interſection of the Plane not being given, nor its Poſition known.

The Center and Diſtance of the Picture are given.

Let A B be the Horizontal Line, and C the Center of the Picture. Fig. 42.

Draw C E perpendicular to A B, and equal to the Diſtance of the Picture; and, through E, draw D E parallel to A B.

Draw E A, making the Angle D E A equal to the Angle which the Interſection of the inclined Plane (with horizontal Planes) makes with the Picture, cuting the Horizontal Line in A, the Vaniſhing Point of the common Interſection.

Draw E B, perpendicular to A E, cuting A B in B; and, through B, draw B G perpendicular to A B, indefinite.

Make B F equal B E; and make the Angle B F G equal to the Inclination of the Original Plane to the Horizon, cuting B G in G, and draw A G, the Vaniſhing Line ſought.

This Problem, for finding the Vaniſhing Lines of Planes, caſually inclined to the Horizon and to the Picture, is univerſal, and applicable in all Caſes, when the Angles are determinable; as in the Roofs of Buildings, Pediments, &c. which, being frequently neceſſary, in common Subjects, could not be diſpenſed with

Plate IX. Fig. 42. with here; otherwiſe, I ſhould have omitted it till the laſt Section, which treats more fully on ſuch Subjects. I ſhall therefore reſerve the Demonſtration of it till then; where, every Caſe and Circumſtance, reſpecting inclined Planes, are fully demonſtrated, and exemplified by moveable Planes.

† Th. 7. N. B. The Center of the Vaniſhing Line, AG, is determined by drawing CH perpendicular to the Vaniſhing Line, cuting it in H, its Center†, nor does it differ, in that reſpect, from any other, except in Poſition; for, CH is a part of the Vertical Line of the Original Plane.

And, if CI be drawn, parallel to the Vaniſhing Line, and equal to the Diſtance of the Picture, the Line IH, to the Center, is the Diſtance of the Vaniſhing Line AG; which, Diſtance, is as applicable to that Vaniſhing Line, as the Diſtance of the Picture (IC) to all Vaniſhing Lines which paſs through its Center, i. e. of Planes perpendicular to the Picture.

Fig. 15. EX. In the Apparatus, VW, the Vaniſhing Line of the Plane of the Roof, HIFG, is determined by this Problem; V being the Vaniſhing Point of the common Section, GH, with a Horizontal Plane, DGHK; and EW, i. e. the Radial or Right Line from the Eye, parallel to GF determines W, in RU produced, the Vaniſhing Point of GF and HI, on the Picture MNOP.

For, *E*, in the Horizontal Line, repreſents the Eye tranſpoſed to the Picture; *E*Y being equal to the Diſtance of the Eye from the Point Y; and Y*E*W is equal to the Angle DGF of the Inclination of the Roof to horizontal Planes.

Wherefore, ſince V is the Vaniſhing Point of one Line (GH or IF) in the inclined Plane, and W is the Vaniſhing Point of another Line (GF or HI) in the ſame Plane; conſequently, VW (a Right Line drawn through thoſe Points) is the Vaniſhing Line of the Plane GFIH (Theo. 10th. Cor 1.)

For, a Plane paſſing through the Eye, and thoſe Vaniſhing Points, would be parallel to GFIH, and would cut the Picture in the Line VW; which is, therefore, the Vaniſhing Line of that Plane. Def. G.

SECTION IV.

CONTAINING

The ELEMENTS of PRACTICAL PERSPECTIVE.

HAVING, in the foregoing Section, ſhewn how to find and determine the Interſecting and Vaniſhing Lines, of Planes, in all common Caſes; and alſo the Vaniſhing Points of Lines, in any Plane whoſe Vaniſhing Line is given or found; by means of which Vaniſhing Points, all original Right Lines (not parallel to the Picture) have their indefinite Repreſentations, on the Picture, truly and accurately deſcribed. In this Section I have ſhewn how to cut off certain portions, from the indefinite Repreſentation, which are the perſpective Repreſentations of certain portions, or ſegments, of Lines in the Original Object.

Having well conſidered, that moſt regular Objects are bounded by Planes, and the bounds of Planes are Lines*; it is evident, that to find the Repreſentations of Lines, in all Poſitions, is to find the Repreſentation of the Figure, or Object, bounded or circumſcribed by thoſe Lines. And, ſince the extremes of Lines are Points, it follows, that, if the two Extremes, of a Right Line, be found, the whole Line is determined; and, by finding ſundry Points in curved Lines, the Repreſentation of the Curve is determinable. Wherefore, the whole, of practical Perſpective, conſiſts in finding the Repreſention of a Point, any how ſituated.

But, ſince Points are the interſections of Lines, and, to find the Repreſentation of a Point, in Perſpective, it muſt, neceſſarily, be ſuppoſed in ſome Line; hence it follows, that, to determine the Repreſentations of Lines, in all Caſes, is the whole ſum and ſubſtance of Practical Perſpective.

* See Page 50 and 51. Book II. Section II.

Now, Right Lines can have but three Poſitions, in reſpect of themſelves and of the Picture, *viz.* they muſt be either parallel, perpendicular, or inclined; and, having learnt how to manage Lines in all theſe Caſes, by the following Problems, there remains little more to be done; for, by conſtructing a number of Lines together, properly, an Object is formed.

By Theorem 12. the Indefinite Repreſentation of a Right Line, not parallel to the Picture, is a Line drawn through its Interſecting and Vaniſhing Points. But, ſince the Interſecting Point is not always wanted (nor is it always attainable) if any Point in the indefinite Repreſentation be determined, a Line drawn through that Point, to the Vaniſhing Point, is the ſame; for, it would, if produced, paſs through the Interſecting Point of the Original Line.

In this Section, which contains the whole Subſtance of practical, rectilinear Perſpective, I have ſhewn how the indefinite Repreſentations of Lines, in all Poſitions, are determined; and then, how to proportion them, in any given or known ratio to the Original; and afterwards, how to manage them when the Vaniſhing Point is not within the limits of the Picture, by various Expedients.

Let the Reader take particular notice, that I ſhall, always (to ſave repetition) in the following Problems, ſuppoſe the Center of the Picture to be given, and its Diſtance known; except, in particular Caſes, when it is otherwiſe expreſſed.

The Diſtance of the Picture (being determined) is applied, in Practice, by the ſame Scale of Proportion to which the Picture is delineated.

Let it alſo be obſerved, that I ſhall always (in the Diagrams) make uſe of the initial Letters of the following Terms, *viz.* C for the Center of the Picture, E for the Eye, in its firſt or principal place on the Picture, and *E* for its firſt tranſpoſed place, in any Vaniſhing Line, &c. and E^2 for the next tranſpoſition, &c. and VL for any particular Vaniſhing Line. But, ſeeing that the Ground Line, the Horizontal, and Vertical Lines never vary their Places, and are always ſtronger drawn than the operative Lines, I think it needleſs to particularize them otherwiſe.

PROBLEM VI.

How to find the Repreſentation of a Point whoſe Seat on the Picture is given, and its Diſtance from the Picture known.

Let C be the Center of the Picture, and S the Seat of the Original Point. Fig. 43.

Draw a Right Line CS, through the Center of the Picture, and the given Seat, indefinitely beyond S. Draw CA, at pleaſure; and SB parallel to CA.

Make AC equal to the diſtance of the Picture, and SB equal to the diſtance of the Original Point from its Seat. Draw AB, which will cut CS in b, the Point ſought.

Or, if the Original Point be between the Eye and the Picture, make S*B* equal to its diſtance as before, and draw A*B*, which produce to the Picture, cuting it in *b*. Then is *b* the projected Repreſentation, of the Point *B*, on the Picture.

Compleat the Parallelogram ACSD.

DEM. AC is equal to the Diſtance of the Picture, SB to the Diſtance of the Original Point from its Seat, and AC is parallel to SB (Con.) Conſequently, the Triangles ACb, and bBS are ſimilar. Wherefore, Sb : bC :: SB : AC; and, conſequently, Sb : Sb + bC (i.e. SC) :: SB : SB + AC, equal SD (i.e. BD.) that is, Sb : SC :: SB : BD. Alſo S*b* : SC :: S*B* : *B*D. - Theo. 13.

Now, becauſe SB is the Diſtance of the Original Point, from the Picture, and AC is equal to the Diſtance of the Picture; draw CE and SF, both perpendicular to CS, conſequently parallel; make CE equal CA, and SF to SB, and draw EF; which will cut SC in the ſame Point, b.

For, the Triangles CEb, bFS are ſimilar. Wherefore, Sb : bC :: SF : EC; i.e. as SB : AC.

Plate IX. Fig. 43. Hence it is evident, that the projection of the Point b does not, in the leaſt, depend on the ſituation of the Lines CA and SB, in reſpect of CS, but on their parallelism and proportion to each other; wherefore, if the true Diſtances are not known, but only their Ratio, the Point b, will be projected the ſame. e. g.

Take CI at pleaſure; and make SG to CI, as SB to AC, and draw IG; the Point b will be projected the ſame, on the Picture.

For, ſince SG : CI :: SB : CA, and Sb : bC :: SB : CA; conſequently, Sb : bC :: SG : CI, and conſequently, the Point b is the ſame.

Hence may be ſeen the univerſality of the 13th Theorem. For, conceiving E to be the Eye, and EC the Direct Radial, i. e. the Diſtance of the Picture, and SF the Diſtance of the Original Point from its Seat, i. e. from the Picture, imagine the Triangle ECb to be turned up, on bC, till EC is perpendicular to the Picture, and ſuppoſe bFS turned back, on Sb, till SF is alſo perpendicular to the Picture, on the other Side; then is E in the true place of the Eye, and F is in the Place of the Original Point, and conſequently, EF is a Viſual Ray from the Eye to the Point; which it is evident will paſs through the Picture, in the Point b; and, ſince Viſion is conveyed in Right Lines to the Eye, the Point F will appear, on the Picture, at b; which is, therefore, the perſpective Repreſentation of the Point F. (See App.)

Again, becauſe EC is parallel to SF and cuts the Picture in C, its Center, C is the Vaniſhing Point of SF; (Cor. 2. Def. L.) for, the Line SF is perpendicular to the Picture; and S is its interſecting Point; (Def. K.) wherefore, SC is its indefinite Repreſentation †; and EF is a Viſual Ray from the Eye to the Original Point; (Def. H.)

† Theo. 12. Def. 25.

And, becauſe EC is parallel to SF, and EF cuts them both, they are, therefore, all in the ſame Plane ‡, conſequently, the Viſual Ray, EF, will cut the Picture ſomewhere in SC, the Interſection of that Radial Plane with the Picture, and conſequently in b; making Sb to bC, as SF (or SB) to EC (or AC) or, Sb to SC, as SF or SB, added to EC, or AC; i. e. to BD, or FH §; for, D is the Directing Point of BS, and H of FS.

‡ Ax. 7.

§ Theo. 13. and Th. 11. 8. of El.

SCHOL. Thus, the whole buſineſs of practical Perſpective will be found (when well underſtood) to conſiſt in finding the repreſentation or projection of a Point on the Picture, any how ſituated; i. e. to determine that Point, in which a Viſual Ray, from the Original Point to the Eye, would cut and paſs through the Picture.

For, if the two Extremes of a Right Line are found, the whole Line is determined; and curved Lines can only be repreſented by finding the repreſentations of ſeveral Points in the Original Curve, and joining them carefully, by hand. I would therefore adviſe the young Student, to beſtow the utmoſt attention on this and the following Problems, as they really contain the whole eſſence of Practical Perſpective.

PROBLEM VII.

The Seats, on the Picture, of any two Points in an Original Line being given, and the Diſtance of the Points from the Picture; to find the Inclination of the Line to the Picture, its Interſecting and Vaniſhing Points; to draw its indefinite Repreſentation, and to find the Repreſentation of each Point.

Fig. 44. A and B are the Seats of the Points given, and C is the Center of the Picture.

Draw AB, indefinite; draw AF and BG perpendicular to AB, and equal to the Diſtance of the Original Points from their Seats, reſpectively; and draw FG, meeting AB in the Point I.

Then is I the Interſecting Point of the Line, in which the Original Points are ſituated; and AIF is the Angle of its Inclination to the Picture.

Through C, draw CV parallel to AI, indefinite; draw CE perpendicular to CV, and equal to the Diſtance of the Picture; and, draw EV parallel to IF, cuting CV in V, the Vaniſhing Point.

Draw IV, the indefinite Repreſentation of the Original Line; and laſtly, draw EF and EG, or AC and BC, cuting IV in a and b, the Repreſentations of the Original Points, F and G.

DEM. Becauſe A and B are the Seats of the Original Points, AB is the Seat of the Line they are in, and conſequently, it is the Interſection of a Plane paſſing through the Line perpendicular to the Picture. (See N. B. Art. 7. General Introduction, Page 44.)

Now, ſince AF and BG are perpendicular to AB, i. e. to the Picture, and meaſure the Diſtance of each Point, reſpectively, from its Seat; if the Triangle AFI was turned back, behind the Picture and perpendicular to it, F and G would be in the true places of the Original Points, in reſpect of the Picture, and FG would repreſent the Original Line; which, being produced to the Picture, would cut it in I, in the Line AB, produced. — — — 11. 7. El.

For, AF and BG are parallel Lines, and conſequently, all other Right Lines which cut them both are in the ſame Plane †. † Ax. 7.

Therefore, I is the Interſecting Point of the Line FG, and AIF is the Angle of its Inclination to the Picture, i. e. to its Seat, AB.

Again. Suppoſe the Triangle CEV turned up, perpendicular to the Picture, it will be parallel to AFI; for CV is parallel to DA, and, AFI and CEV are both perpendicular to the ſame Plane.

Now CE is perpendicular to the Picture, and, ſince it is equal to the Diſtance, E is the true place of the Eye; and becauſe EV is parallel to IF, and cuts the Picture in V, V is the Vaniſhing Point of IF *; ſeeing they make equal Angles with the Picture; (EVC equal AIF; by Theorem 11 th.) * Def. 22.

But, I is the Interſecting Point of FG, and V is its Vaniſhing Point; wherefore, IV is its Indefinite Repreſentation; by Theorem 12 th.

And, EF, EG, are Viſual Rays, from the Eye (E) on this Side, to the Points F and G, on the other ſide of the Picture, which would cut and paſs through the Picture at a and b.

For, EV is parallel to IF, and they are both cut by IV and EF; wherefore, they are all in the ſame Plane ‡; and IV is the Section of that Plane with the Picture, becauſe, the Points I and V are both in the Picture; the Line FG cuting it in I, and EV in V, (Def. K and L.) ‡ Ax. 7 & 8.

Now, ſince EF and EG are alſo in the ſame Plane, they muſt cut the Picture, ſomewhere, in the Interſection (IV) of the Plane they are in, conſequently in a and b.

For, the Triangles VEa and aIF, VEb and bIG are ſimilar; and therefore, Ia : aV :: IF : EV as Fa : aE; and, Ib : bV :: IG : EV; as by Theorem 13th.

Secondly. AC and BC alſo determine the ſame Points, a and b.

For, AF and BG are Lines perpendicular to the Picture; therefore, the Center of the Picture is their Vaniſhing Point §; EC being parallel to them ‖; alſo, A and B are their Interſecting Points; therefore, AC and BC are the indefinite Repreſentations of AF and BG; by Theorem 12th. § Cor. to Theo. 4. ‖ Def. 22.

But, the Repreſentation of the common Section of two Lines, is the Point in which the Indefinite Repreſentations cut each other. (Cor. 7. Theo. 12.)

Therefore, a, the Point in which IV and AC cut each other, is the Repreſentation of F; and b of G, in which the Lines BC and IV cut each other.

Alſo, becauſe CE is parallel to AF, the Triangles CEa, aFA are ſimilar; wherefore, Aa : aC :: Fa : aE :: AF : EC, i. e. :: Ia or IF : aV, or EV. Q. E. D.

Or, if the Diſtance of the Points F and G, from the Interſecting Point, are tranſfered to D and H; and the Diſtance of the Vaniſhing Point, EV, be ſet off, from V to *E*; *E*D and *E*H, being drawn, will give the ſame Points; that is, they will cut IV the ſame, in a and b.

For, the Ratio of Ia or Ib, to aV or bV, is ſtill as IF or IG, to EV; i. e. as ID or IH, to *E*V.

Alſo; if AC, or BC, be conſidered as the indefinite Repreſentation of a Line (BG) in which the Point G is ſituated; make BK equal BG, and C*E*² (on the contrary ſide) equal CE; K*E*² will determine the ſame Point, b, &c.

N. B. The Seat of a Point, on any Plane, is the Point where a Perpendicular from the Original Point cuts the Plane, and conſequently meaſures its Diſtance. See Ichnog. and Orthog. Sect. II. Page 46.

SCHOL. The Diſtance of any Point, in an Original Object, from the Picture, may be obtained when the Interſecting Point, of the Line it is in, and its Diſtance from the Interſecting Point, cannot, for various reaſons; and, ſince it is demonſtrated (Theo. 13.) that the Diſtances of ſeveral Points in a Line, reſpectively, from the Picture, are in the ſame Ratio as their Diſtances from the Interſecting Point of that Line; conſequently, having their Seats on the Picture, and Diſtance from their Seats, reſpectively, their Repreſentations, on the Picture, may be determined without the Interſecting Point. (Cor. 1. Th. 13.)

SCHOL. 2. If the Original Points are in a Plane which is perpendicular to the Picture, the Seats of thoſe Points are in the Interſection of that Plane with the Picture.

Wherefore,

Plate IX. Fig. 44. Wherefore, if F and G are two Points in a Line on the Ground Plane, the Seats of thoſe Points are in the Ground Line (I D) where the Perpendiculars F A and G B cut it. A and B are the Seats; and their Diſtances A F to B G, are as the Diſtance of the Point F is to the Diſtance of G, from the Interſecting Point of the Line F G, where it meets A B produced.

COR. I D being the Interſection of a Plane in which the Line F G is, or may be ſituated, V C is its Vaniſhing Line (Theo. 2. and 10.) and, becauſe it paſſes through C, the Center of the Picture, it is, conſequently, the Vaniſhing Line of a Plane perpendicular to the Picture (Theo. 4.) whoſe Diſtance is E C.

COR. 2. If the Interſection, (I D) and Vaniſhing Line (C V) of any Plane, with its Center, (C) and Diſtance, (E C) be given, and any Line (F G) in the Original Plane; its Interſecting Point (I) is found by producing the Line; and its Vaniſhing Point (V) by drawing E V parallel to F G; or, by making the Angle E V C equal to A I F. (Theorem 11.)

PROBLEM VIII.

The Repreſentation of a Line and its Vaniſhing Point being given or found, to divide the Line, perſpectively, in any known Proportion.

Fig. 45. A B is the Line given, and V is its Vaniſhing Point.

Draw A H, at pleaſure, and E V parallel to A H.

Make A F, F G, and G H in the ratio, or known proportion, of the Original.

Draw B H, and produce it, through B, cuting V E, in E; and draw E F, and E G, cuting A B in C and D.

Then is A B divided, in C and D, perſpectively, as A H is divided; in F and G.

† Cor. to Theo. 3. DEM. If V E be conſidered as the Vaniſhing Line of a Plane in which the Original of A B is ſituated, A H may be conſidered as its Interſection; and E F, E G and E H repreſent parallel Liues †; for they are all in the ſame Plane, whoſe Vaniſhing Line is E V, and E is their Vaniſhing Point.

2. 6. El. Wherefore, ſince A B H is a Triangle, and C F and D G are ſuppoſed parallel to B H; conſequently, A B and A H are divided in the ſame Ratio, in the Points C, D, and F, G ‡.

N. B. If a B had been drawn through the other Extreme (B) inſtead of A H, and a B divided, in the ſame Ratio as A H, that is, in the given Proportion; A a being drawn, cuting E V in E, and E f, E d, produced, will cut A B in the ſame Points, C and D.

Any other Line (A *H*) being divided in the ſame Ratio as A H, and *E* V being drawn parallel to A *H*; *E H* drawn, through B, and *E F*, *E G* alſo drawn, will give the ſame Points C and D, which may be demonſtrated in the ſame manner.

SCHOL. By this Problem, any Right Line in any Object, ſketched by hand and depending on the Eye, may be divided, with certainty, in the proportion of the Original, being known, no regard being had to what Plane the Line is in, nor its Vaniſhing Line.

By the ſame means, a Line drawn in Perſpective may be biſected, as A D, or C B, by making A F, F G, or F G, G H equal; conſequently, if the Parts A F, F G, and G H, are all equal, A B is triſected, perſpectively.

COR. If A V be the indefinite Repreſentation of a Line, and A C repreſents ſome certain portion of the Original; and it is required to cut off another Portion, as C D or C B, in a known Ratio to that repreſented by A C.

Draw A H, and V E parallel to it, at diſcretion; aſſume any Point (E) in E V, and draw E F, through C, cuting A H in F.

Make F G, or F H, to A F in the known proportion of the Original, and draw E G, or E H; which will cut off C D, or C B, in the proportion required.

COR. 2. If A C represents a certain portion, and it is required to cut off, from any other Point, as D, a part (D B) which shall be to A C in a certain Ratio.

Proceed as before, and, from any Point (E) in E V, draw E F, E G through C and D. Make G H to A F in the known proportion, and draw E H cuting A V in B, the Point sought.

PROBLEM IX.

The Vanishing Line of a Plane being drawn, and the Representations of two Lines in that Plane; if one of the Lines be divided any how, how to divide the other in the same Ratio.

Let A B and C D be the given Representations, of which, let A B be divided, any how, in E; it is required to divide C D in the same Proportion. Fig. 46.

G H is the Vanishing Line of the Plane they are in.

At any Distance, at discretion, draw a d parallel to G H; and, from any Point (G) in the Vanishing Line, draw G A, G E, and G B, and produce them till they cut a d, in a, e, and b.

From the same Point (G) or any other, in the Vanishing Line, as H, draw H c, H d, through C and D.

Divide c d, in f, as a b is divided, in e; i. e. make c f to c d, as a e to a b†. † Prob. 32. Geometry.

Draw H f, which will divide C D, in F, in the same Ratio as A B is divided.

This Problem may be performed as well by one Point (O) as by two; if the Line, *C D*, be so situated, as to come within compass. From which construction it is evident, that A B and *C D* represent equal Lines, as well as being equally divided, *a b* and c d being equal, and the two Lines having the same Vanishing Point.

For, they represent parallel Lines, and they are both cut by parallel Lines, *a* O, *b* O, c O, &c. at equal Distances, *a b* equal c d, &c.

Or, it may be done by drawing two Lines, A I and C K, which are parallel to G H, through either Extremes (A and C) of the given Lines; by which Construction, the Demonstration evidently appears from the last; seeing that, A I, and a b, or *a b*; C K and c d are all divided in the same Ratio.

After the same manner, C D may be divided, perspectively, into as many parts as A B is divided into, and in the same ratio to each other, respectively.

The general utility of this Problem is much the same as the other; the former being, how to divide a Line, perspectively, in the proportion of the Original; and this shews, how to find the Proportion of the Original from the perspective Representation; by which means, any other Line may be divided the same.

In the former, the Vanishing Line was not necessary, only the Vanishing Point of the Line, which may be in any Plane whose Vanishing Line would pass through V, its Vanishing Point, as V E or V *E*. But, in this Case (the Lines having different Vanishing Points) the Vanishing Line, of the Plane they are in, must be had; and, it must also be observed, that both Lines are in the same Plane, or the Operation cannot be performed by one Vanishing Line.

PROBLEM X.

The Vanishing Line of a Plane being given, and the Representations of two Lines in that Plane, having different Vanishing Points, to cut off, from any Point in one of them, a portion equal to that represented by the other, or in any known Ratio.

The different positions or situations of the Lines, to each other, may make this Operation appear very different; for which reason I shall give it variously, of which, the first is according to Brook Taylor.

Plate X. Fig. 47. Let A B be a given Repreſentation of a Line, and F G the indefinite Repreſentation of another Line. N M is the given Vaniſhing Line, and C its Center.

It is required to cut off (from the Point F) a portion, which ſhall be to that repreſented by A B in a known Ratio.

Produce A B to its Vaniſhing Point, D; and, through F, draw A H, to the Vaniſhing Line, cuting it in H.

Draw C E perpendicular to the Vaniſhing Line and equal to its Diſtance, and join E D and E G, by Right Lines.

Make E K and E L in the Ratio required; i. e make E K to E L as the Original Line, repreſented by A B, is to the other; and draw E M, or E N, parallel to K L.

Draw B H and F D, interſecting in I; draw I M, or N I, cuting F G, in O or P.

Then, F O, or F P, repreſents a Line having that Proportion, to the Line which A B repreſents, as E L to E K.

DEM. If N M be conſidered as the Vaniſhing Line of a Plane perpendicular to the Picture; imagine the Triangle N E M turned up perpendicular to the Picture; or, if the Plane in which the Original Lines are ſituated, be inclined to the Picture, let N E M be ſuppoſed parallel to it; then, E is in the true place of the Eye.

Now, E D and E G are the Radials of A D and F G, producing their Vaniſhing Points, conſequently
† Def. 22. they are parallel to them, reſpectively; † and make the ſame Angle (D E G) at the Eye, as the Original Lines make with each other. (Cor. 1. Theo. 6.)

And, becauſe E M is parallel to K L, D, G, and M are the Vaniſhing Points of the three Sides of a Triangle (E K L) the Radials, E D, E G, and E M being parallel to them, reſpectively.

But, the three Lines, F I, F O, and I O, vaniſh in thoſe Points, reſpectively; conſequently, F I O repreſents a Triangle ſimilar to E K L; the Angle D F G repreſents the Angle D E G (Prob. 4.) and conſequently, I M repreſents a Line parallel to E M; i. e. to K L.

Therefore, the other two ſides, F I and F O, or F P, have that Proportion to each other, perſpectively, as E K to E L.

But, becauſe A B and F I, A F and B I have the ſame Vaniſhing Points, D and H, reſpectively, A B I F repreſents a Parallelogram. (Cor. 1. Theo. 3.) conſequently, F I repreſents a Line equal to
‡ 15. 1. El. the Original of A B; ‡ and therefore, F O, or F P, repreſents a Line which has that Proportion to the Original of A B, as E K to E L.

N. B. If the ſituation of the Line A B was ſuch, that a Right Line, joining A and F, was parallel to the Vaniſhing Line N M, then B I muſt be drawn parallel alſo.

But, if a b, the given Line, be ſo ſituated, that the Line a F is ſo much inclined to the Vaniſhing Line as not to reach it within the compaſs of the Picture, take any Point (J) in the Vaniſhing Line, and draw J a, J b, indefinite; draw A D, at diſcretion, cuting them in A and B, and proceed as before.

For, A B and a b having the ſame Vaniſhing Point (D) alſo, A a and B b have the ſame Vaniſhing Point (J) a b B A repreſents a Parallelogram; and conſequently, A B, a b, repreſent equal Lines.

Fig. 48. Caſe 2nd. Let the given Lines (A B and F P) be ſo ſituated as to croſs each other; and, it is required to cut off, from the Point (I) of their Interſection (or any other) a part (I F or I P) equal, or in any Ratio to the Original of A B.

Draw A c parallel to the Vaniſhing Line (M D) indefinite; and, from the Point H, or any other, at diſcretion, in the Vaniſhing Line, draw H I and H B, cuting
§ Prob. 8. A c in c and b; then, A b is to b c in the Ratio of the Original of A B to B I §.

Make a c equal to A b and draw a H, cuting A B in d; d I repreſents the ſame meaſure as A B, *viz.* a c equal to A b.

Produce A B to its Vaniſhing Point (D) and F I, to H; if it be not already done.

C being the Center of the Vaniſhing Line, draw C E perpendicular to H D, and equal to its Diſtance; and draw E D, E H.

Make E K, E L, equal, or in the Ratio required, and join K L; to which, draw E G parallel, giving the Vaniſhing Point G.

Draw G d, and produce it, cuting I F in F; then, if E K be equal E L, I F repreſents a Line equal to the Original of d I, that is, of A B, as it was required; or, whatever Ratio E K has to E L, F I repreſents the ſame to A B.

Again. Draw m n, through I, parallel to A c, cuting a H in m.

Make I n equal I m; draw n H cuting A D in O; and draw O G, cuting F H in P.

I P repreſents a Line equal to I F, equal A B; i. e. to their Originals; or in the Ratio of E K to E L, whether equal or otherwiſe.

DEM.

DEM. Becauſe a m and F I have the ſame Vaniſhing Point (H) they repreſent parallel Lines; and, becauſe m n is parallel to a c, and to the Vaniſhing Line H D, a m l c repreſents a Parallelogram; conſequently, the Originals of m l and a c are equal.

But, I n was made equal I m, and n H repreſents a Line parallel to a H; conſequently, I O and I d repreſent equal Lines. (See Problem 8th.)

And, ſince O P and F d have the ſame Vaniſhing Point (G) they alſo repreſent parallel Lines §; and conſequently, I O P, I d F repreſent equal Angles †; wherefore, they are ſimilar Triangles, for the Angles P I O, d I F, are equal ‡.

§ C. 1. T. 3.
† 4. 1. El.
‡ 2. 1. El.

But, they alſo repreſent congruous Triangles, for I P : I F : : I O : I d : : P O : d F; and I O repreſents a Line equal to I d; conſequently, I P repreſents a Line equal to I F, and P O to d F.

If it had been required to cut off, from the Point J, in F H, a portion, in a certain Ratio to A B; draw B J.

If B J be parallel to H D, draw D J cuting Ab in a; then J a repreſents an equal Line as A B.

But, if B J be not parallel to the Vaniſhing Line, produce it to its Vaniſhing Point, and proceed as by Fig. 47, uſing J a for A B, as I d for A B, before.

N. B. If K L was parallel to the Vaniſhing Line, then P O and d F would alſo be parallel; for E G would be parallel, and conſequently, could not produce a Vaniſhing Point.

Suppoſe it was required to cut off, from the Point a (Fig. 47.) in the Line a F, a part, equal, or in any known Ratio, to the Original of A B.

Draw A a till it cuts the Vaniſhing Line in J, and draw J B; alſo draw a D to the Vaniſhing Point of A B. Then a b repreſents an equal meaſure as A B, and a F may be cut in any Ratio to a b, as in the next Caſe; which is the 3rd Example, Prop. 16, of Brook Taylor, mentioned in the Preface to this Book.

Caſe 3rd, Let A C or C B be the given Line. It is required to cut off from the Point C, a part, which repreſents an equal meaſure as the other. Fig. 49.

D and G are the Vaniſhing Points of the two Lines, and E is the Eye.

Draw E D and E G, and biſect the Angle D E G, by the Line E H.
Draw A H, which will cut C G in the Point B, as required; or C A in A.

This Caſe, or this poſition of the Lines, is but another Example of this Problem, which Brook Taylor calls, finding the Repreſentation of a Circle from the repreſentation of one Radius given. It is certain that the whole Circle may be completed, from a Radius given, though not by this Caſe only, but by both, as I ſhall exemplify.

Let A D and F G be drawn indefinite; C, their Interſecting Point, is the repreſentation of the Center, and C B is the Radius given, in the Poſition by Brook Taylor.

He ſays, "make C A to repreſent a Line equal to that repreſented by C B (by the 15th)" *viz.* by the firſt Caſe of this; "i. e. biſect the Angle E O D;" and, to make it ſtill leſs intelligible, in his Diagram, the Angle D E G (i. e. E O D, in his Treatiſe) is not biſected, nor triſected, but is cut, very nearly, in the Ratio of 2 to 5; which negligence, in a Perſon of his ſagacity, is to me ſurprizing.

It is demonſtrated in the 3rd of the 6th of El. that, if any Side of a Triangle be divided in the Ratio of the other two Sides, a Right Line joining the oppoſite Angle and the point of Section will biſect that Angle. But, there may, very juſtly, be an exception to this; for, it is neceſſary that the greater Segment be contiguous to the greater Side, and conſequently the leſſer Segment to the leaſt Side.

In the Triangle D E G, if D G be divided, in H, in the Ratio of D E to E G, then, E H biſects the Angle D E G; becauſe G H, the greater Segment, is contiguous to the greater Side, E G; which, it is obvious, could not be otherwiſe.

The Conſtruction, for proof of the Aſſertion, is to produce either Side, as G E, and make E J equal to E D, and join J D; which is proved to be parallel to E H.

For, becauſe E D = E J the Angle E J D = J D E (9. 1. El.) and, becauſe D E G is equal to them both (10. 1.) and D E H = H E G (Con.) J D E = D E H (Ax. 4.) therefore, E H is parallel to J D (4. 1.)

But, J E is the Radial of C B, produced, and E D of A C; and E J is made equal to E D; alſo, E H is a Right Line from the Eye, parallel to J D (i. e. to K L, as in the former Caſe) producing the Vaniſhing Point of the Line A B (i. e. of I O, and F d, in the two former Caſes); by means of which Vaniſhing Point (H) C A and C B are cut, repreſenting equal portions of thoſe Lines.

Hence may be ſeen the affinity between the two Caſes, in biſecting the Angle D E G.

For

Plate X. Fig. 49.

For E J = E D, i. e. E K to E L, and E H is parallel to K L; conſequently, A C B repreſents a Triangle ſimilar to K E L, or J E D; and conſequently, D, G, and H are the Vaniſhing Points of the three Sides A C, C B and A B; which is Iſoſceles, by Conſtruction.

And, ſince D C G repreſents an Angle equal to D E G (Prob. 4.) conſequently, A C G repreſents an equal Angle to D E J †; therefore, the Sides, containing the external Angle D E J, are, in this Caſe, made in the ratio (E K to E L) of the Originals, which A C and C B repreſent; and ſince they are equal, conſequently, A C and C B repreſent equal Lines.

† 1. 1. El.

Now, C A and C B repreſent equal Lines, from the ſame Point C; wherefore, if C be conſidered as the Center of a Circle, in Perſpective, C A and C B are Radii of the Circle; and this is all that Brook Taylor has done towards finding the whole Repreſentation, which is far from being ſufficient.

In B C produced, make C F repreſent a part equal to what C B repreſents, and C I to C A, or C B; then, B F, and A I repreſent Diameters of the Circle.

Proceed as in Caſe 1ſt. by making E *K* equal to E L, and draw E N parallel to *K* L; draw A N and B N, cuting B F and A D in F and I, the Points ſought.

Or, having drawn A N, only, cuting B F in F; draw F H, cuting A D in I.

The four Points, A, B, I, and F, are all in the Circumference of a Circle, whoſe Center is C; for, C A, C B, &c. repreſent equal Lines.

But, theſe four Points are not ſufficient, for compleating a Circle in Perſpective.

Draw H C and N C indefinite, and cut off, from the Center (C) C a, C b, C i, and C f, repreſenting alſo equal meaſures, to thoſe repreſented by C A, &c. e. g.

Make E M equal to E L; join M L, and draw E O parallel to M L.

Alſo, make E *L* equal E *K*; and draw E P parallel to *K L*.

Draw A O, O I, B P, and P F, cuting the indefinite Lines in a, i, b, and f.

The Angle N E H being biſected, by E Q, ſhews the Affinity to the firſt Caſe.

Through the eight Points, A, a, B, b, I, i, F, and f, an Ellipſis may be deſcribed, which will be the repreſentation of a Circle in Perſpective; from the given repreſentation of one Radius, A C or C B.

PROBLEM XI.

From three Points given, in the Circumference of a Circle, to find the Repreſentation of the Circle; having the Vaniſhing Line of the Plane the Circle is in, and the place of the Eye.

Fig. 50.

A, B, and C are the three given Points, L M is the Vaniſhing Line, and E is the place of the Eye. E S is the Diſtance of the Vaniſhing Line.

Produce A B and C B, cuting the Vaniſhing Line in I and K; draw E I and E K.

Make the Angles K E L and I E M each equal I E K, producing the Points L and M, in the Vaniſhing Line.

Draw A K and A M, C L and C I, interſecting in D and F; the Points D and F are in the Circumference, which paſſes through A, B, and C.

DEM. Becauſe the Angles K E L, K E I, and I E M are equal (Con.) and E is the Eye, the Angles L D K, K B I, and I F M repreſent equal Angles (Prob. 4.) and conſequently A D C, A B C, and A F C alſo repreſent equal Angles ‡.

‡ 2. 1. El.

Therefore, thoſe Angles touch the Circumference §; for they are in the ſame Segment, or ſtand on the ſame Ark, A b C.

§ 10. 3. El.

Now, here are five Points, A, D, B, F, and C, in the Circumference; but they are not ſufficient for deſcribing the true Curve of the Ellipſis.

Therefore, draw A H at pleaſure, cuting the Vaniſhing Line, in H.

Make the Angle H E J equal I E K, and draw C J, cuting A H in G, which is another Point in the Circumference of a Circle.

Thus may as many Points be found in the Circumference as are neceſſary to deſcribe the whole Curve; which, paſſing through all the Points, A, G, D, B, &c. will be an Ellipſis; for, it is the Repreſentation of a Circle in Perſpective, ſeeing that thoſe Points are all in its Circumference.

If there be too much Space between any two Points, to deſcribe the Curve with greater accuracy, another Point may readily be found; as a, or b, between A and C.

Draw a B, or b B, at pleaſure, cuting the Vaniſhing Line in S, or O. Join S E, or O E, and make the Angles S E H and S E N, or O E P, equal to I E K, cuting the Vaniſhing Line in H, N, or P, reſpectively.

Draw H D, or N F, cuting S a in a; or P F, cuting O b in b; which Points, a and b, are alſo in the Circumference.

For, becauſe A D and C B have the ſame Vaniſhing Point (K) alſo, A B and C F having the ſame Vaniſhing Point I, the Originals of the Arks, D B and B F, are each equal A C †; for A D and C B repreſent parallel Lines ‡; conſequently, B a D or F, and B A D or F, or B b F, repreſent equal Angles §, each being equal to the Original of A B C; i. e. I E K.

† Cor. to 10. 3. El.
‡ Cor. 1. Theo. 3.
§ Cor. 2. 9. 3. El.

Othewiſe; draw M C, indefinite; produce M E, and make the Angle Q E R equal to I E K; E R, being produced, will cut the Vaniſhing Line, ſomewhere, if it be not parallel to it.

If E R be parallel to the Vaniſhing Line, draw A a alſo parallel; or to the Point R, in which E R would cut the Vaniſhing Line, cuting M C in a.

For, the Angle Q E R + R E M = two Right Angles †. And, the Angle M a R (i. e. A a C) repreſents the obtuſe Angle M E R; and, A B C repreſents an Angle equal to Q E R.
But, the oppoſite Angles of every Quadrilateral, inſcribed in a Circle, are equal to two Right Angles ‡.
Conſequently, A a C repreſents an Angle in the oppoſite Segment; for, A a C and A B C are oppoſite Angles, in the Quadrilateral A B C a.

† 1. 1. El.
‡ 11. 3.

There is not, perhaps, in the whole Book, a more elegant Problem than this, which induced me to give it a place, and to perfect it. Its utility is not ſo great; yet it may frequently be applied, by thoſe who do not care to be confined ſtrictly to the Rules of Perſpective, in every reſpect. For, having, by any means, obtained the repreſentations of three or more Points, which are known to be in the Circumference of a Circle, the whole may, readily and accurately, be determined; if they know the Diſtance of the Picture, and the Vaniſhing Line of the Plane of the Circle.

Brook Taylor has made a lame affair of this elegant Problem, notwithſtanding his Principles are the ſame; by reaſon of the ſhort Diſtance he has taken for the Eye, and the prodigious Dimenſions of the Circle, it is the moſt diſtorted and prepoſterous Diagram in the whole Book. He finds no more than one Point, on each ſide, and leaves his Readers to find out the reſt; nor does he go about it in a proper manner, there being no occaſion (if the Diſtance was ſufficient) to draw another Line (d O) at pleaſure, in order to make another Angle, equal to D O E; but, to make equal Angles with E O and D O, on either ſide, continually. One thing I am much ſurprized at; he ſays, "make the Angle d O e equal to D O E; or, having made an Inſtrument, containing the Angle D E O, turn it round the Center, O, till it comes into the Poſition d O e," &c. which, is ſo ungeometrical, that I could ſcarce conceive it to be the expreſſion of ſo great a Man.

Having had occaſion, in this Problem, for a Vaniſhing Point (J, or R) which was not within the compaſs of the Picture, the next Problem ſhews how to determine the Diſtance of ſuch Vaniſhing Points from the Center of the Picture, or Vaniſhing Line, and alſo from the Eye.

PROBLEM XII.

The Vaniſhing Line of a Plane being given, and the place of the Eye, with a Line, from the Eye, inclining to the Vaniſhing Line; to find the Center of the Vaniſhing Line, and to determine the Diſtance of that Point, in which the inclined Line would cut the Vaniſhing Line, from the Center, and from the Eye.

Plate X. Fig. 51.

A B is the Vaniſhing Line given, and E D is the inclined Line from the Eye, at E. D b is conſidered as the bounds, or limits of the Picture, and B is the Point in which E D, produced, would cut the Vaniſhing Line, A F, produced.

Draw E C perpendicular to the Vaniſhing Line, cuting it in C, its Center.

Take C G any part of C E, a half, a third, or fourth, as E D is leſs or more inclined to A B, and draw G F parallel to E D.

† 2. and 4. of 6. El.

Then, as C G : C E : : C F : C B †; that is, if C G be a third part of C E then C F is a third part of C B, and G F of E B; or, in whatever Ratio C G is of C E.

Or, drawn E A perpendicular to E D. cuting the Vaniſhing Line in A.

Then, find a third Proportional to A C and C E (Prob. 31. Geo.) it will be C B.

For, A E B is a Right Angle (by Conſtruction) and E C is perpendicular to A B; therefore, as A C : C E : : C E : C B ‡;

‡ 7. 6. El.

Alſo as A C : A E : : C E : E B; i. e. E B is a fourth Proportional to A C, C E, & A E.

In Numbers, they are thus determined, by a Scale of equal Parts.

Firſt. Take A C and C E by the Scale; ſquare C E, and divide the Product by A C, which, will give C B §; for A C, C E, and C B are three Proportionals.

§ 9. 6. El.

Or, if the Triangle C F G be uſed, multiply C F by C E, and divide the Product by C G; the Quotient will be C B †. For, C G, C F, C E, and C B are four Proportionals.

† ſame.

Secondly. Since A C : C E : : A E : E B, conſequently they are four Proportionals. Wherefore, multiply C E into A E, and divide the Product by A C, the Quotient will be E B, the Diſtance of the Vaniſhing Point, B, from the Eye.

If the Diſtance of the Picture be known (equal C E) and the Angle of the inclination to the Picture, of one Side of a right angled Object, be determined; the Vaniſhing Points, and their Diſtances, are determinable.

Let A B be the Horizontal vaniſhing Line, and C the Center of the Picture.

Draw C E perpendicular to A B, and equal to the Diſtance given, or determined.

Make the Angle C E A equal to the Angle given (or to its Complement, according to which ſide the Line inclines) cuting the Vaniſhing Line in A, the Vaniſhing Point of one ſide ‡; from which all the reſt are determinable.

‡ Theo. 11.

Or, if the Diſtance, C E, be ſo great, that it cannot be laid down on the Picture (as is frequently the Caſe) take C G half, a third, a fourth, or any portion of C E, and proceed as before; making the Angle C G H equal to C E A, the given Angle. Then will C H be half, a third, &c. of A C, or whatever portion C G was taken of C E §; by which means, the diſtance of the Vaniſhing Point A, from the Center, C, is aſcertained.

§ 4. 6. El.

Make H G F a Right Angle, i. e. draw G F perpendicular to G H, cuting the Vaniſhing Line in F; then will C F be alſo half, a third, &c. of C B, the diſtance of the other Vaniſhing Point (B) from the Center (C.) Alſo, G F will be the ſame portion of E B.

Thus may the real Diſtances, be found, and the place of the Eye tranſpoſed to the Picture, as it will be exemplified in Practice, when their real places cannot be had thereon; all which may be found arithmetically, as follows.

Let the Diſtance of the Picture, C E, be given, equal 6, 5 (Feet, or what you pleaſe) and let the Vaniſhing Point A be determined as above, or at diſcretion, on the Picture; *viz.* A C equal 2, 6, from which all the reſt may be determined.

Firſt; to find the Diſtance of the Vaniſhing Point, A, from the Eye.

Square A C and C E, which being added together, the ſquare Root of that Product will be A E †; for A E ſquare is equal to A C added to C E ſquare ‡.

† Prob. 3. 7. 6. El.

‡ 20. 1.

Secondly; to find the Diſtance of the other Vaniſhing Point B, from the Center.

§ Prob. 3. 7. 6. El.

Square C E, and divide that Product by A C, the Quotient will be C B §.

For, A C : C E : : C E : C B. Alſo, A C : A E : : A E : A B.

Thirdly; to find the Diſtance of the Point B from the Eye.

Having obtained A E, by the firſt, multiply C E by A E, and divide that Product by A C, the Quotient will be E B; for A C : C E : : A E : E B.

The Diſtance of each Vaniſhing Point, A and B, from the Eye (E) are as neceſſary to be had as their places on the Picture, the uſe of which I ſhall exemplify and explain in its proper place.

First. AC and CE being given, to find AE; the Diſtance of A.

Square AC, *viz.* 2,6;	alſo, ſquare CE, *viz.* 6,5	AE=7
multiplied by 2,6	multiplied by 6,5	7
156	325	ſquared 49
52	390	
6,76 Square of AC.	42,25	
	Square of AC added 6,76	

Sum of both)49,01(7, ſq. Root;=AE.

Secondly. To find CB; AC and CE being given

Square CE; 6,5
6,5

divide by AC=2,6)42,25(16,25=CB

Thirdly. To find EB.

Multiply CE=6,5
by AE=7

divide by AC=2,6)45,5(17,5=EB.

Or; having found CB (by the 2nd) add the two Squares, of CE and CB, into one Sum; the ſquare Root, of which, will be EB. (By 20. 1. El.)

When the two Vaniſhing Points, A and B, are known, to find the Diſtance of the Picture, CE; the Center being given.

Multiply CB by AC, and extract the ſquare Root of that Product, it will give CE the Diſtance of the Picture; by Prob. 4. 7. 6. El.

To find the Diſtance, CE, geometrically; A and B being given, the vaniſhing Points of Lines at right Angles with each other, and C, the Center.

Draw CE, perpendicular to AB; and, having biſected AB, on F (the point of biſection) with the Radius AF, deſcribe the Ark AE, cuting CE in E. CE is the Diſtance required, of the Picture, or of the Vaniſhing Line, AB.

For, AC:CE::CE:CB; conſequently, AEB is a Right Angle; by 7. 6. El.

PROBLEM XIII.

To draw a Line to a Vaniſhing Point which is not on the Picture,* its Diſtance from the Center being given, or ſome Line tending to the Point; the Vaniſhing Line of the Plane it is in, being alſo given.

Let AB be the given Vaniſhing Line, and C its Center.

Let I be the Interſecting Point of ſome Line; or the Repreſentation of any other Point in the Line, given or found. Fig. 51. No. 1.

It is required to draw a Right Line, from the Point I, tending to B, which is out of the Picture; by the known Diſtance of the Vaniſhing Point B, from C, the Center of the Vaniſhing Line. Db is the limits of the Picture.

* By the Vaniſhing Point not being on the Picture is meant, only, that it does not fall within its preſcribed Bounds or Limits. But the Picture may be imagined to be produced, as occaſion may require, ſo that the Vaniſhing Points are always ſuppoſed to be on the Plane of the Picture.

Take

Plate X. Fig. 51. No. 1.

Take C F equal half, a third, or any other equal part of C B.

Draw C I, and F J parallel to C I. Make F J : C I : : F B : C B; i. e. make J F equal half, two thirds, or the ſame Complement of I C, as F B is of C B.

Draw I J which will tend to the Point B.

Or; make C K to C I, as C F is to C B; i. e. if C F be a third part of C B, make C K one third of C I. Join K F, and draw I J parallel to K F.

DEM. Firſt. In the Triangle C B I, becauſe J F is parallel to I C, C F : F B : : I J : J B. - 2. 6. El.
Conſequently, C F : C F + F B (equal C B) : : I J : I J + I B (equal I B) i. e. as J F is to I C. 2. and 4. 6.
Therefore, I J B is a Right Line; for C B I is a right lined Triangle.
2nd. In the Triangle C B I, becauſe C K : K I : : C F : F B (Con.) K F is parallel to I B. Q. E. D.

2ndly; by a Right Line tending to the Point B; the Diſtance not being known.

Let I J be a Line tending to the Point B, in the Vaniſhing Line, A B.
It is required to draw, from the Point a, a Line tending to the ſame Point.

Draw a C and a I, at pleaſure, making an Angle I a C, cuting the Vaniſhing Line in any Point (C) and the given Line at I. Draw I C.

Take any Point (K) in I C, and draw K F parallel to I J, cuting the Vaniſhing Line in F. Draw K L parallel to I a, and join L F.

A Right Line, a b, drawn through a, parallel to L F, will tend to the Point B.

Note. The Point is given at a and *a*, both without and between the given Lines; and the proceſs would be the ſame if the Point was given on the other Side of the Vaniſhing Line; or any other Line, inſtead of the Vaniſhing Line, which tended to the ſame Vaniſhing Point.

DEM. In the Triangles I a C, I *a* C, becauſe K L is parallel to I a, C L : L a : : C K : K I. 2. 6. El
But, C K : K I : : C F : F B; for K F is parallel to I B, by Conſtruction.
Conſequently, C L : L a : : C F : F B; therefore, L F is parallel to a B. Q. E. D.

Or, the ſame thing may be done in another manner.

No. 2. Let A B and C D be two given Lines; E is a Point given, through which it is required to draw a Line tending to that Point, in which A B and C D would interſect.

Draw A C, at pleaſure (through E) and B D, parallel to A C, at diſcretion.

Draw either Diagonal, as A D; draw E F parallel to C D, and F G to A B, cuting B D in G; a Right Line, drawn through E and G, will tend to the ſame Point with A B and C D.

‡ 2. 6. El. DEM. In the Triangle A D C, becauſe E F is parallel to C D, the Sides A C and A D, are cut proportionally, in E and F‡; i. e. A E : E C : : A F : F D; and becauſe F G is par. to A B, A F : F D : : B G : G D; wherefore, B G : G D : : A E : E C; and conſequently, A B, E G, and C D will tend to the ſame Point.
2ndly. When *E* falls without the two Lines. Becauſe *E F* is parallel to C D, A C : C *E* : : A D : D *F*; and becauſe *F G* is parallel to A B, B D : D *G* : : A D : D *F*, i. e. as A C is to C *E*; therefore, &c.

Hence it appears, that nothing more is required than to find a fourth Proportional. For, having drawn a Right Line (A C) at pleaſure, through the given Point, E, cuting any two Lines which tend to the ſame Vaniſhing Point; and, at a proper Diſtance, another (B D) parallel to the former (the farther off the better) B D (or L M) being divided, in G, as A C is divided in E; then, if a Right Line be drawn through the Points E and G (or g) it muſt neceſſarily tend to the ſame Point; for Right Lines, proceeding from the ſame Point, cut parallel Lines proportionally §. This method is the moſt eligible, becauſe it is the readieſt.

§ Cor. to 6. 6. El.

There are various ways of performing this; of which, thoſe by Brook Taylor are very ingenious, and do not require any parallel Lines to be drawn.

No. 3 & 4. Let A B and C D be two given Lines, tending to one Vaniſhing Point (P.)

It is required to draw a Line, through E, which ſhall repreſent a Line parallel to the Originals of A B and C D; and conſequently, it muſt tend to the ſame Vaniſhing Point; ſeeing they are not parallel.

First,

First, when the Point (E) is between the two Lines.

Through the Point E draw two Lines, at pleaſure, cuting the two given Lines, in A, G. C, and F; and, through A, C, and F, G, draw two Lines, meeting at a. No. 3.

Draw a H and a B, at pleaſure, cuting the given Lines in I, H, B, and D; join BI and DH, interſecting at K.

A Right Line drawn through E and K will tend to the ſame Point, with AB & CD.

2nd. When the Point *E* is ſituated without the two given Lines, AB and CD. No. 4.

Draw *EA* and *EF* at pleaſure, cuting the two Lines, B*F* and *CI*, in *C*, *A*, *F*, and *G*; join *AG* and *CF*, interſecting at a.

Draw any other Line, B*K*, indefinite; and, at *D*, where it cuts *CI*, draw *DH*, through a; alſo, draw B*I*, through a.

Laſtly, through *H* and *I*, draw *HK*, cuting B*D* produced, at *K*.

A Right Line, drawn through *E* and *K*, will repreſent a Line parallel to the Originals of *AF* and *CI*; and conſequently it will tend to the ſame Van. Point, P.

Although Dr. Taylor has given Demonſtration of all the Problems, previous to this, he has not favoured us with a Demonſtration of it. But a little conſideration will make it very obvious, on inſpection of the Figure.

In No. 3, becauſe AB and CD have the ſame Vaniſhing Point, they repreſent parallel Lines; and if a be conſidered as the Vaniſhing Point of AC, FG, &c. they, conſequently, repreſent parallel Lines.

Wherefore, AFGC and FHBD repreſent Parallelograms; and conſequently, E and K, the Interſections of their Diagonals AG, GF, &c. repreſent the Centers of thoſe Parallelograms †. † 16. 1. E.

And ſince the Parallelograms are between the ſame Parallels, their Centers are, conſequently, equally diſtant from each; therefore, EK repreſents a Line parallel to the Originals of AB and CD.

In No. 4; becauſe B*F* and *CI* repreſent parallel Lines, the Lines *AG*, *CF*, &c. which paſs through the Point a are all cut proportionally in that Point; wherefore, the Originals of B*H* and *ID*, of *AF* and *CG* are in the ſame Ratio reſpectively, i. e. the Originals of B*H*: *DI*:: *AF*: *CG*; conſequently, *EA*: *EC*:: *EF*: *EG*, i. e. as *KH*: *KI*; for, *KH*: *KI*:: *HB*: *ID*; i. e. as *AF*: *CG*; *viz.* as *EA* is to *EC*. N.B. Cor. 1. 2. 6. El.

Wherefore, ſince *EA*: *EC*:: *KH*: *KI*, conſequently, *EC*: *CA*:: *KI*: *IH*; or, as any other Line drawn from *K*, whoſe Original is parallel to *EA*, would be cut, by B*F* and *CI*.

But, if *KH* be ſuppoſed parallel to *EA*, and *AH* to *CI*, *CAHI* repreſents a Parallelogram; wherefore *IH* is equal *CA*; conſequently, *IK* repreſents a Line equal to *EC*; and conſequently, *EK* repreſents a Line parallel to *CI*. For, *ECIK* alſo repreſents a Parallelogram.

5. If it be required to draw ſeveral Lines to a Vaniſhing Point which is out of the Picture, from various Points, in a given Line.

Let A, B, and C, be three Points in the Line, AC; let DE and FG be two Lines which tend to the ſame Vaniſhing Point, P. No. 5.

At any Diſtance, at diſcretion, draw HL parallel to AC, and draw AH, DI, BK, and CL parallel to either Line (as FG) cuting HL, in the Points H, I, K, & L.

Aſſume any Point (G) in the ſame Line, FG, and draw GI, cuting DE in E; and, through E, draw a c parallel to AC and HL; laſtly, draw GH, GK, and GL, cuting a c, in the Points a, b, and c.

Then, if Right Lines A a, B b, and C c are drawn, they will tend to the ſame Point, P.

For, conſidering the Originals of AC and HL to be parallel to the Picture, and FG as the Vaniſhing Line of ſome Plane; becauſe AH, DI, &c. are parallel to FG, and H a, IE, &c. have the ſame Vaniſhing Point (G) they are all in parallel Planes, of which FG is the Vaniſhing Line; and, becauſe a c is parallel to AC and HL, the Original of a c is parallel to their Originals, and conſequently may be in the ſame Plane with either (Ax. 5.) wherefore, A a, DE, &c. repreſent parallel Lines, in the ſame Plane, A a c C, or in parallel Planes A Ha, DIE, &c. of which FG is conſidered as the Vaniſhing Line; and conſequently, they vaniſh in the ſame Point in that Line.

There are various other Expedients might be given in this Section; but, I ſhall omit them, till they occaſionally occur in the Work; when they will be more intelligible and better underſtood; and, being immediately applicable, in Practice, they will, at the ſame time that we acquire them, ſhew their uſe; by which means, they will be deeper rooted in the Mind, and the application of them, in future Examples, will be more familiar.

SECTION V.

Of the PRACTICE of PERSPECTIVE, reſpecting PLANE FIGURES.

IN the laſt Section, I have exemplified and illuſtrated the Elements of Practical Perſpective, according to Brook Taylor; which, notwithſtanding they are ſo excellent in themſelves, and of great utility in Practice, yet they do not lay a foundation whereon to begin; but teach, only, how to proportion one Part by another, either given or found, on the Picture; ſo that, a Novice, in theſe matters, cannot poſſibly apply them to real uſe. Nor, in my Opinion, would any Perſon ever be made a Practitioner, from that Treatiſe, unleſs he was endowed with an extraordinary Talent, and a very comprehenſive Capacity; being quite converſant in Geometry, and particularly acquainted with the Doctrine of Proportion; having a clear Idea of Planes and their Interſections with each other, and of Right Lines cut by Planes.

Before I proceed to Figures, I have ſhewn in three Problems, how to find the perſpective Repreſentations of Right Lines, in the three Poſitions, parallel, perpendicular, and inclined to the Picture; having the Interſection and Vaniſhing Line of the Plane they are in (their Diſtance from the Picture being known and the Poſition of the inclined Line; that is, the Angle of its inclination to the Picture, or to the Interſection, and its interſecting Point, or the ſeat of ſome Point in it; or the Repreſentation of ſome Point in the Line.) How to proportion them, that is, to cut off ſuch Portions, or Parts, as are the true perſpective Repreſentations of certain Parts in the Originals, (in which the whole foundation of Practical Perſpective conſiſts) is contained in the laſt Section (Prob. 8th, 9th and 10th) and are exemplified in this.

Prob. 6th ſhews how to find the repreſentation of a Point on the Picture, any how ſituated; its Seat on the Picture and its Diſtance from its Seat being given; which, in reality, contains the whole; as it will be found hereafter. I ſhall, in the next Problem, give a more familiar and introductory Leſſon, how to find the repreſentation of a Point, ſituated on the Ground Plane; which is, undoubtedly, the firſt Plane to be conſidered. At the ſame time, let it be obſerved, that there is not the leaſt difference, in the Operation, between the Ground Plane and any other, whoſe Interſection and Vaniſhing Line is given, and its Diſtance known; as it will be ſhewn.

Having, therefore, in the preceding, and in the next four, Problems, given the Elements of the whole, and fully demonſtrated it; I ſhall not trouble the Reader with the Demonſtration of every Operation, in the following Work; but only in particular Caſes, which may not readily be deduced from the foregoing; as it would only ſwell the Work to an enormous bulk, and would be of no uſe to the Practitioner. Therefore, where I ſee occaſion, I ſhall refer to the elementary Problems, or Theorems, for Demonſtration, and to ſhew how each particular Problem is applicable in Practice, in various Caſes, in the courſe of the Work.

Let it, here, again, be obſerved, that whenever any Point, Line, or Figure, is given in the Original Plane (the Interſection of the Plane being alſo given) it is ſuppoſed to be ſo ſituated on the other ſide of the Interſection, as it would be, if the Original Plane was turned over, on the Interſection, to the other Side of the Picture, making the ſame Angle with it; and (if it be not perpendicular) inclined towards the ſame Part.

See Fig. 37; the Triangle X Y Z, on the other Side of the Picture, is inverted, on this Side, for conveniency in Practice.

PROBLEM

PROBLEM XIV.

To find the Repreſentation of a Point, ſituate on the Ground Plane, or other horizontal Plane*, its real Place being given thereon; the Interſection of the Plane it is in being given.

There are ſeveral ways to find the repreſentation of a Point.

Plate XI. Fig. 52. No. 1.

Let A be an Original Point in the Geometrical Plane, of which, BD is the Interſection; C is the Center of the Picture, and CE its Diſtance; E is the Eye.

Through C, draw *E*F parallel to BD, the Interſection; then is *E*F the Vaniſhing Line of the Original Plane; which being horizontal, conſequently, *E*F is the Horizontal Line, or Vaniſhing Line of the Ground Plane.

Now, ſince BD is the Interſection of the Original Plane, and *E*F is its Vaniſhing Line, if the Original Plane was produced infinitely, beyond the Picture, its whole perſpective Repreſentation would be between its Interſection and Vaniſhing Line; conſequently, the Repreſentation of any Point, Line, or Figure, in that Plane (beyond the Picture) muſt be ſomewhere between thoſe Lines; to find which obſerve the following Rules. (See Introductory Preface; Page 104.)

Find S, the Seat of the given Point; i. e. draw AS perpendicular to the Ground Line or Interſection, cuting it in S; which is the Seat of the Point A; becauſe it is in a Plane perpendicular to the Picture.

Draw SC, and AE cuting it, in a, the Point ſought. (See Problem 6.)

For, EA is a Viſual Ray from the Eye, at E, to the Point A, which would paſs through the Picture at a, making Sa to aC, as AS to CE†; therefore, a is the Repreſentation of the Point A. † Theo.

METHOD 2nd. On C deſcribe a Semicircle, with the Radius CE, cuting the Horizontal Line in *E* and F; which are the tranſpoſed places of the Eye (E) to the Vaniſhing Line. (See Fig. 37, No. 1, the Vertical Plane (V) being turned down, on either Side, into the Picture.)

Therefore, having made C*E* or CF, on either Side, equal to the Diſtance of the Picture, make SB, or SD, equal to SA, and draw D*E* or BF, both which will cut SC in the ſame Point, a, as before.

For C*E* is parallel to SB or D, and, C*E* is to SB or SD, as CE to SA, *viz.* as the Diſtance of the Picture, to the Diſtance of the Original Point from its Seat, i. e. from the Picture. (See Prob. 6.)
If C*E*², and S*F* be drawn perpendicular to SC, or any how, parallel between themſelves, it is ſtill the ſame. For *F*a is to a*E*², as Sa is to aC, as before.

Note. This Method is the moſt eligible of all other; for, the Diſtance of the Picture being known (as it is always ſuppoſed to be) and being placed on either Side of the Center, and SB, or SD, equal to the Diſtance of the Point, on the other Side, *E*D or BF, is a Viſual Ray from the Eye to the original Point, as EA before.

N. B. If the Original Point be ſituated in the Station Line, as at *A*, the firſt method cannot be applied; becauſe its Seat, *D*, being alſo in the ſame Line, *D*C and *E A* are one Line, and cannot interſect; in which Caſe, the Eye being tranſpoſed to *E*, on either Side, and *A* to *B*, then is *E B* a Viſual Ray, from the Eye, on one ſide of the Picture, to the Point *A* (i. e. *B*) on the other Side (ſuppoſing the Vertical Plane, in which the Viſual Ray muſt be, turned on C*D*, its Interſection with the Picture, direct before the Eye) in its true, geometrical proportion, or length.

* If the Interſection and Vaniſhing Line, with its Center and Diſtance, be given or found, of any Plane whatever, whether perpendicular or inclined to the Picture, the Proceſs is the ſame, in every reſpect.

If

Plate XI. Fig. 52. No. 1. If ſ be the Seat, on the Picture, of a Point in any other Plane, (ſuppoſe Vertical) then ſG being drawn, parallel to CE, is its Interſection; and, having drawn ſC, make ſG equal to its Diſtance, and draw GE, cuting ſC in g, the Repreſentation of G; i. e. of a Point ſituated beyond the Picture at the Diſtance ſG.

Thus the Repreſentation of any Point may be obtained, knowing its ſituation and Diſtance from the Picture.

No. 2. METHOD 3rd. Through the given Point, A, draw AB and AD, at pleaſure, cuting the Interſection in B and D.

From E, the Eye, draw EF and EG, parallel, reſpectively, to AB and AD, cuting the Vaniſhing Line in F and G, the Vaniſhing Points of AB and AD. (Def. 1.)

Draw BF and DG, interſecting in a, the Repreſentation of A.

For, BF is the indefinite Repreſentation of BA, and DG of DA; conſequently, the Point, in which they interſect, is the Repreſentation of the Point in which the Lines BA and AD interſect. * Cor. 7. Theo. 12.

N. B. The 2nd Method will be found, on inſpection, to be the ſame as this; for, if E*E* and AD be drawn, they will be parallel, and EC is parallel to AS; alſo C*E* is parallel to SD; conſequently, C and *E* are the Vaniſhing Points of AS and AD reſpectively. So that, to find the Repreſentation of a Point is to find the Repreſentation of a Line; ſeeing, it muſt be ſuppoſed in ſome Line.

No. 3. METHOD 4th is rather a matter of curioſity than real uſe, being ſeldom, if ever, practiſed; it may be performed without either Interſection or Vaniſhing Line; nothing more being required than the Place of the Eye, and the Original Point; their Diſtances from the Picture being known, or the Ratio of their Diſtances.

On A the given Point, with the Radius AS, its Diſtance fram the Interſection, or from the Picture, deſcribe an Ark, BSD; and, on E, with the Radius EC, the Diſtance of the Vaniſhing Line, or of the Picture, deſcribe the Ark FCG.

Draw the Tangents, BF and DG, interſecting in a, the Repreſentation of A.

Or, if the Radius be taken, in the ratio of the Diſtances, reſpectively, it will be the ſame, i. e. the Tangents will cut each other in the ſame Point, a.

DEM. Draw AB and AD, EF and EG perpendicular to the Tangents, BF and DG.

Then, the Trapezium ABaD is ſimilar to aFEG; for aB=aD and aF=aG; (C. 2. 16. 3. El.) alſo AB=AD, and EF=EG; the Angles at B, D, F, and G are Right, and BaD=FaG (2. 1. El.) conſequently, BAD=FEG. (See Th. 1. 10. 1. El.)

Wherefore, Ba:aF::AB:EF; i. e. as AS:EC; or, as Aa is to aE (which are correſponding Diagonals) as in the firſt Method (No. 1.) for, AE is a Viſual Ray, in both, and a is the Point in which it cuts the Picture.

PROBLEM XV.

To find the Repreſentation of a Line perpendicular to the Picture; its Place being given in the Original Plane, the Interſection and Vaniſhing Line of the Plane it is in, and the Place of the Eye.

Fig. 53. Let AB be the Original Line; AD is the Interſection of the Plane it is in, and *E*CE is the Vaniſhing Line; E is the place of the Eye.

Draw EC perpendicular to the Vaniſhing Line, cuting it in C, the Center; and, becauſe AB cuts the Interſection, A is its Interſecting Point.

Draw AC, and BE cuting it in b; then is Ab the Repreſentation of AB.

Or (having drawn AC) make C*E* equal to the Diſtance of the Picture, and A*B* equal to the Original Line (AB) and draw *BE*, cuting AC, in the ſame Point, b. In which proceſs, the meaſures, it is obvious, muſt always be placed on contrary Sides of the indefinite Repreſentation, or the Viſual Ray cannot cut it.

DEM. Because AB, the Original Line, is perpendicular to the Picture. EC, the Direct Radial, is its Radial; wherefore, C, the Center of the Picture, is its Vanishing Point †; and, because A is its Intersecting Point, AC is the indefinite Representation of AB, infinitely produced (by Theorem 12th.) † Def. 22.

But, EB (or *EB*) is a Visual Ray, from the Eye (E) to some Point (B) in the Line; which must cut the Picture somewhere in the Line AC; which is the Section of a Radial Plane, passing through the Original Line and the Eye ‡; and consequently it will cut it in b; making Ab : bC :: AB : CE. ‡ Theo. 8.

Therefore, since A, one extreme of the Line AB, is in the Picture, and b is the Representation of B, another Point in the Line, Ab is the perspective Representation of AB. Q. E. D.

To illustrate this, ocularly (See Fig. 37, No. 1.) turn up the Picture AIKB, vertical; also turn over the Plane W, till *C* coincides with the Center of the Picture, then *E* will be in the true place of the Eye, and *EC* is perpendicular to the Picture, consequently parallel to all other Lines, which are perpendicular to the Picture; therefore, to AC and FD, &c. on the other side of the Picture; consequently, C is their Vanishing Point. (Def. L.)

Turn over the Plane AFDC, or X, till it coincides with W; then, the Plane WX is a Radial Plane, passing through the Original Line, AC, and the Direct Radial, *EC*, its Parallel, consequently through the Eye; producing, by its Section with the Picture, the Right Line RC; which is the indefinite Representation of AC; (Theo. 12) for, it passes through R, the intersecting Point of AB, and also through C, its Vanishing Point.

Now, EA, EB, and EC, are Visual Rays, from the Eye to the Points A, B, and C; and since they are Right Lines (Def. H) and the two Extremes, E, A, &c. are in the Radial Plane, WX, consequently, the whole Line, EA, &c. are in that Plane (Ax. 1.) and consequently, they will cut the Picture, somewhere in RC, the Intersection of the Plane they are in.

Therefore, in a, b, and c; as by Theorem 13th.

N. B. Any Point (B) in the Line AB, may be found, by drawing B*B*, at pleasure, and E*E* parallel to it; cuting the Intersection and Vanishing Line, in *B* and *E*, and drawing *EB*; as in Prob. 14.

Or, having drawn B*B*, at pleasure, and E*E* parallel to B*B*; make Ba to Ee, as AB to EC (equal or otherwise) and draw ae; which will cut AC in the same Point, b.

If the given Line does not cut the Intersection, as FG, it must be produced till it does, as at D; and then proceed the same as before; making D*G* equal DG, and D*F* equal DF; draw DC; and *EF*, *EG*, cuting DC in f and g: fg is the Representation of FG.

If I be the Intersecting Point of a Line, perpendicular to the Picture, whose Representation is required; draw IC; and IH, or IK, parallel to either the Horizontal or Vertical Line; make IH, or IK, equal to the Distance of any Point, in the Original Line, from its intersecting Point, and draw H*E* or KE, cuting IC in *a*. I*a* is the Representation of a Line, perpendicular to the Picture; from its intersecting Point, I, equal to IH or IK.

PROBLEM XVI.

To find the Representation of a Line, parallel to the Picture; in a Plane which is perpendicular to the Picture.

Let AB be an Original Line, in the Ground Plane, *E*CL is the Vanishing Line of the Ground Plane, and DI is its Intersection with the Picture. Fig. 54.

If the Original Line was applied close to the Picture (as DF) it would have its full Dimensions delineated thereon, and be its own Representation.

But, if AB be situated at some Distance from the Picture (as AD) draw AD and BF, perpendicular to the Intersection, cuting it in D and F, the Seats of the extreme Points, A and B. Draw DC and FC.

Plate XI. Fig. 54. Having made C *E*, or C E, equal to the Diſtance of the Eye, and D *a* equal D A, draw A E or *a E*, cuting D C in a, the repreſentation of A.

A Right Line, a b, being drawn, parallel to the Interſection, or Vaniſhing Line, till it cuts F C, is the Repreſentation of A B.

DEM. Becauſe A D and B F are Lines perpendicular to the Picture, C, the Center, is their Vaniſhing Point (Prob. 15.) and, D and F are their Interſecting Points †; conſequently, D C and F C are the indefinite Repreſentations of A D and B F ‡.

† Theo. 10.
‡ Theo. 12.

But E A or *E a* is a Viſual Ray, from the Eye to the Original Point A; which, it has been proved, would cut the Picture, ſomewhere, in the Indefinite Repreſentations, D C, making D a : a C :: A D : C E, &c. by the thirteenth Theorem.

§ Theo. 9.

But, the Repreſentation of a Line, parallel to the Picture, is parallel to its Original §; and conſequently to the Vaniſhing Line of the Plane it is in; and has that proportion to the Original, as the Diſtance of the Picture, to the Diſtance of a Plane paſſing through the Original Line parallel to the Picture; which ſhall be proved, here.

‖ 15. 1. El.

Now, D F = A B ‖; and, in the Triangle D C F, becauſe a b is parallel to D F, the Triangle a C b is ſimilar to D C F; wherefore, C a : C D :: a b : D F, i. e. as a b is to A B, equal D F. (2 & 4. 6. El.)

But C a : a D :: C E : A D; conſequently, C a : C D (i. e. C a + a D) :: C E : C E + A D.

But, C E is equal to the Diſtance of the Picture, and A D is equal to the Diſtance of the Original Line, beyond the Picture; wherefore, A D + E C is equal to the Diſtance of a Plane, paſſing through the Original Line, parallel to the Picture.

Therefore, a b, the Repreſentation, is to A B, the Original Line, as the Diſtance of the Picture, C E, is to C E + A D. Q. E. D.

N. B. The Repreſentations, a and b, of the Extremes, A and B, of any Line, may be found, by any of the Methods for finding the Repreſentation of a Point, in Problem 14th.

Any other Line, as G H, parallel to the Picture, is found by the ſame means.

Or, being ſituated in any other Plane, as *G H*, whoſe Interſection, *a b*, and Vaniſhing Line, E C G, is given, or found; whether the Plane it is in be perpendicular to the Picture, or inclined, having the Center and Diſtance of the Vaniſhing Line, or of the Picture, the Proceſs is the ſame.

a and *b* are the Seats of *G* and *H*, and *E H* or E *c* are Viſual Rays; the reſt is obvious, on inſpection of the Figure. But, if the Original Plane be inclined to the Picture, *a* and *b* are not the Seats of *G* and *H*.

PROBLEM XVII.

To find the Repreſentation of a Line, any how inclined to the Picture, ſituated in a Plane perpendicular to the Picture; having the Interſection and Vaniſhing Line given.

Fig. 55. Let A B be an Original Line in the Ground Plane, whoſe Interſection is L I, and Vaniſhing Line, V C; C is the Center of the Picture.

Draw C E, perpendicular to the Vaniſhing Line, and equal to the Diſtance of the Picture. Produce B A, to its interſecting Point, I; and draw E V, parallel to A B, cuting the Vaniſhing Line in V, the Vaniſhing Point of A B.

Draw I V, the indefinite Repreſentation of A B; and the Viſual Rays, A E, B E, being drawn, will cut it in a and b, the Repreſentations of the two Extremes, A and B. a b is the finite perſpective Repreſentation of A B.

Or, the Seats of the extreme Points being found, by drawing A d and B D, perpendicular to the Interſection; d C and D C, being drawn, project the ſame Points a and b, as before.

Or, make V *E* equal to V E, I G equal I A, and I F equal to I B.

Draw G *E* and F *E*, cuting I V in the ſame Points, a and b.

DEM.

DEM. Becauſe EV is a Right Line, from the Eye, parallel to the Original Line, and cuts the Picture in V, V is the Vaniſhing Point of the Line AB†; and I being its interſecting Point, IV is its indefinite Repreſentation‡; a and b are the Interſections of the Viſual Rays, with the Picture, as proved in the former Problems. Therefore, a b is the Repreſentation of AB.

† Def. 22.
‡ Theo. 14.

GE and FE alſo project the ſame Points, a and b.

For VE is parallel to FI, and equal to EV, the Diſtance of the Vaniſhing Point; alſo, IG is equal to IA, and IF to IB; by Conſtruction.

Wherefore, if EE and BF be drawn, they will be parallel; for the Triangles, EVE and BIF, are ſimilar Iſoſceles; and ſince IF, IB are reſpectively parallel to EV and VE, conſ. BF is parallel to EE. Therefore, E is the Vaniſhing Point of BF; and conſ. Fb : bE :: Bb : bE, i. e. as Ib : bV, &c.

The laſt Method of proportioning inclined Lines will be found the moſt convenient, in Practice, of all other; and it may alſo be obſerved, that if the Repreſentation, a, of any Point, A, in the Original Line, be given or found, and the Interſection of the Plane it is in; the Repreſentation of any other Point (B) in that Line, may be determined, its Diſtance from A being known, without the Interſecting Point I.

Alſo, the Inclination of AB being known (equal BIF) the Vaniſhing Point, V, may be found; by making the Angle HEV, with the Parallel of the Eye (HJ) equal BIF; or CEV equal to its Complement; or, if EE be the Radial of any other Line whoſe Vaniſhing Point is E, make the Angle EEV equal to the Angle which the two Lines make with each other; all, which, produce the ſame Point V.

Then, having made VE equal VE, the Diſtance of the Vaniſhing Point from the Eye; and, if a be the known Repreſentation of ſome Point A, in the Original Line (AB) and FI the Interſection of the Plane it is in, draw the indefinite Repreſentation, aV, from that Point, and draw Ea cuting the Interſection, in G; make FG equal to the Diſtance of the Point B from A, and draw EF, cuting the indefinite Repreſentation, aV, in b, the Repreſentation of B.

If the Vaniſhing Point, V, be out of the Picture, having found its Diſtance from the Center, C, and from the Eye, E (by Prob. 12) CE will be equal to the Difference; i. e. make CE equal VE, leſs by VC.

Or, when the Vaniſhing Point is in the Picture, and the Point E cannot be had thereon (as is frequently the Caſe, when the Vaniſhing Point falls near the Center of the Picture, as E) take any Portion of VE, *viz.* VK half VE; draw Ka, cuting the Interſection in *a*; make *ab* equal half AB; or the ſame Portion VK was taken of VE, draw K*b*, which will project the ſame Point, b, for the Repreſentation of B, as before. This Expedient will be found, frequently, neceſſary in Practice.

If the Repreſentations, a and b, of any two Points, A and B, in any Line, be found; their Seats, d and D, are aſcertained by drawing Ca and Cb, from the Center, to the Interſection (FI) of a Plane perpendicular to the Picture, in which the Original Line is ſituated, cuting it in d and D.

By which means, the Repreſentation of any other Point, in that Line, may be found; making DL to Dd, as the Diſtance of the Point from B, is to AB, and draw CL, cuting aV in c, the Point ſought. (See Prob. 8.)

SCHOL. After ſome one or other of theſe Methods, all Lines inclined to the Picture are drawn on the Picture, and divided perſpectively, as the Original is divided; either by the real meaſures, being applied, or by the known ratio of the Originals. Various Expedients for performing the ſame thing will be found neceſſary, becauſe the ſame Method cannot always be applied, in all Caſes; for which reaſon, I have given all the Methods, I would deviſe, which are really uſeful.

Theſe Methods of proportioning inclined Lines are general and applicable in all Caſes whatever; and, on inſpection of the Figures, they will be found the very ſame as for Lines perpendicular to the Picture. For, the Center of the Picture being the Vaniſhing Point of all ſuch Lines, its Diſtance is laid down, on either Side as is moſt convenient, on the Vaniſhing Line, from the Center; and the Lines, which vaniſh in the Center, are proportioned by the ſame means. (See Prob. 15, Fig. 53.)

When

Plate XI. Fig. 55. When the Vaniſhing Point, V (being remote) cannot be had, on the Picture, the tranſpoſed place of the Eye, *E*, may be thus found.

The inclination of the Original Line, to the Interſection, being known, ſubtract it from 180 Degrees, or two Right Angles; take half the ſum of the remainder, and make the Angle V E *E* equal to it.

Or, through E, draw H E J, the parallel of the Eye, parallel to the Vaniſhing Line; make H E k equal to the Angle of Inclination of the Original Line; biſect the remainder, k E J, by the Line E *E*, cuting the Vaniſhing Line, at *E*, the tranſpoſed place of the Eye, for the Vaniſhing Point V.

§ 9. 1. El. For, the Triangle E V *E* is Iſoſceles (E V being equal V *E*) conſequently, the Angle V E *E* = V *E* E § equal *E E* J, by conſtruction, and by 4. 1. of Elements.

Hence it is evident, that I b is the perſpective Repreſentation of I B.

† Cor. to Theo. 1. For, ſuppoſe B I F to be the Original of the Triangle b I F; one, Side (I F) being in the Picture, therefore, it has, no Vaniſhing Point†. The Triangle B I F is Iſoſceles, and ſimilar to E V *E*.

‖ Def. 22. Now, E V is parallel to the Side I B, and E *E* to B F; wherefore, V and *E* are the Vaniſhing Points of thoſe two Sides‖, and I V, F *E* their indefinite Repreſentations; conſequently, I b and I F repreſent equal Lines. I F, being in the Picture, retains it full dimenſions, and is its own Repreſentation on the Picture; and, F b repreſents the other Side, F B; therefore, I b repreſents I B; for, b repreſents B‡.

‡ C. 7. T. 12.

PROBLEM XVIII.

How to find the perſpective Repreſentation of a Triangle, having the Original given, in any Plane, whoſe Interſection and Vaniſhing Line is alſo given, and the Place of the Eye.

Fig. 56. A B C is the Original Triangle, D G is the Interſection, and L H the Vaniſhing Line; E is the Eye.

Produce the three Sides, A B, A C, and B C, to the Interſection, cuting it in D, F, and G, the interſecting Points of the Sides, reſpectively.

Draw E H, E I, and E L, reſpectively parallel to A B, A C, and B C, cuting the Vaniſhing Line in the Points H, I, and L, their Vaniſhing Points.

Draw the indefinite Repreſentations, D H, F I, and G L, cuting each other in the Points a, b, and c, and giving the repreſentation of the Triangle by their Interſections.

This is manifeſt, ſeeing that the Sides of the Triangle, are in the Lines A D, A F, and B G; conſequently, their Repreſentations muſt be in the indefinite Repreſentations of thoſe Lines, and conſequently, between their Interſections (by Theorem 8. and 12.)

† Cor. 3. 1. 1. El. N. B. The Angles L E I, I E H, and H E K, at the Eye, are equal, reſpectively, to A C B, B A C, and A B C; and they are alſo equal to two Right Angles†; and ſo are the three Angles of the Triangle‡.

‡ 10. 1. El. Hence, if any one Side of the Triangle, as a b, be given, on the Picture, the Vaniſhing Points, I and L, of the other two Sides, are determined by Problem 4th; and, by drawing I a and L b, interſecting at c, the Triangle is completed.

See this illuſtrated, by moveable Planes, in Fig. 37.

Raiſe up the Picture A I K B perpendicular to the Ground Plane, and let the Horizontal Plane be placed parallel to the Ground Plane, i. e. perpendicular to the Picture; or, if the Picture be inclined to the Ground Plane, the Horizontal Plane making an equal Angle with the Picture, as that makes with the Ground Plane, will be parallel to the Ground Plane. In which Caſe, the Lines E N, E O, and E L are the Radials of the three Sides of the Triangle X Y Z, on the other Side of the Picture; being reſpectively parallel to them§ Conſequently, the Points N, O, and L, are their Vaniſhing Points‖; B, P, and § are their Interſecting Points, and B N, P O, and § L are their indefinite Repreſentations, producing the perſpective Repreſentation, x y z, by their Interſections.

‖ Def. 22.

Let the Picture be turned down, and imagine the Geometrical Plane turned over on its Interſection, A B, to the other Side; in which Caſe, the Triangle is inverted. Turn down the Horizontal Plane into the Picture; the Radials E N, E O, and E L, are ſtill parallel to the three Sides of the Triangle, and produce the ſame Vaniſhing Points, N, O, and L; alſo, the Viſual Rays E X, &c. being drawn on the Picture, paſs through their reſpective repreſentations, x, y, z; as it is obvious they would paſs through the Picture, in thoſe Points, to the Original Points X, Y, and Z, on the other ſide.

N. B.

N. B. The Angle NEO, which the Radials EN and EO make, at the Eye, is equal to the Angle Y, of the Triangle; for, they are reſpectively parallel to them. EO and EL, the Radials of YZ and XZ, make the Angle OEL equal to the Angle Z; and if NE or LE was produced beyond the Eye (E) they would make an Angle equal to X; which is conſonant to Cor. 1ſt, Theorem 6th.

PROBLEM XIX.

How to find the Repreſentation of a Square or other Rectangle, given in the Geometrical Plane; one Side being parallel to the Picture.

If the 15th and 16th Problems are well underſtood, there will be little occaſion to explain the Method of proceeding in this; for, if any Rectangle have one Side parallel to the Picture, the other two adjoining Sides muſt, neceſſarily, be perpendicular to the Interſection of the Plane it is in, if not to the Picture.

Let ABFD be a Square; one Side, AB, lying cloſe to the Picture, AD and BF are therefore perpendicular to it. C is the Center of the Picture, or of the Vaniſhing Line, *EE*, of the Plane of the Square, AB, is its Interſection. Plate XII. Fig. 57.

Now, AD and BF are perpendicular to the Interſection, therefore they vaniſh in the Center of the Vaniſhing Line. (See Problem 15.)

Draw AC and BC, their indefinite Repreſentations; make C*E*, on either Side, equal to the Diſtance of the Picture, or Vaniſhing Line, and draw B*E* (or A*E*) diagonal Ways, cuting AC in d, the Repreſentation of D, (or BC in f.)

Draw df parallel to AB. AdfB is the repreſentation of the Square A*D*FB.

For, B*E* is the indefinite Repreſentation of the Diagonal, BD, *E* being its Vaniſhing Point; for E*E* is parallel to BD, (C*E* being equal to CE, and AD to AB) and DF being parallel to the Picture, its Repreſentation, df, is conſequently parallel to the Interſection. (Prob. 16.)

2. If another ſquare be required, draw f*E*, cuting AC in g, and draw gh parallel to df. By which Expedient any length of AC may be obtained.

If the length AG had been required, equal twice AD; A*G* on the Interſection being made equal twice AB, and *GE* drawn, gives the ſame Point g; as before.

Or, if C*E* be biſected, at K, BK is the indefinite repreſentation of the Diagonal BG; for EK is parallel to BG; the Angle CEK is equal GBH, and, K is the Vaniſhing Point of that Diagonal; draw BK, cuting AC in g, as before.

3. If the Square be at ſome Diſtance from the Picture, as *ABFD*, produce the two perpendicular Sides to the Interſection, cuting it in a and b; alſo, produce the Diagonal *FA* to I, its interſecting Point; or, make aI equal a*A*, and proceed as in the former Caſe. The Figure explains the reſt.

COR. Hence, a Pavement of Squares, having their Sides parallel to the Picture, may be delineated, with great facility.

Let AB be the Ground Line, and ECD the Horizontal Line. No. 1.

Take the geometrical meaſure of a Square, and apply it, on the Interſection, AB, as often as it is required, from A to B; as a, b, &c.

From each Diviſion, draw Right Lines to the Center, AC, aC, &c. and draw a Diagonal, from A or B, to the Eye, at E or D, cuting each indefinite Repreſentation, in the Points *a*, *b*, *c*, &c. through which, draw the Lines FG, HI, &c. parallel to the Interſection, or Vaniſhing Line, and the ſeveral Figures X, Y, Z, &c. are the Repreſentations of Squares on the Picture; which may be repeated, by drawing other Diagonals to any length or width.

Plate XII. Now, if one of theſe Squares (as Y or Z) be conſidered ſingly, it has ſcarce the appearance of a Square, having no other, contiguous to it, to biaſs the judgment; but, by the affinity of the whole, the Eye (being accuſtomed to ſee Objects as they appear, in all ſituations) is not offended, and readily gives the aſſent; although it is certainly capable of determining, that the ſeveral Repreſentations of Squares, are not Squares, but have the appearance, only, of Squares, in certain Poſitions and Situations.

PROBLEM XX.

To find the Repreſentation of a Square, or other Rectangle, whoſe Sides are all equally inclined to the Picture.

Firſt, by the Original Figure being geometrically drawn, in its determined Poſition to the Picture; the Interſection of the Plane it is in being given.

Fig. 58. Let A B C D be a Square, the Sides of which are equally inclined to the Picture. C is the Center of the Vaniſhing Line *E E*, or of the Picture; *E*, *E*, are the tranſpoſed places of the Eye, to the Vaniſhing Line, *viz.* C *E* equal C E.

As one Angle of the Square (A) touches the Interſection, it is, conſequently, the Interſecting Point of the two Sides A B and A D.

Let the other two Sides be produced to the Interſection, at F and G.

Draw A *E*, both ways; alſo, draw F *E* and G *E*, diagonal ways, cuting each other. A b c d is the Repreſentation of A B C D.

For the Point A being in the Picture, is its own Repreſentation, A *E*, A *E* are indefinite Repreſentations of A B and A D; and F *E* and G *E*, of F C and C G (Theo. 12) conſequently, they cut each other in the repreſentations of the ſeveral Angles, B, C, and D. (Cor. 7. Th. 12.)

If the Rectangle H I K L be at ſome Diſtance from the Picture, the Sides being produced to their interſecting Points, *a*, *b*, *c*, *d*, and the indefinite Repreſentations, *a E*, *b E*, &c. being drawn (as in the Figure) give the Repreſentation h i k l of that Rectangle; its Sides being parallel to the Sides of the Square.

E V, parallel to the Diagonal, I L, produces its Vaniſhing Point.

Note. In this Caſe, it may be obſerved, that there is no neceſſity for the Eye, i. e. the Diſtance of the Picture or Vaniſhing Line, being placed above it, but placed equally on either Side of its Center, C, in the Vaniſhing Line, as C *E*. Squares, or other Rectangles (wherever they are ſituated in the Original Plane) having the ſame Poſition to the Picture, have their Sides parallel, and conſequently, they have the ſame Vaniſhing Points.

N. B. The Diagonals of the Square are, in this Caſe, the one (B D) parallel, and the other (A C) perpendicular to the Picture; conſequently, their Repreſentations are either parallel, as b d, or vaniſh in its Center, as A c.

Let it be, here alſo, particularly noticed, that *E*, being the place of the Eye, in the Vaniſhing Line (commonly called the Point of Diſtance) is the Vaniſhing Point of the Diagonal of a Square, whoſe Sides are parallel and perpendicular to the Picture. Conſequently, if the Diagonals are parallel and perpendicular to the Picture, they are the Vaniſhing Points of its Sides.

By which Points, all Lines perpendicular to the Picture are proportioned, perſpectively; as it may be obſerved in the preceding Problems.

2nd. How to find the Repreſentation of a Square. in this Poſition, not having the Figure drawn out geometrically, only its meaſure and place known.

Let A be the Interſecting Point of an Angle of the Square, ſituate on the left ſide of the Station Line; at the Diſtance A J.

Make A F and A G each equal to the Diagonal, and proceed as before.

Or, by the meaſure of its Sides.

Make

Make *EF* equal to the Diagonal of a Square, whoſe Side is CE; i. e. make *EF* equal to the Diſtance of the Vaniſhing Point *E*, from the Eye, equal *E*E.

Make Af equal to AB and draw f*F*, cuting the indefinite Repreſentation A*E* in b, the Repreſentation of the Angle B.

Draw bd parallel to FG, the Interſection, cuting A*E* in d; and, laſtly, draw b*E* and d*E*, diagonal ways, cuting each other, which compleats the Figure.

COR. Hence, a Pavement of Squares, diagonal-ways, may be delineated.

Fig. 58. No. 1.

Let AB be the Ground Line and ED the Horizontal vaniſhing Line.

Make Aa, ab, &c. equal to the Diagonal of the Square; make CE, and CD each equal to the Diſtance of the Picture (C being the Center.)

Draw AE, aE, &c. and AD, aD, &c. cuting each other in the repreſentations of Squares, placed diagonal-ways.

Through d, where AD and BE interſect, draw ef parallel to AB; and where it cuts the ſeveral Lines, drawn from a, b, c, &c. *viz.* in e, *a*, *b*, &c. draw *a*D, *b*D, and E*d*, &c. contrary ways; by which means they may be continued at pleaſure.

PROBLEM XXI.

To find the Repreſentation of any Rectangle, obliquely ſituated to the Picture.

Firſt, by having the Original drawn in the Geometrical Plane, in its true place and poſition, in reſpect of the Picture and of the Eye.

Fig. 59.

ABCD is the Rectangle to be delineated; FI is the Interſection of the Plane it is in; KL is the Vaniſhing Line, and *C* its Center. The Diſtance is known.

Make *CE*, equal to the Diſtance, and perpendicular to KL.

Draw EK and EL, parallel to the Sides of the Rectangle, AB and BC, reſpectively, producing their Vaniſhing Points, K and L.

The Original being at ſome Diſtance from the Picture, produce every Side, DA, CB, &c. to the Interſection, cuting it in F, G, H, and I, their Interſecting Points.

Draw the indefinite Repreſentations FL, GL, HK, and IK; which, by their mutual Interſections, give the Repreſentation, abcd, of ABCD.

METHOD 2nd. By the Seats of every Angle, on the Picture, and their Diſtances from their Seats, reſpectively.

If the Original be in a Plane perpendicular to the Picture, the Seats of all its Angles are in the Interſection of the Plane they are in.

Let *a*, *b*, *c*, and *d*, be the Seats of the Angles A, B, C, and D, reſpectively.

Draw *aC*, *bC*, &c. to the Center; make *ab* equal to the Diſtance of the Point A, from its Seat, *be* equal to the Diſtance of B, *cd* of C, and *df* of the Angle D.

Make *CE* and *CF* each equal to the Diſtance of the Picture, and draw *bE*, *eF*, *dF*, and *fE*, cuting *aC*, *bC*, &c. in the Points a, b, c, and d, reſpectively; which are the repreſentations of the ſeveral Angles A, B, C, and D. (Pr. 14. Meth. 1.)

Join the Points a and b, b and c, &c. as in the Figure; and the Quadrilateral, abcd, is the repreſentation of the Rectangle, ABCD, ſituated as in the Figure.

Note. This Method, is uſed by all the old writers on Perſpective; particularly in the Jeſuits.

In which proceſs it may be obſerved, that there is no occaſion for the Vaniſhing Points, K and L; for if they made uſe of Vaniſhing Points, at all, they were found by producing the Sides ba, bc, &c. to the Vaniſhing Line; i. e. to the Horizontal Line, for they knew no other Vaniſhing Line. Which Vaniſhing Points, ſo produced, they called accidental Points.

Plate XII. Fig. 60. METHOD 3rd. By the known Proportion of the Original; its Poſition, Situation, and Diſtance being determined.

The Original Figure, ABCD, is drawn out in the Geometrical Plane, to ſhew how the meaſures, &c. are applied, but it is, otherwiſe, of no uſe in the operation; as the careful and accurate obſerver will perceive.

IN, the Interſection; KL, the Vaniſhing Line; *C*, its Center; and *CE* its Diſtance, are given, the ſame as before.

Through E, draw HI, parallel to the Vaniſhing Line; draw EK and EL, making the Angles, HEK, and IEL, equal to the known Inclination of the Sides of the Rectangle (AB and BC) to the Interſection, reſpectively; i. e. make HEK equal to ABF, and IEL equal to CBG; FG being parallel to the Interſection, the Angles ABF and CBG are equal to their inclination to it†.

† 4. 1. El.

Make *DB* equal to JB; i. e. to the diſtance of the neareſt Angle (B) to the Station Line, and draw *BC*; alſo, make *Bd* equal to its diſtance from the Interſection (equal B*B*) and, *CE* being made equal to CE, draw *dE*, cuting *BC* in b, the Repreſentation of the Angle B. (Prob. 14th.)

Draw the indefinite Repreſentations, bK and bL, from the Point b.

It remains, now, to cut off (from b) ba and bc, in the perſpective Repreſentations of AB and BC, the Originals.

Make K*E*¹ equal to KE, and L*E*² equal to LE; which Points are uſed for proportioning all Lines which vaniſh in K and L, reſpectively. (Prob. 17.)

Draw *E*¹b, cuting the Interſection in *b*; make *ab* equal AB, and draw *aE*¹, cuting bK in a, the Repreſentation of A.

Alſo draw E²b cuting the Interſection in *b*² and make *b*² *c* equal BC (the other Side, whoſe vaniſhing Point is L) and draw *cE*² cuting bL in c, the Repreſentation of C, another Angle of the Figure.

From the Points a and c, draw aL and cK, interſecting at d, which compleats the Figure; for, the Originals of thoſe Sides, being parallel to the Originals of ab and bc, they have, conſequently, the ſame vaniſhing Points, reſpectively.

SCHOL. This Method, which is perfectly conſonant to the new Principles by Brook Taylor, may appear to ſome Perſons more difficult than either of the other, particularly the firſt, which is performed, alſo, entirely on the ſame Principles; the difference is very obvious, notwithſtanding the effect is the ſame, as it may be ſeen, by comparing the Figures; or by comparing both, with the 17th Problem, which contains all the various Methods of proportioning inclined Lines, in general.

In the firſt, there is a neceſſity for having the Original Figure drawn geometrically, either on the Picture or ſomewhere apart; for which there is not, frequently, room to ſpare on the Picture; and we are liable to errors, in transfering the interſecting Points; but, without the Original being placed in the very Poſition, we cannot draw the Radials, or Parallels from the Eye, producing the Vaniſhing Points; as by that Method.

Whereas, by the laſt Method, the Angles, being known, are made equal to the Originals; in which Caſe, the Radials would be parallel to the original Lines being in their true places.

Or, if there be not room on the Picture, or interfere with the Objects, the Vaniſhing Points and their Diſtances, from the Center and from the Eye, are all determinable, by Problem 12th; the Vaniſhing Points, K and L, and their Diſtances K*E*¹ and L*E*² being aſcertained by it.

There is, likewiſe, no need for the Original Figure on the Picture, but being any where drawn out, geometrically, or its meaſures being known, and the Poſition determined, they are applied to the Picture with the greateſt facility, for which, a little Practice will render it quite familiar; regarding always, carefully, to make uſe of the proper Points for particular Lines, and to diſtinguiſh between the Vaniſhing Points and thoſe which are uſed, for Viſual Rays, i. e. for cuting indefinite Lines in the Ratio required.

PROBLEM XXII.

To find the Repreſentation of a regular Pentagon, one Side being parallel to the Picture.

Pl. XIII. Fig. 61. Let ABCDF be a regular Pentagon, having one Side (AB) parallel to the Interſection (IK) conſequently parallel to the Picture.

LM is the Vaniſhing Line, and E the Eye.

Produce

Produce the inclined Sides, FD, CD, AF, and BC, to the Interſection, cuting it in G, H, I, and K.

Draw EL, EM, &c. reſpectively parallel to them, producing the Vaniſhing Points, L, M, N, and O, of each Side.

Draw the indefinite Repreſentations GL, HM, IN, and KO; producing, by their Interſections, the Figure, fjcd, of the Original FJCD, (No. 1.) and *IfdcK* of *IFDCK* (No. 2) ſo that, the repreſentation of one Side (AB) is wanting, in each; which, on account of its paralleliſm to the Picture, has neither Interſecting nor Vaniſhing Point†; and muſt be found by Problem 16; or, by drawing a Viſual Ray, EA, cuting the indefinite Repreſentation, IN, of the Side AF, in a; or, make NP equal NE, and I*a* equal IA, and draw *a*P. (Prob. 17.) Or, by producing either Diagonal, AD or BD; being parallel, reſpectively, to the Sides BC, and AF, O and N are their Vaniſhing Points.

† Cor. to Theo. 2.

Having, by any of theſe Methods, found a, the Repreſentation of A, draw ab, parallel to the Interſection, cuting KO in b; which compleats the Figure.

Secondly. How to determine all the Vaniſhing Points in this Poſition, without the Original Figure on the Picture.

Through E, draw RS parallel to the Vaniſhing Line; and make the Angles REL, LEO, &c. each of 36 Degrees; i. e. divide the Ark of the Semicircle, R23S, into five equal Parts, and produce the Lines E1, E2, &c. to the Vaniſhing Line, cuting it in L, O, &c. the Vaniſhing Points of the Sides and Diagonals of a Pentagon, having one Side parallel to the Picture.

SCHOL. The reaſon of this is obvious; for three of thoſe Angles, *viz.* REN, is equal to the Angle of a Pentagon; and ſince one Side is parallel to the Picture, RE is parallel to it; and conſequently, EN will be parallel to AF, another Side; the Angle REN being equal BAF.

Alſo, the Angle which any two Sides, not contiguous (as AF and CB) make with each other, *viz.* FJC, is equal to one of thoſe Angles; wherefore, EN being parallel to one of thoſe Sides, EO is conſequently parallel to the other. (By 6. 1. El.) Alſo to the Diagonals AD and BD.

This is fully explained, after Problem 11th, in the 4th Book of Elements.

To find the Repreſentation when every Side is inclined to the Picture (the Original being in its true Place) has nothing particular; for, if every Side be produced to its interſecting Point, and their Vaniſhing Points are found by drawing parallels to every Side, from the Eye, the indefinite Repreſentations being drawn, produce the Repreſentation of the Original; as in the laſt Problem, or Prob. 18.

If the Vaniſhing Point of any Side be found, all the reſt are determined as before; by producing the Radial of that Side and deſcribing a Semicircle; and dividing it into five equal Parts, as in this Figure.

PROBLEM XXIII.

How to find the Repreſentation of a Pentagon, having only one Side given, in its true Place and Poſition, in the Geometrical Plane.

Let AB be the given Side.

It is required to find the Repreſentation of a regular Pentagon, whoſe Side is equal to AB; and inclined to the Picture in the Angle BDG, or to the Interſection GD. KM is the Vaniſhing Line, C is its Center, and CE its Diſtance. Fig. 62.

Through E, draw EH parallel to the Vaniſhing Line, and make the Angle HEN equal to BDG, the given Angle, producing the Vaniſhing Point I.

Produce BA to D; draw the Indefinite Repreſentation DI; and find the finite part ab, the repreſentation of AB (by Prob. 17) i. e. draw the Viſual Rays AE and BE; or, make IP equal IE; and, DF, DG, equal to DA, and DB, reſpectively; draw FP and GP, cuting DI in a and b.

Produce IE, and deſcribe a Semicircle N*ac*O; divide the Ark into five equal Parts, at *a*, *b*, *c*, and *d*; draw E*a*, E*b*, &c. to the Vaniſhing Line, producing the Vaniſhing Points, K, L, &c. of the remaining Sides of the Poligon.

Having obtained ab, the Repreſentation of AB, as above; draw aM and bL the indefinite Repreſentations of two Sides, from thoſe Points; for, IaM repreſents an Angle (IEM) equal to the Angle of a Pentagon; and IbL repreſents an Angle equal to IEL; which is equal to an external Angle of a Pentagon.

Pl. XIII. Fig. 62. Draw a K, cuting b L in c; b c is another Side of the Pentagon; for, a c repreſents a Diagonal (which, in a Pentagon, is parallel to the oppoſite Side) whoſe Vaniſhing Point is K.

Draw I f, through c, cuting a M in f; c f repreſents a Diagonal parallel to the Side a b; and conſequently, a f repreſents another Side.

Laſtly; draw f K, and b M cuting it in d, and draw c d, which compleats the Figure, a b c d f, required.

The Vaniſhing Point Q (which is out of the Picture) of the Side c d, and its parallel Diagonal b f, has not been wanted in this proceſs.

N *c* and O *b* are Diagonals, E N, E *c*, or E O and E *b*, being ſuppoſed Sides of a Pentagon; and E Q, E K are Parallels to them, from the Eye, producing their Vaniſhing Points. (See Prob. 10.)

Note. If a b or b c, or any Side in the Repreſentation had been given, the whole Figure may be completed, as above, without the Interſection.

PROBLEM XXIV.

To find the Repreſentation of a regular Hexagon, ſituated any how to the Picture.

There is no Figure whatever, except a Square, eaſier to deſcribe than a Hexagon.

To have the Original Figure drawn in the Geometrical Plane, it is evident, is the ſame as has already been explained, in the foregoing; I ſhall, therefore, beg leave to paſs over that deſcription.

Fig. 63. Let A B be a Side, given, of a regular Hexagon; the Original of which, is parallel to the Picture. I K is the Vaniſhing Line.

Make I E K an equilateral Triangle, whoſe Perpendicular is *C* E; as follows.

Through E draw H L parallel to the Vaniſhing Line, and, on E, deſcribe a Semicircle; which, divide into three equal Arks, at *a* and *b*.

Draw E *a* and E *b*, and produce them to the Vaniſhing Line, cuting it in I and K; which, are all the Vaniſhing Points required for this Poſition.

Draw A I and B K; make A b and B a each equal to A B, in A B produced, both ways; and draw a I and b K.

Draw B I and A K, or A *C* and B *C*, cuting them in F and G, and join F G, which is parallel to A B, and compleats the Figure. A D F G C B.

SCHOL. A regular Hexagon having an even number of Sides, the oppoſite ones are conſequently parallel; and the Diagonal, which paſſes through the Center, is alſo parallel to them; and becauſe A B is parallel to the Vaniſhing Line, its Original is parallel to the Picture; and conſequently, the oppoſite Side F G is alſo parallel to A B; and, becauſe they are equal, A F and B G are parallel, and conſequently their Originals were perpendicular to the Interſection; therefore they vaniſh in the Center of the Vaniſhing Line. (Cor. to Theorem 11.)

The two Sides, A D and C G, and the Diagonal, B F, repreſent parallel Lines; conſequently they have the ſame Vaniſhing Point, I; alſo B C, A G, and D F; all which may be ſeen in the Hexagon A B *C G F D*, which may be ſuppoſed the Original Figure, one Side A B, being in the Picture.

By drawing the Diagonals A *G*, B *F*, and *C D*, the Hexagon is divided into ſix equilateral Triangles, A O B, B O *C*, &c. and I E K has its three Sides reſpectively parallel to them; conſequently, ſince one Side in each Triangle is parallel to the Picture, and has no Vaniſhing Point (Theo. 1.) E I and E K are parallel to the other two Sides of each Triangle (which are equally inclined) and conſequently, to the Sides A *D*, *C G*, and B *C*, *D F*, of the Hexagon; I and K are, therefore, their Vaniſhing Points.

COR. Hence, a Pavement of regular Hexagons, or equilateral Triangles, may be eaſily delineated.

No. 2. Let A B be the Interſection of the Picture, or the Ground Line of the Pavement; and let A a be the determined meaſure of one Side of a Hexagon.

Make a b, b c, &c. equal to A a, as often as there is occaſion.

If C be the Center of the Picture (its Diſtance being known) find the Vaniſhing Points, D and E, as I and K above.

Draw

Draw AD, aD, bD, &c. and aE, bE, &c. and, where they interſect, draw Lines parallel to the Interſection, as fg, hi, &c. which divide the whole into the repreſentations of equilateral Triangles; ſix of which, around the ſame Point o (as a*cdef*b) form a Hexagon; and thus, as many may be deſcribed as are required; for which, the inſpection of the Figure is ſufficient.

Caſe ſecond. When the given Side is inclined to the Picture, in any Angle.

Let AB be the Side given, DF is the Interſection, and GL the Vaniſhing Line of the Plane it is in; E is the Eye. Fig. 64.

Find ab, the repreſentation of AB, by any of the former Problems, its Vaniſhing Point is G; draw GE, and produce it; on E deſcribe a Semicircle, *abcd*.

Make *ab* and *bc* each equal *a*E, and draw E*b*, E*c*, cuting the Vaniſhing Line, in H and I the Vaniſhing Points of the other Sides.

Draw aI and bH, the indefinite Repreſentations of two Sides.

For, baf repreſents the Angle GEI, which is the Angle of a Hexagon; equal *a*E*b*+*b*E*c*, twice 60 Degrees; and abc repreſents the Angle HE*d*, equal *a*E*c*.

Make af and bc repreſent Lines equal AB (by Prob. 10, Caſe 3rd) i. e. biſect the Angle GEH, by the Line EK; or, E*d* being equal E*b*, having drawn *bd*, draw EK parallel to *db*; i. e. perpendicular to EI.

For, EI is the Radial of af; conſequently parallel to its Original; and, EK is the Radial of a Diagonal of two contiguous Sides, which is perpendicular to the Original of af, parallel to *ab* (for *abcd* is half a regular Hexagon in the Poſition of the Original) conſequently, EK is parallel to a Diagonal whoſe Vaniſhing Point is K.

Draw aK cuting bH in c, and draw Gc till it cuts aI in f; for, the Original of the Diagonal cf is parallel to the Original of ab. As *ad* is parallel to *bc*.

Draw cl; aH, and fH; and GD, cuting fH in e, which compleats the Figure.

Note. If the Vaniſhing Point H had coincided with C, the Center of the Picture, G and I would be equally diſtant from C; and the Original of the Hexagon would be regularly poſited; having two Sides and a Diagonal perpendicular to the Picture; or to the Interſection of the Plane it is in.

By which means, a Pavement of regular Hexagons may be drawn in that Poſition.

Let AB be the Ground Line of the Pavement, C the Center of the Picture, and DF the Horizontal vaniſhing Line. No. 2.

On AB, take as many equal diviſions Aa, ab, &c. as you pleaſe, each equal to half the width of the Hexagon required.

Make CE equal to the Diſtance of the Picture, and make the Angles CED, CEF, each of 60 Degrees, i. e. having deſcribed the Ark *abc*, on the Center E, make *ba* and *bc* each equal E*b*, and draw E*a* and E*c*, to D and F; C, D, and F, are the Vaniſhing Points of all the Sides.

From all the Diviſions A, a, b, &c. draw Lines to the Center, C; and from every other Diviſion a, c, e, draw aD and aF, &c. to both the other Vaniſhing Points, D and F; and, where they cut AC and BC, *viz.* in f, g, &c. draw fF, gD, &c. as in the Figure; by which means they may be continued at pleaſure. The reſt is obvious, on inſpection.

PROBLEM XXV.

To find the Repreſentation of an Octagon and of a Circle inſcribed, having only one Side given.

An Octagon, having four of its Sides in a Square, is very readily delineated in Perſpective, ſeeing that, if one Side be parallel to the Picture, two are neceſſarily perpendicular to the Interſection, and four are inclined to it, equally, *viz.* in an Angle of 45 Degrees, half a Right Angle; conſequently, the Vaniſhing Points are the Center, and Diſtance, of the Vaniſhing Line of the Plane it is in.

Pl. XIV. Fig. 65.

Let AB be the given Side of an Octagon, parallel to the Picture; *EF* is the Vaniſhing Line of the Plane it is in, and C its Center.

Make C*E* and C*F* each equal to the Diſtance; alſo make *F*J equal *F*E.

Draw A*E* and B*F*, which are two Sides, indefinite.

For, ſince the given Side, AB, is parallel to the Picture, the adjoining Sides are inclined in half a Right Angle to the Interſection; therefore, *E* and *F* are their Vaniſhing Points. (Pr. 19 & 20.)

Produce AB, both ways; make BD equal AB, and draw DJ, cuting B*F* in G; and draw FG parallel to AB, cuting A*E* in F.

Draw CF and CG, and produce them, till they cut AB, produced, in H and I; draw I*E* and H*F*, cuting HC and IC, in K and L, and draw KL.

Draw AC and BC, cuting KL in M and N; and draw *E*N and *F*M, and produce them, cuting IC and HC in O and P, which compleats the Figure AFPMNOGB, of an Octagon.

Or, without drawing AC and BC, make KM equal NL, where DJ cuts KL, and draw *E*N and *F*M, as before.

Or, from a and b, where the Diagonals HL and IK cut AF and BG, draw aC and BC, cuting the Diagonals, again, in d and c; and through them, draw *E*c and *F*d.

A Curve deſcribed, carefully, by hand, touching every Side of the Octagon, paſſing through the Points a, b, c, and d, will be an Ellipſis; for it is the repreſentation of a Circle, in Perſpective.

Or, for greater exactneſs, through S, where the Diagonals HL and IK interſect, draw e f parallel to AB; and CS to g, biſecting AB and KL, in g and h; by which means there are eight Points obtained in the Circumference.

Note. The Figure below HD (which may be conſidered as the Interſection of the Plane of the Original) is the Original, geometrically drawn. The Italics, correſponding with the Roman Letters in the Repreſentation, ſhew the Original of each Point, or Line; S repreſents *S*, the Center.

To draw an Octagon inclined to the Picture has nothing ſingular or particular in it; as every Line, which is inclined to the Picture, is projected, on the Picture, by the ſame general Rule (Prob. 17) which, having been ſo often exemplified, in the preceding Problems, it would be quite ſuperfluous to repeat.

Of all the various Methods for finding the Repreſentation of a Circle, in Perſpective, there is none preferable to this; which I ſhall exemplify more at large in the 8th Section. For, if any regular Poligon whatever, be drawn in Perſpective, a Circle may be deſcribed through all the Angles, or touching every Side. Six Points are not ſufficient for deſcribing the Curve with accuracy; beſides, an Octagon is readier to deſcribe than a Hexagon, ſeeing that, the Vaniſhing Points, required, are only the Center of the Vaniſhing Line and its Diſtance. The Octagon being inſcribed in a Square, renders it, of all others, the moſt expeditious for finding the Repreſentation of a Circle.

PROBLEM XXVI.

The Interſection of a vertical Plane being given, and its Inclination to the Picture, to find the Repreſentation of any Figure in the Plane.

Fig. 66.

Let AB be the given Interſection, and C the Center of the Picture.

Through C, draw SD perpendicular to the Interſection, and CE perpendicular to SD, i. e. parallel to AB, and equal to the Diſtance of the Picture.

Make CES equal to the Complement of the Angle of Inclination, cuting CS in S; and, through S, draw MN parallel to AB; MN is the Vaniſhing Line, S is its Center, and ES is its Diſtance. (See Prob. 3rd.)

If it be required to find the Repreſentation of a Square, whoſe Side is parallel to the Picture, and equal to AB; at the diſtance A*b* from the Interſection.

Draw AS and BS; make S*E*' & S*E*' equal SE, and draw *b E*', cuting AS in F.

Draw F*E*', cuting BS in H; and draw FG and HI, parallel to AB.

Then, FGHI repreſents a Square, in that Plane, at the diſtance A*b* from the Interſection of the Plane of the Original.

If another Square be required, draw another Diagonal HE' or IE^2.

Or, if a greater length than a Square be required, make SO to S*E* (equal SE) as one Side of the Rectangle, required, is to the other, and draw HO, cutting AS in K, and draw KL. For *E*O, being drawn, is parallel to the Original of HK.

See this illuſtrated by moveable Planes. (Fig. 37, No. 1.)

Turn up the Plane AFDC, (on the other Side of the Picture) at right angles with the Ground Plane, RS is its interſection with the Picture, and AB, with the Ground Plane.

AB and FD, being perpendicular to the Picture, vaniſh in its Center, C; and, if the Vertical Plane, V, be turned up, perpendicular to the Picture, *E*Q will be parallel to the Diagonals, AE and BD, and produce their Vaniſhing Point; and, Q, on the other Side of the Center, C (in the Vertical Line) equally diſtant from C, is the Vaniſhing Point of the other Diagonals, FB and EC.

The reſt is obvious; if the Rectangles AFEB and BEDC were Squares, their Diagonals would vaniſh in E; CE being equal to the Diſtance of the Picture.

Second. Let ab be the given Side of a regular Heptagon (in the Plane AIHB) parallel to the Horizon; its Vaniſhing Point is S; it is required to deſcribe the Heptagon. S*E* is perpendicular to the Vaniſhing Line MN.

Having made S*E* equal SE, make the Angles S*E*M, S*E*N each equal to the external Angle of a Heptagon, and biſect them, by the line *E*O and *E*P; alſo, make the Angles M*E*Q, N*E*R, equal M*E*O, &c. i. e. having deſcribed a ſemi-circle, on *E*, divide the Semi-circumference into ſeven equal parts, and make ab, bc, and cd, each equal one ſeventh part.

*E*Q and *E*R, being produced to the Vaniſhing Line, MN, will generate two Vaniſhing Points, equally diſtant from S; which, with M, O, S, P, and N are the Vaniſhing Points of the Sides and Diagonals; none of them being, in this Caſe, parallel to the Picture.

Draw aM; and Nc, through b, indefinite; draw Pc, through a, cuting Nc in c, and draw cS, cuting aM at d. bc and ad are Sides of the Heptagon, whoſe Vaniſhing Points are M and N.

Draw cQ, and Rd indefinite (by Prob. 13.) and, through a, draw Ne, cuting cQ at e; and draw eS, cuting the oppoſite Side, at f; and, laſtly, draw eO and Pg, through f, cuting eO at g, which compleats the Figure afecb, the repreſentation of a regular Heptagon, whoſe Sides are all inclined to the Picture.

PROBLEM XXVII.

To find the Repreſentation of any irregular Figure (entirely on the Principles of Brook Taylor) from the known dimenſions of the Figure, and the Angles which every Side makes with the adjoining Side or Diagonal; the Place, Poſition, Situation, and Diſtance of the Figure, in reſpect of the Picture and of the Station Line, being given; in a Plane which is inclined to the Picture, in a given Angle, the Center and Diſtance of the Picture being given, and the Interſection of the Plane of the Figure.

Note. The Original Figure (A*BDFG*) being geometrically drawn, in the Original Plane, is of no other uſe (in the following Operation) than to determine its Situation and Poſition; otherwiſe it is not neceſſary. The Place of the neareſt Angle (A) and the Inclination of a contiguous Side (A*B*, or A*G*) in reſpect of the Interſection, being all that is wanted; the reſt are determinable, in reſpect of themſelves; as it will be exemplified. Fig. 67.

C is the Center of the Picture, *HI* is the Interſection of the Plane of the Original Figure, and X is the Angle of its Inclination to the Picture.

Pl. XIV. Fig. 67. A is the place of the neareſt Angle of the Figure, and JA*B* is the Angle of the inclination of the Side A*B*, to the Interſection; AB is the Diſtance of the Angle A from the Interſection, and A e is its Diſtance from the Station Line.

Through C, draw EC perpendicular to the Interſection, which is the Vertical Line of the Original Plane of the Figure. (Def. D.)

Draw CE' perpendicular to EC, and equal to the Diſtance of the Picture.

Make the Angle CE'*C* equal to Z (the Complement of X) cuting EC in *C*; and, through *C*, draw DG, the Vaniſhing Line of the Plane of the Figure; *C* is its Center, and *C*E' its Diſtance. (Def. 19 and 20.)

*C*E, on the Vertical Line, being made equal to *C*E'; through E, draw ER parallel to the Vaniſhing Line; the Parallel of the Eye of that Plane.

Draw ED; making the Angle RED, equal JA*B* (i. e. to *BKH*, the Inclination of the Side A*B* to the Interſection) cuting the Vaniſhing Line in D; the Vaniſhing Point of A*B* †, and make the Angle DEG equal *BAG*, cuting the Vaniſhing Line in G, the Vaniſhing Point of A*G* ‡.

† Prob. 2.
‡ Prob. 4.

Alſo, make the Angle DEH, equal to A*BH*, the Supplement of A*BD*; and H is the Vaniſhing Point of *BD*; by the ſame.

Make the Angle GEI equal A*GI*, the Supplement of A*GF*; cuting the Vaniſhing Line in I, the Vaniſhing Point of *FG*.

Laſtly, make the Angle IEF equal to *DFG*; EF is the Radial of *FD*; which, if produced, would cut the Vaniſhing Line, in the Vaniſhing Point of *FD*.

Thus, are the Vaniſhing Points D, G, H, and I, produced, by the known Inclination of one Side to the Interſection, and the Angles which each Side makes with the adjoining Side, (by Prob. 4.)

The place of the Angle A being given in the Geometrical Plane (which is inclined to the Picture, in the Angle E'*G*C; and to the Horizon, in the Angle *C*E'C) find its repreſentation on the Picture (Prob. 14.)

Draw AB, perpendicular to *HI*, and draw B*C*. Make B*L* equal to BA, and *CE* equal to *C*E', and draw *LE*, cuting B*C* in a, the repreſentation of the original Point A.

Or, draw B*A* parallel to *C*E', and make B*A* equal BA; produce AB, and draw *AS* parallel to *HI*, or perpendicular to *BS*; *S* is the Seat of the point A, on the Picture; and *SA* its diſtance from its Seat.

§ Prob. 6. Draw SC, and *A*E' cuting it in a, the Repreſentation of A§, as before.

Draw aD and aG the indefinite Repreſentations of A*B* and A*G*, from the Point a; which, if produced, would paſs through their Interſecting Points, *K* & *L*.

Make D*E*' equal DE; and, through a, draw *E'a* to the Interſection, cuting it in *a*.

Make *a b* equal A*B*, and draw *bE*', cuting aD in b; ab repreſents a Line equal to A*B* (Prob. 7, or 17.) Draw bH.

Produce DE, and make EK and EL reſpectively equal to A*B* and *BD*; or in the ratio of A*B* to *BD*; and draw EM parallel to KL, cuting the Vaniſhing Line, in M, the Vaniſhing Point of the Diagonal A*D*.

Draw a M. cuting bH in d. bd repreſents a Line in the ratio to that which ab repreſents, as EK to EL, i. e. as A*B* to *BD*. (Prob. 10*.)

For, EK is parallel to A*B*, and EL to *BD*; wherefore KL, conſequently EM, is parallel to the Diagonal A*D*; therefore, M is its vaniſhing Point ‖; and abd repreſents the Triangle A*BD*. (Pr. 18.)

‖ Def. 22.

Now, F, the Vaniſhing Point of *FD* (being much inclined) is not in the Picture.

Draw d*f* at pleaſure, cuting the Vaniſhing Line, and EF, in *e* and *f*; and draw *g i* parallel to d*f* (at any diſtance, at diſcretion) and make *h i* to *g h*, as *e* d is to *ef*.

* From the Poſition of A*B* and *BD*, the meaſure of A*B*, or its Ratio to *BD*, is taken beyond the Eye, in the Radial of A*B*, produced; as in the third Caſe, of that Problem.

e. g. Draw the Diagonal *fh*; and draw d*k* parallel to the Vaniſhing Line, cuting *fh*, produced, in *k*; draw *ki*, parallel to FE, cuting *gi* in *i*; *hi* is a fourth Proportional to *fe*, *e*d, and *gh*; i. e. *hi*: *gh*:: d*e*: *ef*. Draw d*i*; which will tend to the Vaniſhing Point F; (by Prob. 13, No. 2.)

Produce GE, and make EN equal *AG*; and, on EI, take EO equal *GF* (or, make EN and EO in the ratio of *AG* to *GF*) and draw EP, parallel to NO, cuting the Vaniſhing Line in P; the Vaniſhing Point of the Diagonal *AF*.

Draw aP, cuting d*i* in f; and, laſtly, draw If, and produce it till it cuts aG, in g, which compleats the Figure.

N. B. If the Sides of the Original Figure be produced to their interſecting Points, *H*, *I*, *K*, and *L*, the truth of the proceſs may be perceived, by its affinity and agreement with the former methods, *viz.* when the Original Figure is drawn in the Geometrical Plane, its Vaniſhing Line, Center, and Diſtance being given; which, in this Problem, are given only by Poſition and Diſtance; the Original Figure being uſeleſs in the Operation, otherwiſe than to ſhew the meaning of every ſtep; and that, if the Original Figure be ſuppoſed in its true Place, and Poſition, each Line, from the Eye, producing a Vaniſhing Point, would be parallel to the correſponding Side, or Diagonal of the Figure.

In this Problem, I have ſumed up moſt of the Rules given in the two laſt Sections; which are called elementary, becauſe, by them, the whole of practical Perſpective is performed.

It will, I know, appear, to young practitioners in Perſpective, ſomewhat difficult; yet, if they do but ſee the Principles on which it is performed, and will be at the trouble to go through the Proceſs a ſecond time, and compare each Operation with the Problem it refers to, that apparent difficulty will vaniſh; and they will find it of great advantage to them, in applying all the elementary Problems to real uſe, in delineating Plane Figures.

This Proceſs, difficult and operoſe as it may appear, at firſt, is infinitely preferable to any other Method of performing it; nay, it is ſcarce poſſible to perform it at all, by any Rules given by the old writers on Perſpective, or indeed by any; and, conſidering that I have given the whole procedure, from the begining to the end, it is not a long one. I have, not only, referred to the foregoing Problems for each particular ſtep in the Operation, and (as is cuſtomary) left the Reader to apply it; but, I have gone on with him, ſocially, hand in hand; and, like a truſty Guide, pointed out every ſtep he ſhould take.

I have, alſo, drawn every Line in the whole Operation, which remains in the Plate; yet there is not the leaſt confuſion, the uſe of each being obvious, on inſpection; but, being taken regularly, in the courſe of the Work, every apparent intricacy is unravelled.

At the ſame time, let it be obſerved, that there is no need for half the Lines to remain at once; e. g. after having found the Vaniſhing Line, DG, its Center and Diſtance, and the Vaniſhing Points of all the Sides of a Figure, all the operative Lines may be rubed out before we begin to draw the Figure; alſo, finding the Seat of A, is not abſolutely neceſſary, but is only another Method to find the repreſentation of the Angle A; but they muſt all remain in the Diagram; which circumſtance frightens many, with apparent intricacy; which, in reality, is but apparent.

In order to compare the difference, I will ſhew, how it may be performed otherwiſe. But, as a Figure of ſo many Sides would be ſomewhat confuſed, I have made choice of a Triangle, in which, the Proceſs is more diſtinct, and will be better underſtood.

Let ABD be the given Figure, in the Geometrical Plane; of which, GI is the interſection, and JI*M* is its inclination to the Horizon; let NI be conſidered as a vertical Section of the Picture; NIM is the angle of Inclination of the Plane of the Figure with the Picture. Fig. 68.

Produce NI; draw AK, BL, and DM, parallel to GI, cuting IM in K, L, and M.

Make I*K*, I*L*, and I*M* reſpectively, equal to IK, IL, and IM; and draw *MN*, *LO*, and *KP* perpendicular to the Picture; NI being perpendicular to JI.

Draw AF, BG, and DH, perpendicular to GI, and produce them, till they cut *KP*, *LO*, and *MN*, produced, in *A*, *B*, and *D*, the Seats, on the Picture, of the Angles of the Triangle. *ABD* is its orthographic Projection.

Having thus found the Seats of the three Points A, B, and D, the Repreſentation of each may be found by Prob. 6. as a, the Repreſentation of A, in the preceding Figure; by the ſecond method.

C being

Pl. XIV. Fig. 68.

C being the Center of the Picture, draw C E at pleaſure, and equal to the Diſtance of the Picture; and, having drawn *A* C, *B* C, and *D* C, draw *A* k, *B* l, (or *B l*) and *D* m (or *D m*) parallel to C E (or C *E*) and equal to the Diſtance of each Point from its Seat, reſpectively; *viz.* *A* k equal P *K*, *B* l (or *B l*) equal O *L*, and *D* m (or *D m*) equal N *M*.

Draw k E, cuting *A* C in a; l E (or *l E*) cuting *B* C in b, and m E (or *m E*) cuting *D* C in d.

The three Points a, b, and d, are the repreſentations of the Angles A, B, and D; which, being joined by Right Lines, compleat the Figure.

In this Proceſs, it is obvious, there is neither Vaniſhing Line nor Vaniſhing Point required, the whole being performed by the 6th Problem. If the Interſection, GI, be parallel to the Horizon, E C E may be conſidered as the Horizontal Line. But, if the Interſection of the Plane of the Figure, be given, it is immaterial what Poſition it has to the Horizon; the Center of the Picture and Diſtance, the Place and Poſition of the Object, and the Inclination of the Plane it is in, being alſo given.

Although this method is perfectly conſonant to the Principles of Brook Taylor, yet it is by no means ſo maſterly or correct, that is, it is more liable to error, than the other, by means of Vaniſhing Points; which, in complex Objects, having various Lines in various Planes, is preferable to all other. In ſhort, it is truly Perſpective, which cannot be ſaid of the old Authors; having few or no elementary Principles, to ſupport or demonſtrate the Rules they preſcribed, the Operation is, merely, a mechanical Proceſs.

As the old Authors had no Idea of any other Vaniſhing Line than the Horizontal, nor of placing the Eye any where but in the Point of Diſtance, therein; their method of finding the repreſentation of each Angle of the Figure, in a Plane which is inclined to the Horizon, is by means of their Seats on the Ground Plane, and their heights above it; as J of *M*, &c.

As the Proceſs would, in this Place, be confuſed, by mixing it with the other, I ſhall defer it to a more proper Time and Place, in an Appendix to this Work; in which the difference will be clearly and fairly ſtated, and exemplified.

PROBLEM XXVIII.

To find the Repreſentation of any Line, or Plane Figure, by means of the Directing Plane. (See the 14th Theorem.)

Fig. 69.

Let A B be a Line perpendicular to the Picture, or to the Interſection, I K, of the Plane it is in. Let E be the Eye, in its true Place, as in the foregoing Problems.

At the Diſtance E P, equal to the Prime Director *, draw O N, the Directing Line, parallel to the Interſection, I K †.

† Theo. 2.

The Space between O N and I K repreſents all that Space which lies between the Directing Line and the Interſection of the Original Plane with the Picture.

In which Conſtruction, the whole of that Space, together with the Directing Plane, NEO, is ſuppoſed to be turned up, on the Interſection I K, till it falls into the Picture; the Original Figures being turned along with it, into the ſame Poſition as uſual.

Produce B A, cuting the Interſection in I, its Interſecting Point; and the Directing Line in D, its Directing Point. Draw E D, which is the Director of A B‡; to which, draw I C, parallel.

‡ Def. 12.

For, C being the Center of the Picture, and I the Interſecting Point of a Line perpendicular to the Picture, I C is its indefinite Repreſentation; and E C, the Direct Radial, is parallel to A B, and equal to I D, a part of AB, produced; conſequently, I C is parallel to E D. (15. 1. El.)

As E C (Fig. 37) is equal to *B* R, a part of AC, intercepted between the Interſecting and Directing Lines; alſo, E O is equal to G P, and E L to H *S*; and being parallel, reſpectively, the Lines R C and E *B*, E G and O P, &c. are alſo equal and parallel.

* In Theory, the Prime Director is a Right Line joining the Eye, or drawn through the Eye, and the Station Point of the Original Plane. In common Caſes, E P is equal to the height of the Eye above the Ground Plane, always equal to the diſtance between the Interſection and Vaniſhing Line.

If the Sides of the Triangle, FGH, be produced, till they cut the Directing Line, in the Points M*, N, and O, their Directing Points, EM, EN, and EO, are their Directors; to which, their ſeveral Repreſentations are parallel; J, K, and L, are the interſecting Points of thoſe Lines.

Draw Lh parallel to EO, Kh parallel to EN, and Jf parallel to EM; which, by their Interſections, produce the Figure fgh, the Repreſentation of FGH.

Through C, parallel to IK, draw RS, the Vaniſhing Line of the Plane of the Triangle; and, having produced the Indefinite Repreſentations to their Vaniſhing Points, R, S, Q, the Lines ER, ES, and EQ, are the Radials of the Sides of the Triangle, and conſequently parallel to them, reſpectively.

The agreement between theſe two Methods of producing the Repreſentation, fgh, is ſo very obvious, as not to need any further explanation.

See this further illuſtrated, in Fig. 37. Turn up the Picture and Directing Plane.

The Sides of the Triangle ZY and ZX, on the other Side of the Picture, being produced, cut the Interſection (AB) in P and S; and, the Directing Line (HG) in G and H; EG and EH, in the Directing Plane, are reſpectively parallel to the Repreſentations of thoſe Lines, PO and SL, the Directing Plane being parallel to the Picture. For, they are the Interſections of Radial Planes with the Picture and Directing Plane; which are parallel, by Def. 4.

The Planes W and X being turned over, till *E* coincides with E; the Section with the Picture, RC, is the indefinite Repreſentation of AC (Theo. 8 and 12) and E*B*, its Section with the Directing Plane, is the Director of the Line AC (Def. 12) which is parallel to RC (by Theo. 14) being the ſections of parallel Planes, by another Plane.

SECTION VI.

Of the PRACTICE of PERSPECTIVE, applied to PLANE SOLIDS.

THE laſt Section comprehends the whole Art of practical Perſpective, in reſpect of Plane Figures; illuſtrated with variety of Caſes and Examples; in which, I have ſtudied more to render it eaſy and familiar, than to embelliſh the Subject with what is no way uſeful. Every Figure, which I have given, are ſuch as are common, and applicable to Buildings and other Objects; the Pentagon and Heptagon are not ſo uſeful in themſelves, as for the ſake of managing inclined Lines, in all Poſitions; ſo that the Leſſons deduced from them are excellent. By varying their Poſitions and Situations, and diverſifying the Caſes as much as is conſiſtent with utility, I am perſuaded that no Perſon, who has a tolerable Capacity, will be at any loſs to know how to deſcribe any Plane Figure, and in any Poſition whatever.

Although ſome Perſons, of keener Talents, may imagine, that the preceding Section contained Inſtructions ſufficient for every occaſion (it certainly contains the Elements of the whole) yet, I preſume, if no more had been given, it would be found of but little uſe to many; when, by applying thoſe valuable Leſſons to familiar Subjects, what, before, might appear ſomewhat myſterious will, hereafter, be applied with eaſe, as occaſion may require.

* Suppoſe FG produced till it cuts ON, produced, and EM a Line tending to that Point; then EM is the Director of FG.

Plate XV. It may with equal propriety be alledged, that there is no need for ſhewing how plane Solids are formed; ſeeing they are only compoſed of plane Figures, in various Poſitions, all which have, already, been copiouſly treated of. Yet, a few Leſſons, and familiar Examples, will not be found ſuperfluous or unneceſſary. I ſhall not dwell on any thing not really uſeful, but haſten to apply the whole to proper Objects; for which, all the foregoing is but preparatory.

In this Section, I ſhall always conſider the Object as ſituated on a horizontal Plane, and the Picture as vertical; moſt of the foregoing Problems, being general, are applicable in all Poſitions whatever.

In the following part of this Work, I would adviſe the young Student, as he proceeds, to draw every Object, or a ſimilar one, at his own diſcretion. Or, if that be not neceſſary, in ſome Caſes, let him carefully notice what things are given, in the Propoſition, and what is required to be done; and then proceed, ſtep by ſtep; not ſuppoſing any thing as done, but what he has paſſed over; by which means, he will ſee, clearly, how every Line is produced, by which the Objects are formed; but to do it, himſelf, and varying the Poſition, a little, is better.

This Precaution would have been neceſſary in the third or fourth Section, but more particularly ſo here.

PROBLEM XXIX.

How to deſcribe a Cube, perſpectively, or any other right angled Parallelopiped*, having one Side, given, parallel to the Picture; at any Diſtance beyond the Picture, its Situation and Dimenſions being known.

No. 70. ECE is the Horizontal Line, and PM the Interſection or Ground Line.

C being the Center of the Picture, make CE, on either Side, or on both Sides, equal to its Diſtance; and let this be underſtood in future Examples, that CE (E being the Place of the Eye) either on the Horizontal or Vertical Line (the Picture being vertical) always denotes the Diſtance of the Picture; ſometimes C*E*.

Firſt; let the Picture be ſuppoſed applied cloſe to a Face of the Cube; in which Caſe, being parallel to the Picture, it coincides with the Picture.

This is generally the Caſe, when there is but one Object; as it anſwers no purpoſe to ſuppoſe it at any Diſtance, from the Picture, as in Figures 59, 60, &c. or, when there are various Objects to be delineated, it is, moſt commonly, ſuppoſed to be applied cloſe to the neareſt, as in this Example (ſee Fig. 38.) ſo that, the Diſtance of the Picture is the Diſtance of a Plane direct before the Eye, and touching the Object, or the neareſt Object, if there are more than one.

Let AB, in the Ground Line, be the meaſure of a Cube, to be repreſented; having one Face in the Picture.

On AB, deſcribe a Square, ABDF, which, being in the Picture, has the full dimenſions of the Side given; wherefore, ABDF is the repreſentation of a Face of the Cube required. (Cor. 5. Theo. 9.)

Draw DC and FC; and DE or FE, diagonal-ways, cuting FC in G, or DC in H; draw GH, parallel to FD, which compleats the Figure AGHB, the repreſentation of a Cube, in that Poſition, and Situation; having but two Faces ſeen.

Note. Each Side (AF, ED, and DB) of the Square AFDB, repreſenting the front Face of the Cube, is, in this Caſe, an Interſection of the Picture, as well as AB; *viz.* of the adjoining Face.

* See the Difinition of a Cube, &c. in the General Introduction, Section 1, Page 41.

If

If another, of equal dimenſions, be required, ſituated on one ſide of the Station Line, at the Diſtance LJ (as *ABEH*) and diſtant from the Picture L*a*, equal L*A*.

Make LM equal AB; and, on LM, deſcribe the Square LMNO, as before.

Draw OC, NC, &c. and draw *a*E, cuting LC in a, and MC in e.

Draw a b parallel to L M; and ad, b c, to MN; alſo, draw c d parallel to a b.

i. e. having drawn a b parallel to LM, deſcribe a Square on a b; and, at e, deſcribe the Square e f g h, parallel to a b c d, which compleats the Figure.

Or, having drawn LC, MC, and *a*E, compleat the Plan a f e b. (Prob. 19.)

Deſcribe a Square on a b, and draw cC and d C; draw f g parallel to a d, and g h parallel to c d, which compleats the Figure.

DEM. abcd being parallel to the Picture (by Hypotheſis) is ſimilar to the Original, LMNO†; and its Sides have that proportion to the Originals, as the Diſtance of the Picture (CE) to the Diſtance of the Plane abcd; viz. as EC+L*A*; i. e. a b (or a d) : LM : : EC : EC+LA. (Theo. 9.)

† Cor. 5. Theo. 9.

But, LMNO is a Square; therefore, abcd is a Square.

And, ſince it is parallel to the Picture, the Originals of the Sides a f, d g, &c. are perpendicular to the Picture; conſequently they vaniſh in its Center, C‡.

‡ Cor. to Theo. 4.

Alſo, afgd repreſents a Square in a vertical Plane, perpendicular to the Picture; for LO is the Interſection of that Plane §, and *E*C*E* its Vaniſhing Line ‖; wherefore, C*E*, being made equal to CE, *E* is the Vaniſhing Point of Lines (in that Plane) inclined to the Interſection in half a Right Angle; conſequently, if one Side (a d) be parallel to the Interſection (LO) and OP being made equal L*a*, P*E* paſſes through a Diagonal of that Side †, therefore, afgd repreſents a Square, ; and O*b* being equal L*a*, *b*E is a Diagonal of the Top, d g g h, which alſo repreſents a Square, parallel to afeb, and ON is the Interſection of that Plane, with the Picture.

‡ Theo. 10. ‖ Th. 2 & 10. † See N.B. Prob. 20.

Therefore, agc repreſents a Cube, having three Faces ſeen, each of which, is the repreſentation of a Square. Q. E. D. for the oppoſite Faces are alſo Squares.

PQ is the Interſection of a Diagonal Plane cdfe, whoſe Vaniſhing Line would paſs through *E*.

The Cube AGHB being ſituated on the Station Line (which, on the Picture, coincides with the Vertical Line) has but two Faces ſeen; the Front AFDB, and Top, FGHD, the Eye being above it.

The other Faces, AFGK and BDHI, ſeeing they apparently incline towards the Vertical Line, are conſequently loſt to ſight in this Poſition; as is obvious, ſuppoſing the Solid tranſparent.

EXAMPLE I.

To draw the Repreſentation of a high Wall, the End being parallel to the Picture, and at ſome Diſtance from it.

This Object, being a right angled Parallelopiped, is delineated as the Cube. Fig. 71.

Let PJ be equal to the Diſtance of the Wall from the Station Line, on the Left.

Make P Q equal to its thickneſs, and draw P C and Q C; make P*d* equal to the diſtance of the end of the Wall, from the Picture; draw *d*E cuting P C in R, and draw R S parallel to P Q.

Now, ſince P is the interſecting Point of R F, the common Interſection of the Plane of the Wall (which is ſuppoſed vertical) with the Ground; P T, perpendicular to the Ground Line, or parallel to the Vertical, is the Interſection of that Plane†.

† Cor. 1. Theo. 2.

And, ſince the Wall is perpendicular to the Horizon, conſequently, its Angles or Corners, R U and S V, are parallel to P T or Q X the Interſections of the Planes thoſe Lines are in (for they are parallel to the Picture, being vertical) therefore, draw R U and S V parallel to P T, i. e. to the Vertical Line.

Make P T equal to the known height, and draw T C, cuting R U in U, and draw V U parallel to R S; then R U V S, repreſents the End of the Wall, which is parallel to the Picture (on the Ground Line).

Laſtly. Make P M (on the Ground Line) equal to the Diſtance of the farther end of the Wall from the Picture, or *d* M equal to its known length.

Draw

Plate XV. Fig. 70. Draw E M, cuting P C, the indefinite Repreſentation of its common Section with the Ground Plane, in F; and draw F Y, parallel to R U, which compleats the Wall.

Or, if C *F* (on the Vertical Line) be made, to C E or *E*, as the height of the Wall to its Length, *F* will be the Vaniſhing Point of the Diagonal, R Y, which cuts T C, in the Point Y, for its length; draw F Y parallel to R U.

Let it be obſerved, here, that if there be not room enough, on the Interſection, to ſet off the whole length of the Wall, equal *d* M, take half the Diſtance, P J (equal P f, in the Plan below) make C e equal half C E (the Diſtance of the Picture) and draw J e, projecting the ſame Point F.

If the half meaſure be too much, take a third, or fourth, or any equal portion, of P M (equal P m) the ſame part being taken of C E will anſwer the ſame purpoſe (ſee Prob. 17.)

EXAMPLE II.

How to repreſent ſeveral Parallelopipeds, as Blocks of Stone, &c. for the Baſement of a Building, ranged in a Right Line, perpendicular to the Picture.

Fig. 71. W, X, Y, and Z, are the Plans, or Seats of the Blocks on the Ground Plane.

Let A B D, the Front of the firſt, be cloſe to the Picture, i. e. in the ſame Plane; conſequently, in its geometrical Proportion, of height and width.

Draw A C, B C, and D C. Make A *a*, on the Interſection, equal to the length of the firſt, *a b* equal to the ſpace between them, and *b c* equal to the ſecond.

Draw *a* E, *b* E, and *c* E, cuting A C in F, G, and H; draw F I, G K, and H L perpendicular, cuting B C; and l M, K N, and L O, parallel to B D, cuting D C, in M, N, and O, and compleat them, as in the Figure; having regard to the firſt hiding part of the next, and that of the third, &c.

Now, in order to continue them further, to any length, this Expedient may be uſed, when the meaſures exceed the limits of the Picture, and, conſequently, cannot be applied to the Interſection, or Ground Line.

Draw H g, from the fartheſt corner of the ſecond Parallelopiped parallel to the Interſection, and draw *a* C and *b* C, cuting it in e and g.

If it be required to continue them of the ſame dimenſions, and ſpace, make H f equal e g; or, A *d* being made equal to *a b*, draw *d* C, cuting g H, at f, and draw f E and g E, cuting A C in a and b. Compleat the Parallelopiped a b c d as in the Figure.

After the ſame manner they may be repeated as often as you pleaſe by drawing b *g* parallel to g H, cuting *d* C and *b* C in *f* and *g*, and then proceed as before.

This Expedient is, in many caſes, better than applying the half-meaſures, or other portion, on the Ground Line; as they may be continued infinitely, within the compaſs of A *b* (one Block and Space) by drawing G h (inſtead of g H) which, being cut by *a* C, at i, needs not to be tranſpoſed, as e to f.

It muſt be obvious, that G h (or g H) is cut in the ſame proportion to the ſecond Block (at the hither end, or the farther) as A *b* is to the firſt; and conſequently, i E, h E, &c. will cut A C in the ſame Points, as if the full meaſures were applied on the Ground Line; and ſo of *g* b, to the next.

EXAMPLE III.

To draw ſeveral long Parallelopipeds, parallel amongſt themſelves, and perpendicular to the Picture; repreſenting large Joiſts, ſupporting a rough Floor (over Head).

Let W, X, Y, and Z be the ends of the Joiſts, of equal dimenſions, and equally ſpaced; and, ſince they are ſuppoſed to be parallel to the Picture, therefore, they have their geometrical proportion on the Picture. (Cor. 5, Th. 9.) Fig. 72.

Let them be drawn accordingly, by a Scale of equal Parts.

Then, becauſe the Joiſts are perpendicular to the Picture, in reſpect of their length; (i. e. they are horizontal, and at right angles with the Picture) draw a C and b C, &c. from every Angle, neceſſary, as in the Figure; and determine their length, by Prob. 15, i. e. make A B, or *A B*, equal to the required length, and draw B E, cuting A C in D; or *A* C in *D*.

Through D, draw F G parallel to the Vaniſhing Line (C E) which determines the under Sides, F e, &c. and the perpendicular Lines, d e, &c. compleat them.

The diviſions of the Boards are determined by Prob. 8th; i. e. take any meaſure, *A a*, and repeat it, on *A B*, as often as there is occaſion, if the Boards are ſuppoſed equal; otherwiſe *A a*, *a b*, &c. muſt be made in the ſame ratio as the Boards, to each other; and at the laſt diviſion, *e*, draw *e D*, cuting the Vaniſhing Line in H; and draw *a* H, *b* H, &c. cuting *A D* in 1, 2, 3, &c. through which, draw Right Lines parallel to *A B*, i. e. to the Vaniſhing Line.

Or, divide *A B* into the ſame number of Parts (as *A* 1, 12) and draw Lines to E, which will produce the ſame, as is obvious; ſeeing that it does not depend on the dimenſions of the Parts, but on their number and ratio to each other. (See Prob. 8.)

Note. *A* 1, 12, on *A B*, being each equal to a fifth Part of *A B*, is the geometrical width of the Boards, in proportion to the ends of the Joiſts, and to *A c*, the ſpace between them.

PROBLEM XXX.

To draw the Repreſentation of a Cube, or other right angled Parallelopiped, any how inclined to the Picture.

Firſt. Let the Square A B C D be the Plan of a Cube, in the Geometrical Plane, equally inclined to the Picture, and to the Interſection, at the Angle A. Fig. 73.

Lie down the Diſtance of the Picture, C E', from the Center, as uſual; E and E are the Vaniſhing Points of all the Lines of its Sides. (Prob. 20.)

Draw A E, both ways, A being the interſecting Point of two Sides, A B and A D.

Find the perſpective Plan *A B C D* (by Prob. 10) or of the two Sides, A B, A D, only.

Draw A F perpendicular to the Ground Line, and equal to A B, a Side of the Cube, and draw F E, both ways.

Draw *B* G and *D* H, parallel to A F; and draw G E and H E diagonal-ways, cuting each other, at I, which compleats the Cube, A G H.

Otherwiſe, without drawing the perſpective Plan, by means of the Vaniſhing Lines of its Faces.

Draw A F perpendicular to the Ground Line. Then, A F is the Interſection of two Faces of the Cube. Make A F equal to a Side, A B.

Draw A E and F E, both ways; and through E, on either ſide, draw the Vaniſhing Line of the contiguous Face of the Cube. (Prob. 3.) E *E*, or E F.

Make E *E* or E *F*, on either Side, equal E E', and draw A *E* or A *F*, cuting the indefinite Repreſentation F E, in G or H, and draw *B* G, or *D* H, perpendicular.

Draw *B D* parallel to the Interſection, and *D* H parallel to A F; and laſtly, draw G E and H E, interſecting at I, as before.

Plate XV. Fig. 73. *A B G H D* is the Repreſentation of a Cube, in the poſition required; ſituated on a horizontal Plane, and its vertical Faces equally inclined to the Picture.

DEM. For, ABCD the given Plan is a Square, wherefore *A B C D* repreſents a Square (Prob. 20) and FGIH alſo repreſents a Square, in a Plane parallel to *A B C D*. (Th. 3. and Cor. 1.)

And, they are between the ſame Parallels, AF, *B G*, &c. therefore equal - - - (18. 1. El.)

But, AF is the Interſection of the vertical Planes, *A B* GF and *A D* HF; and *E E* is the Vaniſhing Line of *A B* GF (Theo. 2 and 10) E is its Center (Def. 19) and E E' its Diſtance (Cor. 2. 7.)

Wherefore, *E* is the Vaniſhing Point of Lines inclined to the Interſection of that Plane, in half a Right Angle or 45 Degrees; conſequently, it is the Vaniſhing Point of the Diagonal of a Square, in that Plane, whoſe Sides AF and *B* G are parallel to the Picture (Prob. 19;) and, *A B* and FG, perpendicular to them, vaniſh in E, the Center of the Vaniſhing Line, E *E*. - - - (Cor. to Th. 11.)

Therefore, AFG *B* repreſents a Square, at right Angles with *A B C D* and AFH *D*, which (after the ſame manner) may alſo be proved to be the repreſentation of a Square; and their oppoſites, *D* HI *C* and *C* I G *B*, being between the ſame parallel Planes, are alſo repreſentations of Squares; conſequently, *A B* GH *D* repreſents a Cube. (See Defin. Page 41.) Q. E. D.

a c f g repreſents another Cube of the ſame dimenſions, and in the ſame poſition to the Picture; whoſe ſituation in reſpect of the former, is as the Plan, W, to BD; which, on account of the ſhort Diſtance (CE) has a diſtorted appearance, being remote from the Station Line, though wholly within the Radius of the Diſtance.

By means of the Interſection (*A F*) of the Face a d f g, or *a f* of the Face a b c d (*A F*, or *a f*, being made equal to the Side of the Cube) the proportion of the parallel Lines (a d and f g, or b c) in thoſe Planes are determined. For, *A F*, a d, and f g; alſo *a f*, a d, and b c are in the ſame Plane, and are all parallel to the Picture; conſequently, they are parallel between themſelves, and to their Originals†.

† Theo. 9. Cor. 1.

EXAMPLE IV.

How to draw Parallelopipeds ranged in a Right Line, and inclined to the Picture obliquely.

Fig. 74. Let X and Z be the Plans of right angled Parallelopipeds; their inclination to the Picture is the Angle A I *A*, which they make with the Ground Line, *C K*.

C being the Center of the Picture, and C*E* its Diſtance, find the Vaniſhing Points, G and H, of the Sides and Ends of the Parallelograms, X and Z, (Prob. 2, and 4.) and lie down their Diſtances on the Horizontal Line (Pr. 12.) at E^1 and E^2.

Tranſfer all the meaſures, I A, I B, &c. of the Objects, and the ſpaces between them, to the Interſection, at *A*, *B*, and *C*; and, having drawn the indefinite Repreſentation, I G, of the whole length, draw AE^1, BE^1, &c. cuting it in a, b, and c.

Becauſe the Angle I touches the Picture, I F, perpendicular to the Ground Line, is the Interſection of the Plane of the Fronts, and alſo of the End, I K.

Make I D equal to the height of the Objects, and draw D G, D H, and I H.

Make I *K* equal I K, and draw KE^2 cuting I H; I k repreſents I K. (Pr. 17.)

Draw a d, b e, and c f, parallel to I D; and, where they cut D G, in d, e, and f, draw d H, e H, and f H; and from i, where k i cuts D H, draw i G, cuting them in j, h, and g, which compleats the upper faces.

h l, parallel to I D, &c. cuting k G, compleats the End b e h l.

If another Object be required, in the ſame Line, ſeeing that the meaſures on the Ground Line, would exceed the limits of that Picture, take G E^2 half G E^1, and draw E^2 c till it cuts the Interſection, at *a*.

Make *a B* equal to half the ſpace between the Objects, and *B c* equal half the width of the next Object, and draw *B* E^2, and *c* E^2, cuting I G in *b* and *c*.

If it be required higher than the reſt, make I F equal to its known height, and draw F G. Draw perpendiculars from *b* and *c*, cuting F G in *g* and *h*.

Draw

Draw *b*H cuting kG at *d*, and draw *g*H; and *di* parallel to *bg*, which compleats that Object.

Or, if c n be drawn parallel to the Ground Line, and *B*G cuting it in n; divide cn, at m, as *AC* (equal AC) is divided in *B*; and, from m and n, draw to *E*' cuting IG in the ſame Points, *b* and *c*, as before.

By either of theſe Expedients they may be continued to any length.

Note. G, the Vaniſhing Point of the front Line IC, and the parallels to it (being much inclined) may be ſuppoſed to be beyond the limits of the Picture; conſequently, Ic, Df, &c. are tending to a Vaniſhing Point which is not on the Picture, and muſt, therefore, be drawn by means of the Expedients, in the 13th Problem; which are very convenient when but few Lines are wanting.

But, a better Expedient than them all is (when the Point is not very remote) to fix a Lath to the Drawing Board, or ſtraining Frame of the Picture, and continue the Vaniſhing Line as far as is requiſite, on the Lath; fix a Pin in the Vaniſhing Point, and, with a long Ruler, draw the Lines IG, DG, &c. for no other Expedient, whatever, can be ſo true, i. e. it cannot be performed ſo accurately as to have the Vaniſhing Point itſelf. For large Work (as in Scenery) a fine, ſmooth Cord, fixed in the Vaniſhing Point, is a good Expedient.

Priſms of all kinds may be repreſented after the manner of the foregoing, which are right angled Priſms, having found the Plans of their Baſes, on the Ground Plane or other Plane, whether they be triangular, quadrangular, or multangular; of which one or two Examples will be ſufficient.

EXAMPLE V.

Pl. XVI.

To repreſent a hexagonal Priſm, perpendicular to the Ground Plane.

The Situation, Diſtance, and Poſition being determined, or AB being a Side, given or found, in Perſpective, whoſe Radial is EV, find the Vaniſhing Points, I and K, of the other Sides, and compleat the Plan, ABCDEF, of its Baſe (by Pr. 24.) Fig. 75.

Draw Aa, Bb, &c. perpendicular to the Ground Plane, i. e. to its Vaniſhing Line (IK) or Interſection (G*D*.)

Produce any Side, as AB, to its interſecting Point, (G) and draw GH, perpendicular to the Ground Line.

GH is the Interſection of the Plane of the Face Aa b B, or g h of B b c C (Pr. 3) for it is vertical; and G, or g, is the Interſecting Point of one Line in the Plane §. § Th. 2. C. 1.

Make GH equal to the known height of the Priſm, and draw HV † cuting Aa and Bb in a and b; the Original of a b being parallel to the Original of AB. † Prob. 13.

Then, becauſe Aa and Bb are the common ſections of the contiguous Faces with Aa b B, draw aI and bK, cuting the other Perpendiculars Ff and Cc, in f and c; which compleat as many Faces as can be ſeen.

The other Faces, which are all Parallelograms (See Def. Page 41) Fe, Ed, and dC are ſuppoſed to be ſeen through, the Object being imagined tranſparent; which, in ſome Caſes, is a neceſſary Expedient; by which means, the connection of the ſeveral Parts are more accurately determined.

EXAMPLE VI.

To repreſent a pentangular Priſm, laid along on the Ground Plane; the Interſection and the Seat of the Object, on the Ground Plane, being given.

AB is the given Side of the Pentagon, and BD the Seat of one Face of the Priſm. Fig. 76.

C being the Center of the Picture, and CE its Diſtance, find the Vaniſhing Points, I and K, of AB and AD (Prob. 2) by making the Angles JEI and OEK

Pl. XVI. Fig. 76. equal, reſpectively, to B*A*B and *D A*D; or, having determined one, find the other, by Prob. 4. making the Angle IEK equal BAD, a Right Angle.

Make a b to repreſent AB (Prob. 17.) and, on a b, conſtruct a Pentagon, in a vertical Plane, on the ſame Principles as in Prob. 23.

I being the Vaniſhing Point of a b; FG, paſſing through I, is the Vaniſhing Line of the Plane of the end of the Priſm (being perpendicular to the Ground Plane) I is its Center, and E I its Diſtance. (Prob. 3.)

As there is not room, on the Picture, beyond the Vaniſhing Line, F G, to find the Vaniſhing Points of the Sides of the Pentagon, take I*E* on this Side equal to I E, through which draw MN (the parallel of the Eye, of that Plane) parallel to F G, and, on *E* deſcribe a Semicircle; which, divide into five equal parts.

Make *a b*, *b c*, and *a d*, *d e*, each equal to a fifth part, and draw *E b*, *E c*, &c. which, produce to the Vaniſhing Line, producing the Vaniſhing Points, F, G, & H, by which, the Pentagon a b c d e is completed, as in the Figure (by Prob. 23.)

Draw aK indefinite, of the Side AD; and make a f to repreſent a length equal to AD (Prob. 17) alſo, draw e K and d K.

Draw H g, through f, and g F, cuting d K in h, which compleats the Figure.

For, a f g e and e g h d repreſent Parallelograms, which are Faces of the Priſm a f h d b; and a b c d e repreſents an End which is a regular Pentagon; to which the other Faces are perpendicular, I E K being a Right Angle (which IaK repreſents, Prob. 4) equal BA D.

s

The Expedient of turning over the Vaniſhing Plane, F*E* H, to find the Vaniſhing Points of the Side of the Pentagon, is a very uſeful and neceſſary Expedient; becauſe it often happens, that there is not room for turning it over on the other Side of the Vaniſhing Line, if it be remote from the Center. And, notwithſtanding it interferes with the Object in this Diagram, yet being done firſt, and the Vaniſhing Points found (as in the Figure) all the operative Lines (for finding them) are rubed out before we begin to draw the Object; or rather, they are never drawn, for, the Vaniſhing Points are determined (as F.) by applying a ſtreight Ruler to *E* and *b*, &c.

In Fig. 37, the horizontal Vaniſhing Plane, N*IK* L, or the vertical Plane V, being turned over on their Interſections with the Picture (which are the Vaniſhing Lines they produce) it is obvious that the Angles, formed by the Radials, E N, E O, &c. are the ſame on both Sides, and conſequently, they produce the ſame Vaniſhing Points, N, M, &c. whether the Vaniſhing Plane be turned up or down, to the Right or to the Left.

EXAMPLE VII.

To repreſent a quadrangular or hexagonal Pyramid; or any other.

Fig. 77. Let W and Z be the Plans of the Baſes of two Pyramids, whoſe Altitudes are known, the one a ſquare Baſe, the other a Hexagon; two Sides, of which, are parallel to two Sides of the Square.

Find the perſpective Plan ABCD, of the Baſe, W (by Prob. 21.)

If the Original Figure be drawn in the Ground Plane, the ſhorteſt way is by producing its Sides to the Interſection, AK; and having found the Vaniſhing Points, M and N, of the Sides, draw their indefinite Repreſentations (Method 1ſt.)

Draw the Diagonals, AC and BD, interſecting at H, the Center of the Baſe.

Draw HI perpendicular; and where the Diagonal AC cuts the Interſection, at A, draw AF, alſo perpendicular to the Ground Line.

Make AF equal to the height of the Pyramid, and draw FG, cuting the Perpendicular HI at I, and draw AI, BI, and DI, which compleat the Figure.

If (as in this Example) on account of the Diagonal, AC, having very little inclination to the Ground Line, the Altitude cannot be determined with accuracy; and the other Diagonal, BD, is ſo much inclined as not to cut the Interſection within the Picture; it may be thus determined.

From

From either Vaniſhing Point, M or N, draw a Right Line through the Center of the Baſe, till it cuts the Ground Line, at a; draw a f parallel and equal to A F, and draw f N, cuting H I in the ſame Point, I, for the Vertex of the Pyramid, as before. Or, as in the following.

Second. Having found the perſpective Plan ABCDFG, of the hexagonal Baſe, Z. (Prob. 24) M and O being two Vaniſhing Points; the other is out of the Picture. From any Point, P, in the Vaniſhing Line, draw P H, cuting the Ground Line, in K.

Draw K L perpendicular to the Ground Line, and H I parallel to K L.

Make K L equal to the known Altitude of that Pyramid, and draw L P, cuting H I in I; and draw A I, B I, &c. as before, which compleat that Object.

Priſms and Pyramids are very uſeful Subjects for practical Leſſons. By the firſt, we learn to form Pavillions, Temples, Cupolas, Alcoves, &c. which are, generally, of ſome priſmatic Form; by the other we may form a Spire, Obeliſk, or Pyramid, of any Figure.

As thoſe Objects admit of infinite variety (as many as there may be various Poligons) it would be to little purpoſe to give more Examples of them, ſeeing that, if a Poligon of any number of Sides be drawn in Perſpective, for the Baſe (by the various Problems of the laſt Section) the reſt is the ſame as in the Examples given.

It may, I preſume, have been obſerved, how inconvenient it would be to have the Ground Plans of all the Objects, to be delineated, geometrically drawn, in the Geometrical Plane; beſides, the impoſſibility of having room for them, when the Objects extend to any conſiderable Diſtance; as the Plans, W, X, Y, and Z, of the Parallelopipeds in the ſecond Example (Plate XV.) evince.

It may alſo be obſerved, that they are of no other uſe than to know how the Objects are ſituated, in reſpect both of the Picture and of each other; but ſince it is obvious, from what has been done, that they are not abſolutely neceſſary to be drawn, ſeeing that, if their meaſures are applied to the Interſection of the Plane they are in, it will anſwer the purpoſe; therefore I ſhall, in future Examples, where they are not abſolutely neceſſary, do without them. Yet it muſt not be underſtond, that we can do entirely without, which is impoſſible; but, in many Caſes, their Meaſures and Diſtances, only, are requiſite. In others, the Situation and Poſition of one Object to another, being irregular, require a true and correct Plan to be firſt drawn (but not on the Picture) from which all the Vaniſhing Points may be aſcertained; as every Line, in Perſpective, is determined from its geometrical Proportion, and the poſition of the Object to the Picture; conſequently, they muſt be known or imagined, otherwiſe we have no foundation to build on.

EXAMPLE VIII.

To repreſent Steps with Kirbs at the ends; as at the entrance into a Houſe, &c.

Firſt, when they are parallel to the Picture; and the Picture cloſe to them.

Fig. 78 is a Section of the Steps, in their geometrical proportion, with the Inclination of the Kirbs to the Horizon, and to the Picture.

Take A B, on the Ground Line, equal to the width, or more properly, to the length of the Steps; and, A D and B J to the thickneſs of the Kirbs. Fig. 79.

Make A F equal to their height, and deſcribe the Rectangles A F G D, and B H I J, geometrically; which are ſuppoſed to be in the Picture.

C is the Center of the Picture, and C E the Diſtance. Draw A C and B C.

Make B *a* equal to the Diſtance of the firſt Step from the Front of the Kirb; and *a b*, *b c*, and *c d*, each equal to the breadth of a Step, and draw *a* E, *b* E, &c. cuting B C in a, b, c, &c. the perſpective breadths of the Steps, on the Ground.

Make B *e*, *e f*, &c. each equal to the height of a Step, and draw *e* C, *f* C, &c. cuting the Perpendiculars, from a, b, c, &c. at l, m, &c. and transfer them, by the parallel Lines a f, b g, &c. on the Ground, to the other end, cuting A C at f, g, h, &c. from which Points, draw the Perpendiculars f r, g s, &c. and, from l, m, &c. draw Lines parallel to A B, cuting them at r, s, &c. draw r C, s C, &c. cuting them again, at *s*, *t*, &c. and the parallel Lines *l r*, *m s*, &c. at the internal Angles, being drawn, compleat the Steps.

Pl. XVI. Fig. 79. The upper Step having a greater breadth than the other, make *d*D equal to its breadth, and draw DE cuting BC at e; the Perpendiculars d o and e p cut *b*C in o and p; and o u, p v, being drawn parallel to the Ground Line, cut the corresponding Perpendiculars i u and k v, which determine it; *viz.* o p v u.

If the Kirbs are ſquare, they are right angled Parallelopipeds, and are delineated, as deſcribed in Problem the 29th. AFGD and BHIJ are the Fronts, AFvk and BHpe are the Sides, which incloſe the Steps; and, FvwG and HpqI are their upper Faces.

If they incline with the Steps (as is common) make the Rectangles AK and BL equal to the Ends; ſuppoſe, equal to the height of a Step.

Draw C*C* perpendicular to the Horizontal Line; and, CE being equal to the diſtance of the Picture, make the Angle CE*C* equal to the inclination of the Steps, cuting C*C* at *C*, which is the vaniſhing Point of their Inclination.

Draw K*C*, L*C*, &c. cuting G*C*, I*C*, &c. in x y, &c. which compleat the inclined Faces of the Kirbs, as in the Figure; by drawing x u and o y.

In this Example, the front of each Step, being parallel to the Picture, are ſimilar to each other; and have that Proportion to each other, reſpectively, as their ſeveral Diſtances, i. e. the ſecond Step, is, in length and height, to the firſt, as CE is to CE+*ab*; and, the ſecond is to the firſt, as CE is to CE+*ac*, &c. †

† Theo. 9.

VL, paſſing through *C* parallel to the Horizontal Line, is the Vaniſhing Line of a Plane inclined to the Horizon, in the Angle *C*EC, equal E*C*L, and *C* is its Center (ſee Fig. 15, No. 3, P. 72.)

For C*C* is perpendicular to VL; and E*C* producing the point *C*, is the Radial of ſuch Lines in the inclined Plane as are perpendicular to its Interſection KL; therefore, *C* is their Vaniſhing Point. (Cor. Th. 11.)

Case 2nd. EXAMPLE IX.

When the Steps are inclined to the Picture; the Inclination being determined, alſo their Situation, in reſpect of the Station Line.

In this Example I ſhall ſuppoſe, that there is not room on the Picture for aſcertaining the Vaniſhing Points and their Diſtances, as uſual; in order to apply the 12th Problem.

Fig. 80. C being the Center of the Picture, and KCL the Horizontal Line, draw CE perpendicular, and take CE any equal part (ſuppoſe one third part) of the determined Diſtance of the Picture.

Make the Angle CED equal to the Inclination known; and CD will be one third part of the diſtance of the Vaniſhing Point L, of one Side; which being determined, all the reſt are determinable, arithmetically (by Prob. 12.)

Or, make DEF a Right Angle; CF will be a third part of CK; and FE will be a third part of its Diſtance, by the ſame, &c.

Having thus obtained the Vaniſhing Points, K and L, and their Diſtances E^1 and E^2 being laid down (CE^1 being equal to three times the difference between FC and FE; and CE^2 to three times the difference between CD and DE) then, proceed as uſual; A*E* being the Interſection of the inward Angle of the Kirb.

Draw AK, the indefinite Repreſentation of the Front, and AL of the End.

Let AJ be the length of the Steps (eq. AB, Fig 79) draw JE^1 cuting AK at *A*.

Make A*a* equal to the diſtance of the firſt Step; *ab*, and *bc*, &c. each equal to the breadth of a Step, and draw *a*E^1, *b*E^2, &c. cutting it in a, b, &c. and, ſeeing there is not room on the Interſection A*c*, to apply the meaſure of the top Step, which, ſuppoſe equal to A*c*, draw d*f* parallel to A*b*, and *c*L cuting it in *f*; and draw *f*E^2 cuting AL at B.

Draw the Perpendiculars aj, bf, &c. indefinite.

On A*E*, take A*B*, *BC*, &c. each equal to the height of a Step, and draw *B*L, CL, &c. cuting the Perpendiculars from a, b, &c. as in the Figure.

Draw

Draw aK, and *A*L cuting it in k; draw kl perpendicular, and jK cuting it in l; draw lL, and eK cuting it in m, and fK cuting the Perpendicular, m n at n, and proceed, after the ſame manner, throughout.

Or, having drawn all the Lines eK, fK, &c. indefinite, draw lL; and m n perpendicular to the Ground Line; then n L, and o p, &c. by proceeding after that manner the Steps are completed.

The Kirb being equal to BJ (as in the former Figure) draw B*E*', cuting AK at *D*; *AD* repreſents the thickneſs of the Kirb, at that end.

The Original of the Point, G, being ſuppoſed on this ſide of the Picture, and being in the Ground Plane, the Repreſentation (G) is conſequently below the Ground Line, AB. (See N.B. Prob. 8. Fig. 45. The meaſure being applied on this ſide, i.e. below AB, as AH, ſuppoſes AB proportioned perſpectively; but, being at the other Extreme, as aB, it is projected; and is, conſequently, larger than the Original.)

Make A*d* equal to BJ, and draw *E*"*d*, cuting KA produced, in G; AG is the projective repreſentation of the end of the Kirb, the Original, of which, is equal to A *d*, equal BJ.

Draw the Perpendiculars GH, *AF*, and *DG*, indefinite; and, A*E* being made equal to the height of the Kirb, draw *E*K, cuting them in *F*, and *G*; and, being produced, it cuts GH at H.

*E*L cuts a Perpendicular from B, at I; and, if *F*L, *G*L, and HL be drawn, KI, cuting them, compleats the upper Faces, *F*J and *EIh*H. Draw *gh* perpendicular, cuting GL, at *g*, which compleats the End.

If the Kirbs are inclined with the Steps, as in the former Caſe, draw LV, perpendicular to KL; which is the Vaniſhing Line of the Planes A*E*IB, *AF*J, &c.

Make L*E*' equal to its Diſtance, and draw *E*'V, making the Angle L*E*'V equal to the inclination of the Steps, cuting LV in V, the Vaniſhing Point of the inclined Sides*. KV is the Vaniſhing Line of the Inclination. (Theo. 10. Cor. 1.)

A*B* being made equal to the height of the Front, draw K*B*, cuting *DG*, *AF*, and GH, in *K*, *H*, *B*, and *L*; from which Points, draw *KV*, *HV*, &c. cuting the horizontal Lines *G*L, HL, &c. in s, r, t, and u; through which, draw Ku, or join the Points rs, tu, only.

Note. The Picture might, with equal propriety, have been ſuppoſed on this ſide of the Kirb, entirely; but, as it frequently happens, in Practice (the Picture being fixed inadvertently) that ſome parts of the Object would project on this ſide; I thought it neceſſary to give an Example how to proceed in that Caſe.

SCHOL. In this Example, it is obvious that, the Plane A*E*IB, in which, the Steps are perſpectively proportioned, being farther removed from the Vaniſhing Line, VH, is fitter for the purpoſe than GHL; and, if the Original of GH was in the Picture, the meaſures on A*E* would be leſs than the full meaſure of the Steps, ſeeing it would be beyond the Picture.

A*Bob* is the Section made by the Picture, with the inclined Kirb, and ſhews how much is ſuppoſed to project on this ſide. A*b* is the ſection of the Bottom, conſequently, parallel to the Horizontal Line; *ob* is the ſection of the End, which is vertical, therefore parallel to VL its Vaniſhing Line; and *Bo* is the Section with the inclined part, conſequently, parallel to the Vaniſhing Line, KV, of the inclination of the Steps. (Theorem 2nd.)

* This will be clearly demonſtrated and made manifeſt, in the 12th and laſt Section of this Book.

N.B. The Vaniſhing Line (VW) of the Roof of the Object in the Apparatus, is determined in this manner (as KV) and is in a ſimilar Poſition, on the direct Picture, MNOP.

V, the Vaniſhing Point of HG, anſwers to K, in this Figure; Y anſwers to L, and W to V.

SECTION

Plate XVII.

SECTION VII.

Of the application of Perſpective, to MOULDINGS, &c.

THE foregoing Section contains many uſeful Leſſons in plane Solids, which may be conſidered as parts of Buildings, &c. In this, I intend to ſhew how the more decorative parts are formed, ſuch as Pedeſtals for Columns, Cornices and Entablatures, &c. of the various Orders; without which embelliſhments, a Building ſeems naked and deſtitute of Ornament.

Theſe Leſſons, of Steps, are very neceſſary to the art of delineating Mouldings; which, when we know how to manage well, particularly ſuch Steps as return at the Ends, or on four Sides, forming mitre Angles, the greateſt difficulty is ſurmounted. For Mouldings, breaking round a Pedeſtal, or the internal and external Angles of a Cornice, &c. properly conſidered, are but ſo many Steps, one above or below another, of different dimenſions; formed by the Fillets, between the cylindrical parts, which are, properly, the Mouldings; and are effected only by Light and Shade. The Fillets, between them, are narrow Planes, cuting the curved Surfaces in parallel Lines; which being deſcribed, by the Rules given, and the mitre Angles of the Mouldings drawn, the buſineſs of the linear part is done, and nothing remains but to give the appearance of ſolidity, convexity, and concavity, by a proper diſpoſition of Light and Shade.

EXAMALE XI.

How to delineate ſquare Steps, returning on every Side.

Fig. 81. Firſt, when they are parallel to the Picture. AB is their length, ABCD is half the geometrical Plan of the firſt Step; FGHI of the ſecond, and KLMN of the third; and let OP be the meaſure of a right angled Block, on the upper Step.

C being the Center, draw AC and BC; and AE, which is a Diagonal, repreſents AQ, in which are the Seats of G, L, and O, the Corners of the Steps.

Compleat the Square A*DC*B (Prob. 19) and draw the other Diagonal B*D*; which, it is evident, tends to a Point on the left hand of the Center, equal to CE, the Diſtance of the Picture.

Produce FG, KL, &c. to the Ground Line, cuting it in *g*, *l*, &c. i. e. make A*g*, *g l*, B*h*, &c. equal to the breadth of the Steps, and draw *g*C, *l*C, &c. cuting the Diagonals at *G*, *L*, *H*, *M*, &c. the perſpective Seats, on the Ground Plane, of the ſeveral corners of the Steps.

Draw AF perpendicular to AB, and make A a, a*b*, and *bc*, equal to the height of the Steps, as in the former Examples; and, having drawn the Perpendiculars from *G*, *L*, &c. draw aE, *bE*, and cE, cuting them at c and e, g and i.

Draw ab, cd, ef, &c. parallel to AB, cuting Perpendiculars from B, *H*, &c. at b, d, f, &c. which give the repreſentations of the Fronts, correſponding with GH and LM. AabB, being in the Picture, has its full dimenſions, in height and length; the other two are in proportion to their Diſtances.

Having obtained the Fronts (by means of an imaginary Plane paſſing through the Diagonal A*C*, which A*cFC* repreſents) finiſh the ſquare of the Top, iklm†. † Prob. 29.

Then, draw fC and bC, till they cut hk and df, which repreſent the return of the Steps on that Side, ſo far as they can be ſeen; on the other Side they are ſeen to their full extent, the Point of view being on that Side.

Draw aC and eC, &c. cuting Perpendiculars from *D*, *I*, &c. at *f*, j, and m; and, having compleated the Face g m i, which is ſomewhat ſeen, from j and *f*, draw Lines parallel to the Ground Line, cuting the adjacent Step, which terminate their appearance, and compleat the Steps.

To repreſent the Block on the upper Step; draw the Perpendiculars *O*r, and P q indefinite; make *cd* (on the vertical Interſection, AF) equal to its height, and draw *d*E, cuting *O*r, in r; *c*E cuts it at o.

Draw op and rq, parallel to the Steps, which cuting a Perpendicular (*P*q) compleat the Front, orqp. OC and rC, cuting *N*s, finiſh the End, orsn, as in the Figure.

I preſume that the Reader will, ere this, have diſcovered, how inconvenient it is to have the perſpective Plans, of complex Objects, in the very place where the Object itſelf is drawn; which, I am ſenſible, muſt render it, to young practitioners, perplexing and intricate, to diſtinguiſh one part from another. That difficulty ſhall be obviated in the next Example; but I muſt obſerve, that the connection of the ſeveral parts, and the agreement between the Plan and Elevation, would not have been ſo diſtinctly ſeen, without the Plan in its true place; hereafter we ſhall place it either above or below, as is moſt convenient.

Let it be obſerved, notwithſtanding, that this apparent intricacy will vaniſh at a ſecond proceſs; and let it be noticed, that, as the Side B*C*, of the ground Plan, paſſes through the Angles, d and h, of the Steps, it is not ſo neceſſarily; as it muſt be obvious, if the Center of View (C) was moved, on either hand, it would not. The Line Bp, being conſidered as a diagonal Line, paſſing through the Angles of the Steps, d and h, cannot be in the Ground Plane, as B*C*, a Diagonal of the Square, but making an Angle of elevation with the Diagonal BD; and conſequently, its Vaniſhing Point cannot be at C, in the Horizontal Line, but above it, in the Vaniſhing Line of a Diagonal Plane, paſſing through B Q.

Above the Vaniſhing Line, let *AB* be drawn parallel to it, which let be equal to AB; the Point *A* perpendicularly over A, and *B* over B. Fig. 81. No. 2.

Then, if *A*C and *B*C be drawn, and the Diagonal *A*E, cuting *B*C in *C*; and, *CD* being drawn parallel to *AB*, compleats the Plan of a Square, *ABCD*, correſponding with A B *C D* below.

And, if the meaſure of the Steps be ſet off, from *A* and *B*, at *a*, *b*, *c*, and *d*, *e*, *f*; *a*C, *b*C, &c. being drawn, cut the Diagonals *AC* and *BD*, in *g*, *h*, &c. which correſpond with *G*, *H*, &c. below; as may be ſeen by the doted Lines *gG*, *hH*, *mM*, *pP*, &c.

Wherefore, if there be room above, and not below, the Plan may be formed above; by means of which, the Object may be completed; drawing the Perpendiculars *g*c, *l*g, &c. cuting aE, *b*E, and *c*E; and *h*d, *m*h, &c. cuting the parallel Lines ef, &c. which terminate the other ends of the Steps.

EXAMPLE XII.

Is an Expedient, by means of which, Mouldings, &c. may be delineated, without having the perſpective Plan in the Work.

Let ABCD be a perſpective Plan of a Square, whoſe meaſure is AI, on the Interſection (deſcribed by Prob. 20 and 21) F and G are the Vaniſhing Points of the Sides of the Square, and H of the Diagonal; conſequently, FG is the Vaniſhing Line of the Plane of the Figure; S is its Center. Fig. 82.

The Diſtance of the Vaniſhing Line, F G, is a mean Proportional between S F and S G (Prob. 12) the Point E, which is the Diſtance of the Vaniſhing Point F, from the Eye, is determined by the ſame; and, the Vaniſhing Point, H, of the Diagonal, by biſecting the Angle, which the Radials of A B, and A D, make at the Eye; or, by making F H to H G, as the Radial of A B is to the other (3. 6. El.)

See Prob. 10, Caſe 3rd, Fig. 49. EH, biſecting the Angle D E G, made between the Radials of A C and C B, gives the Vaniſhing Point H; by which, the Lines A C and C B are cut, perſpectively, repreſenting equal Lines. As, in this Example, by drawing A H (H being obtained, as above) A B being made to repreſent a meaſure equal to A I (Prob. 17) and B G drawn; B G is cut, in C, repreſenting an equal Line as A B repreſents, i. e. equal to A I (Prob. 10.) Then, draw A G; and F C, till it cuts A G, at D; A B C D repreſents a Square, whoſe Side is equal A I.

It is required to draw, within the Square ABCD, the repreſentations of ſeveral leſs Squares, of certain dimenſions in proportion to the other, about the ſame Diagonals (AC and BD) and conſequently, having the ſame Center; i. e. to draw the repreſentations of Borders or Margins within it.

Plate XVII.

Make A*a* and *ab* respectively equal to the breadths of the Margins.

Draw *a*E and *b*E, cuting AB in *c* and *d*; then, draw *c*G and *d*G, cuting the Diagonals in e and h, i and m; and draw eF, iF, &c. cuting the Diagonals again, at f, k, &c. and join fg and kl; or, draw fG and kG.

efgh and iklm are the representations of Squares, having the same Center, o; and are about the same common Diameters, AC and BD.

Now, because the Intersection (AI) of the Plane those Squares are in, is so near its Vanishing Line (FG) the Sides of the Squares, AB, ef, &c. cut the Diagonal BD very oblique; for which reason, the Points of section, f and k, cannot be ascertained with accuracy; nor can the true breadths of the Margins be determined. Therefore, either below or above the Vanishing Line, draw *A*B, or a*B* parallel to AI; in which, take *A* or a, in a perpendicular Line from A, and make *A*B, or a*B* equal AI; and make *A*1, or a1, and 12, respectively equal to A*a* and *ab*.

Draw *A*F and *A*G, or aF and aG; and BE, 1E, 2E, cuting *A*F in *B*, p, and q.

Compleat the Square *ABCD*, or abcd, by the same Vanishing Points, F, G, and H, and draw the Diagonal *BD*, or bd. Draw pG, and qG, cuting the Diagonals, in *E*, *H*, *i* and *m*; from which Points draw *E*F, *i*F, &c. and finish the interior Squares, *EFGH*, and *iklm*, as in the Figure.

If Perpendiculars are drawn from the Points *E*, *F*, or *e*, *f*, *i*, *k*, &c. they will cut the Diagonals AC and BD in the Points, e, f, i, k, &c. as it is obvious from inspection of the Figure.

For, if AI be the Intersection of the Plane of ABCD, with the Picture, *A*B, a*B* may be considered as the Intersections of other Planes, parallel to the former, and therefore, they have the same Vanishing Line, FG†; and consequently, *AB*, AB, and ab; also *AD*, AD, and ad, and all other Lines, whose Originals are parallel to their Originals, have the same Vanishing Points, F and G‡; notwithstanding, the Lines are in different Planes; and, H is the Vanishing Point of the Diagonals, *AC*, AC, and ac, seeing that, they represent parallel Lines, and in parallel Planes.

† Theo. 3.
‡ Cor. to Theo. 3.

Therefore, *AB*ba*dD* represents a right angled Parallelopiped or Prism, whose Base, *ABCD*, and Top, abcd, are Squares; consequently, they are equal and parallel to each other§; and similarly posited.

§ See Def. Page 41.

ABCD may be considered as a Section of that Prism, by a Plane parallel to its Base; and consequently, it is also a Square, equal, similar, and parallel to the other.

The same may be said of all the interior Squares; which, with the Perpendiculars *Ee*, *Ff*, &c. also form Prisms. The Border, around, represents the edge of a hollow Prism.

In Example 5, the hexagonal Prism has its Base and Top similar Figures, and similarly posited; and being in parallel Planes, their Sides are parallel, and consequently, they have the same Vanishing Points.

Hence it is manifest, that, either the extra Plan *ABCD*, or abcd, may be used, for the purpose of drawing perpendicular Lines, in order to determine the extremes of Steps, &c. instead of the real Plan ABCD; in which, on account of their being farther from their Vanishing Line, the Intersections are determined with greater exactness, and without incommoding the Object.

EXAMPLE XIII.

How to represent square Steps, obliquely situated to the Picture and having a Pedestal situated on them.

Fig. 83.

Let AB be the Ground Line, and DF the Horizontal vanishing Line, C is its Center; A is the intersecting Point of the nearest Angle of the Ground Plan.

The Distance of the Picture being known, and the Inclination of the Sides of the Object determined, find the Vanishing Points, D and F, of the Sides (by Prob. 12) and G of the Diagonal (as O, Fig. 51; EO bisecting the Angle AEB) also, find E, making DE equal to the Distance of the Vanishing Point D.

Draw AD and AF, indefinite; make AB equal to the Side of the Square of the Steps, and draw BE, cuting AD in d.

Draw *AB* parallel to AB, at any distance at discretion.

Draw

Draw A *A* perpendicular to AB; and at the Point *A*, deſcribe the repreſentation of a Square *ABCD* (by means of the Vaniſhing Points D and F; and O, of the Diagonal) whoſe Sides *AD*, &c. repreſent the meaſure AB; as Ad, Ab, above.

Make *Aa*, *ab*, each equal to the width of the Steps; and draw *a*E, *b*E, &c. cuting *AD* in *g* and *h*.

Draw the Diagonals *AC* and *BD*; and draw *g*F and *h*F, cuting them in *F*, *G*, &c. and compleat the interior Squares, as in the laſt Figure.

Draw AH, perpendicular to AB. Take Aa and ac equal to the height of the Steps, and draw aD and aF, terminating at b and d, where Perpendiculars from *B* and *D* cut them.

Draw aG and cG, cuting a Perpendicular from *F*, at e and f.

Draw eD and fD, eF and fF, cuting Perpendiculars from *G* and *H*, at g and h, which terminate that Step; bD and dF cuting the next Step, at g and h, compleat the firſt; and, by the ſame means, the next is compleated.

It remains, now, to delineate a Pedeſtal upon the upper Step.

Set off the geometrical projectures of the Mouldings, from *b* to *c*, on the Interſection; and transfer them to *AD*, by means of the Point E; and draw *i*F, cuting the Diagonals at *L* and A; and draw *L*D, cuting the Diagonal *DB* at *ƒ*.

Then, on the Interſection AH, make cH equal to the height of the Pedeſtal; make ci equal to the Plinth, ik to the Baſe Moulding, kl to the Dado, lH to its Cornice or Sirbaſe, and Hj to the Plinth of the Baſe of the Column; which, notwithſtanding it is a part of the Baſe, always goes with the Pedeſtal. Fig. 83.

At X and Z, on the right hand, the Profile of the Mouldings and Steps are deſcribed, geometrically; from which they are transfered by means of parallel Lines, to the vertical Interſection AH.

Draw cG, iG, kG, &c. cuting Perpendiculars from *K* and *L*, in m, n, o, p, and q; the repreſentations of the correſponding Angles in the Profile.

Draw mD, nD, mF and nF, &c. cuting Perpendiculars from *ƒ* and *I*, in r, s, t, *m*, *n*, *o*, &c. and having divided ik into the ſmaller diviſions of the Mouldings (as at Z) draw to G, cuting Perpendiculars from the correſponding parts in the Ground Plan, at J*IK*, by which means, the whole is compleated; as in the Figure.

Now, when this Object is well conſidered, what are the ſeveral Parts which compoſe the Pedeſtal, but ſo many Steps of different heights and breadths. The Plinth, m n *n m*, is in the place of the next Step, about twice the height; and the Fillet, which follows it, is another of a much leſs height; the Dado, after that, is another ſtill higher, equal kl; and the Cornice, v x, is but Steps reverſed.

The Curve Lines s t, n o, and *n a*, joining the Angles of the ſeveral Fillets, are all that have any linear curviture; although n o *o n* repreſents a cylindrical Surface, yet the appearance of it is effected by Light and Shade only; which, according as it is ſuppoſed to be ſituated, in reſpect of the Light, will appear either convex or concave; to which, the curve Lines, as its extremes, do not at all contribute.

And now I might take leave of the Reader, and of this Section, and endeavour (as others have done) to perſuade him, that I have, in this Leſſon, given him ſufficient inſtructions for delineating all kinds of rectilinear Mouldings; but, I have not quite ſo good an opinion of his Capacity (if it be his firſt eſſay) as to ſuppoſe any ſuch thing; my intent in this, is only to give him ſome little Idea of it, by comparing the Proceſs with that of delineating Plane Steps. There remains yet ſomewhat more to be ſaid on the Subject; therefore I ſhall not treſpaſs on his time, but give ſome other Examples for his further Inſtruction.

EXAMPLE XIV.

How to repreſent Mouldings around the Baſement of a Building, being a regular Pedeſtal, with an internal and external Angles; whoſe Planes are parallel and perpendicular to the Picture, and alſo one to another.

Let AB be the Ground Line, and AD the Interſection of the Picture with the vertical Plane of the Dado, b c g *e*; on which, ſet off all the meaſures of the heights of the Mouldings, and their diſtances from each other. Fig. 84.

Plate XVII.

A a is the height of the Plinth, a b of the Baſe moulding; b c of the Dado, and c D of the Sirbaſe, or Cornice of the Pedeſtal.

Let each Moulding be drawn, geometrical, c d D the Cornice, and a b *c* the Baſe; draw the Perpendicular *a c*, and draw the diagonal Lines b *c* and c d.

C being the Center of the Picture, and E C the Diſtance, draw A C indefinite.

Make A F the meaſure of the firſt break, and draw E F, cuting A C at G.

Draw G I parallel to A B; make A F equal to the projecture of the Pedeſtal, and draw F C, cuting G I, at I.

Make I J to G I as the front of the Pedeſtal to its projecture; that is, as F B to A F; and draw J E, cuting F C, at K; or B E, cuting A C, at H, and draw H K parallel to A B, cuting F C.

From the ſeveral Angles, G, I, and K, thus obtained, draw G r, I s, and K t, perpendicular to the Ground Line; which conſtitute the ſeveral Planes, from which the Mouldings are ſuppoſed to project.

Draw b C, c C, and D C, cuting G r, at *e*, g, and e; draw *e f*, g m, and e k, parallel to the Ground Line, and where they cut I s, at *f*, m, and k, draw Lines to the Center, cuting K t, at *g*, n, and o.

Now, D e g c, e k m g, and k o n m, are the Seats of the Cornice on thoſe Planes; c g *e* b, &c. is the Dado or plane part between; and A b *e* G, G *e f* I, and I *f g* K are the Grounds of the whole Baſe, thereon.

Draw *a* C, cuting E F at *b*; draw *b h* parallel to A B, cuting E I produced, at *h*; and draw *h* C, indefinite.

The Angle *k* is determined, by drawing a Line from K to the Eye, on the other Side of the Center, equal C E; for K *k* is a Diagonal of a Square; whoſe Vaniſhing Points are, in this Caſe, the Points of Diſtance. (Prob. 19.)

Or, if the Vaniſhing Point of the Diagonal K *k* is not on the Picture, draw K *d* parallel to the Ground Line; on which deſcribe the Square K *i k d* (by Pr. 19) which will give the Angle, *k*, required; i. e. draw *d E* cuting K C; and *i k* parallel to K *d*, cuting *h* C at *k*, the Angle ſought.

From the ſeveral Angles, *b*, *h*, and *k*, draw the Perpendiculars *b* f, *h* l, and *k* p; and, through e and k, draw E e, and E k, cuting the two firſt, at f and l.

Or, from *c* and d, draw Lines to the Center, cuting the firſt at *d* and f; from which, draw *d B* and f l, cuting the ſecond, at *B* and l; and from them draw Lines to the Center, cuting *k* p, at *l* and p; which give all the extreme Angles of the Mouldings.

Draw the diagonal Lines *e d*. *f B*, and f g, &c. and obſerve how the parallel Lines in the geometrical Mouldings, cut their Diagonals; as, at h, the great projecture of the Facia goes beyond it.

Draw the lower Line of the Cimma or Ogee, at h; and, where it cuts the Diagonals, at i and j, draw the perpendicular lines of the Facia, which will terminate it, at the ſeveral Angles; by drawing h i, tending to the Center, and i j parallel to the Vaniſhing Line, &c. The reſt is obvious on inſpection of the Figure.

The Fillet at z between the Ovolo and Cavetto, touches the diagonal Line, c d; by which, it is eaſily deſcribed, as above.

As the Fillets of the Baſe Moulding touch the diagonal Line b *c*, they are very readily returned at the internal Angle, *e d*, to the external, at *f B*, and from thence to the other, at *g l*.

It may perhaps be imagined, that b *c*, and c d are mitre Angles, at the Corner A D, which they are not. For, ſuppoſe A D v u to be a returning Plane, parallel to the Picture, and at right angles with A D e G. Then, becauſe A D was conſidered as the Section of that Plane, A D v u is in the Picture; and conſequently, A b *c a* and c d D are geometrical ſections of the Mouldings, by the Picture; and if they were returned on the Plane A D v u, they would project on this Side of the Picture.

Draw E A, meeting *b a*, produced, at *n*; and draw *n* x parallel to A D.

Produce

Produce fd to x; alſo produce *dc*, till it cuts the Perpendicular *n*x at *o*; and draw *op* and *nq* parallel to the Ground Line; a diagonal line, *o*b, is the mitre Angle of that Moulding.

It may be obſerved, that, if AD paſſed through E, the mitre Angles would be in that Line entirely; indicating the Eye to be in the diagonal Plane of that Angle; if AD was beyond E, the mitre Angle would be on the other hand of it, when it becomes greatly diſtorted.

AB being the Interſection of the Ground Plane with the Picture, all the Mouldings on this ſide project through the Picture, and they are conſidered as projected to the Picture.

Dq is the height of the Plinth of the Baſe, which determines it, at r, s, t, on the Pedeſtal.

EXAMPLE XV.

Is a general and univerſal Rule for delineating Mouldings, parallel to the Picture; whether they are above or below the Eye.

At either extreme of the Picture, if there be room (or it may be done on another drawing Board, and the Scale Line transfered, carefully, to the Picture) deſcribe the Cornice, &c. geometrically; as at No. 1.

If the Student be not acquainted with Architecture, or has not a knowledge of the geometrical proportions of Mouldings, let him not attempt the delineation of them in Perſpective, before he has ſtudied, and practiſed himſelf a little in that kind of Projection; as he will but loſe his time; it being impoſſible to ſucceed in one without the other; ſeeing that, the projection of Mouldings, in Perſpective, depends wholly on the geometrical Profile.

Let AB be the height, and BD the projecture of a Cornice, intended to be delineated; to which let there be added an Architrave, FG; between which, the plane part, AF, is called the Frize; the whole together compoſes an Entablature. Fig. 85.

Let the ſeveral Mouldings be geometrically deſcribed, according to their heights and projectures; at a, b, c, &c. and, CD being drawn perpendicular, may be conſidered as a vertical Section of the Picture, applied cloſe to the greateſt projecture of the Cornice, at D.

According to the height which the Cornice is ſuppoſed to be above the Eye, take the Point C, and draw CE perpendicular to CD, i. e. horizontal; and equal to the Diſtance of the Picture.

Now, E being conſidered as the Eye of a Spectator, viewing the Cornice, AD, &c. EA, ED, EF, &c. are Viſual Rays, in which direction the ſeveral parts are ſeen; and, CD is ſuppoſed to be a Plane, interpoſed, cuting thoſe Rays; conſequently, the ſeveral Points, *A*, *a*, *b*, &c. projected by them, are the apparent places of the ſeveral edges, or parallel Lines of the Mouldings, on the Picture.

C is the Center of the Picture, and EC is its Diſtance; by which, the perſpective repreſentation of the Mouldings muſt be projected; provided that, the drawing be required to the Scale CD, of the Profile, ABD.

But, it may, from the ſame Profile, be made either greater or leſs. e. g.

If the Rays ED, &c. be continued, beyond D to H, and HI be conſidered as the Plane of the Picture, the projection of the Cornice is JH, which is greater than *A*D, and ſo of the reſt; EI is the Diſtance of that Picture.

Or, if any other Plane be interpoſed, as at Kd, or *CH*; the proportions of the ſeveral Parts on each, *viz.* *GF*, *FA*, and *A*D; gf, fa, ad, or *gf*, *fa*, *ad*, are, to each other, as their ſeveral Diſtances, EC, to EK, or EC. (6. 6. El.

Hence it is evident, that, from parallel Sections of the Rays, EA, ED, &c. any where, the Mouldings will have the ſame proportion to each other.

The ſeveral Sections, CD, Kd, and *CH*, are Scales of the perſpective Proportions for each reſpective Diſtance.

It is required to delineate the Entablature (No. 1.) by the Scale *CH*.

Plate XVII.

Let E be considered as the Center of the Picture or Point of View, and E C the Distance of the Picture; the Terms being inverted.

To the farthest Corner (B) of the given Cornice (i. e. to its height, or Seat of the extreme projecture, D, on the Perpendicular A B) draw E B, cuting *C H* at *H*.

Take any length, required, as *C I*, for the Ground Plane of the Cornice.

Draw *I K* parallel to *C H*, i. e. perpendicular to E C, the Horizontal vanishing Line; and from *H*, where E B cut *C H*, draw *H B* parallel to the Horizon, cuting *I K* at *B*. Draw *d D* also parallel; and *C B*, cuting it, at *D*, gives the Angle *D*.

Draw E h and E i; and, from their Intersections, with *C H*, draw Lines parallel to *D d*, till they cut *B D*; and, from *b* and *i* draw Perpendiculars, cuting the Parallels from *c* and *b*, at *k*, which compleats the Corona. *c k b* represents the Planceer, b c, in the Profile. The rest is obvious, for the parallel Cornice.

The projecture of the Architrave, at *F*, is obtained, by drawing *C F*, after the same manner as the Cornice, at *D*.

If the length of the returning Plane exceeds the limits of the Picture, take any equal part of the whole length; as a *B*, half, in *H B*, produced; make *E E* equal to half E *C*, and draw a *E*, cuting E *B*, at b.

To ascertain the mitre Angle, *e*, seeing that the Vanishing Point of the other Diagonal is not in the Picture, draw b d parallel to a *B*; and *D E*, cuting it in e; make c d equal b c, and draw d *C*, cuting *D E* in *e*, the Point sought.

Or, draw b *L* parallel to a *B*, cuting the Diagonal, *D C*, at *L*; bisect *B L* perspectively, at *f*; draw *f* b, and produce it, till it cuts *D* E, at e. Draw e f.

For, because b *L* is parallel to the Picture, and, *C* is the Vanishing Point of one Diagonal of a Square, in that position, *B* b represents a Line equal to b *L*; and *B* b *L*, representing a Right Angle, is bisected by *f* b, half the other Diagonal; which, being produced, cuts *D E* at *e*, the extreme Angle of the Cornice.

Draw E *b* and E *i*, cuting e b at 1 and 2; from which, draw Perpendiculars; and draw Lines to E, from the Corona, at *k*, and all the Fillets, cuting e f; observing, that the extreme Angle of the Corona, goes beyond the Diagonal.

This Method of representing Mouldings is deduced from the most natural and simple Ideas of it, and cannot, I think, fail of being intelligible to the meanest capacity, who has any emulation to attain to the art of drawing in Perspective.

As there are no Modillions, or other Enrichments, of that kind, in this Cornice, there is no apparent distortion; if there were such, they would be distorted towards the extreme *C H*, seeing that, the Mouldings extend to the full Distance (E *C*) by which they are projected.

EXAMPLE XVI.

Plate XVIII.

To represent a Doric Entablature, with an internal and external Angles.

On account of the variety of Parts, and the many breakings of the Fillets, around the Triglyphs, in the Doric Frize, it is necessary to have it described, geometrically; so that, the Parts may be clearly understood. Indeed every Moulding, delineated in Perspective, ought first to be geometrically drawn, to the Scale of the Drawing; although the measures may be applied from a Scale only.

I have shewn, in the 12th Example, how a perspective Plan may be drawn either above or below the Work; by which the projectures of the Mouldings, &c. may be determined at the Angles, as exemplified in the 13th. But, that process (though the most to be depended on, for accuracy) is attended with extraordinary trouble, which may be lessened greatly, when we are tolerably acquainted with Mouldings. A Specimen of both will be shewn, in the following Example.

Fig. 86. No. 1.

Let A B be the height of the Entablature, according to the Scale of the Drawing. Let it be bisected at C; and A C, or C B, is a Module, or Diameter of the Order. A C divided into 60 Minutes, is the Scale; by which the whole is proportioned.

Q M, the projecture of the whole Cornice, is a Diameter; the rest is proportioned as by the Scale A B. D F is the Architrave, F G is the Frize, with its Triglyphs, and G M Q is the Cornice. H I is the projecture of a Mutule or Modilion, and K of the whole Planceer; which, being seen in Perspective (the Eye being below it) has a fine effect, and addsgreatly to its august appearance.

No. 2. is a geometrical Plan of the Cornice, with an internal Angle at S, and an external, at T; shewing the Planceer of the Mutules, and panneling between them; for, unleſs we know the true geometrical form, it is not poſſible to deſcribe any thing, perſpectively. Each return of the Cornice is the ſame, having two Mutules each way, except the laſt, on the Right, which is ſuppoſed unlimited; and, on the Left, it is limited by the bounds of the Picture; which, on account of its diſtance from the Center, would be diſtorted, if it was continued much farther. No. 2.

BAFG is a perſpective Plan of the whole; which is the beſt method of proceeding, if accuracy be required in the ſeveral parts, and we do not grudge the time ſpent in doing it. It is formed as follows. No. 3.

At any Diſtance from the Horizontal Line, *E*CE, either above or below it, draw *AB* parallel to it; one Side, in this Example, being parallel to the Picture.

Having determined on the Place of the Angle a, in the Deſign, take *A* perpendicular over it, and draw *AC*, indefinite; C being the Center of the Picture.

Take *AD* equal ST, in the geometrical Plan; and, having made C*E* in the Horizontal Line, equal to the Diſtance of the Picture (equal ES) draw *DE*, cuting *AC* in *F*; then, *AF* repreſents the length of *AD* (equal ST, No. 2) and *F* is the internal Angle of the extreme Moulding in the Cornice.

Draw *FG* parallel to *AD*, and *DC* cuting it, in *G*, the external Angle; for *FG* alſo repreſents an equal length as *AF*†, each equal to ST. † Prob. 19.

Make *AC* equal MQ (No. 1) the projecture of the whole Cornice, and draw *CC* and *AE* cuting it, at V. The Diagonal *FX* (i. e. *DE*) alſo cuts *CC* at X; from which, draw X Y parallel to *AB*.

The Angles *K*, *L*, and *M*, of the Corona (K, in the Profile) are determined by making *Ad* equal to MN, and drawing *dC*, cuting the Diagonals *AV* and *FX*; by means of which, they are carried around, to *L* and *M*, and, by the ſame means, the whole Plan is compleated, as in the Figure.

To deſcribe every Step, by which the whole perſpective Plan is formed, would be as tedious as it would be uſeleſs; ſeeing that, the various Leſſons, already given, are ſufficient for any right-lined Figure whatever. Fig. 86.

The Plans of the Mutules, Z, Z, are the repreſentations of Squares having one Side parallel to the Picture (found by Prob. 19) their places are determined as follows.

Having made *Ae* equal M O (in the Profile) draw *e*C cuting the Diagonals *AV* and *FX* at *S* and *W*, and draw *WM* parallel to *AB*.

Make *Aa* equal to J M, in the Profile (equal T 1 in the geometrical Plan) make *ab*, *bc*, &c. equal to the width of the Mutules and the ſpaces between them (12, 23, &c. in the Plan) and from each Point, *a*, *b*, &c. draw Lines to C, the Center, cuting a parallel Line from *S*, in *R*, *P*, &c.

For, the returning Side, *AF*, make A*f*, *fg*, &c. equal *Aa*, *ab*, &c. and draw *fE*, *gE*, &c. cuting *AF*, at 1, 2, 3, from which, draw Lines parallel to *AB*, cuting *S*C at *T*, V, &c. and *H*C at *m*, *n*, &c.

For the other parallel Side, *FG*, make *Dm*, *ml*, *lk*, alſo equal to *Aa*, *ab*, &c. from which draw Lines to the Center; cuting *WM*, at *X*, *Y*, *Z*.

For the apparent width of each Moulding, &c. in Front, draw MS perpendicular to the Horizontal Line; take SE equal to *E*C, on the Horizontal Line; and, from every Angle, G, H, K, &c. in the Profile, draw EG, EH, &c. cuting MS in the ſeveral Points, g, h, k, &c. as in the laſt Example.

Being thus prepared, we now proceed with the Repreſentation.

a is the determined Angle of the extreme Moulding; draw the Fillet a z of the upper Moulding (which is in the Picture) geometrical. Fig. 86.

From V, X, and Y (in the perſpective Plan) draw perpendicular Lines indefinite; which are the Angles of the Ground Planes* of the whole.

Then, from the ſeveral Angles *H*, *K*, &c. in the perſpective Plan, draw Lines perpendicular; and, from the correſponding parts on the vertical Section, M S, draw Lines parallel to the Horizon, cuting them. e. g.

* By Ground Planes, here, is not meant the real Ground or Floor, as defined, but the vertical Planes, from which the whole Cornice, &c. project; called ſo by Workmen.

From

Plate XVIII.

From *K* draw a Perpendicular, *K*c, and from k (where KE cut MS) draw horizontal Lines cuting it, at c, the repreſentation of the external Angle of the Corona.

Draw cC, cuting the Perpendicular from *L* at l, the internal Angle of the Corona; and from l, draw parallel Lines, cuting the Perpendicular from *M* at m, the other external Angle of the ſame.

Then, from h (where HE cuts *C*M) draw horizontal Lines cuting the Perpendicular from *H* (in the Plan) at h, the repreſentation of the Angle H.

Draw hC cuting a Perpendicular from *I*, at j, and the horizontal Line j n gives the other external Angle, n.

For the Mutules, draw a Perpendicular from *S*, in the Plan (where they would meet if they were continued to the Diagonal *A*V) and from i in the Scale Line, MS, draw parallel Lines cuting it, at ſ; which muſt be returned at all the Angles, as if it was a continued Facia.

From *O*, *P*, &c. draw perpendicular Lines cuting them, in the ſeveral Points, *o*, *p*, &c. which are the front Faces of the Mutules. Draw *r* C, *q*C, &c. cuting the horizontal Line hh; where they fall againſt that Facia.

In the returning Side, draw Perpendiculars from *TV*, &c. till they cut ſC, the returning Facia, and ſo of all the reſt; which are beſt deſcribed by the Figure; obſerving that, if from the ſeveral parts, in the firſt or neareſt front of the perſpective Plan (*B AV*) perpendicular lines be drawn, and from the correſponding parts in the Section, or Scale Line, MS, parallel Lines cuting them; and, by carefully remarking the mitre Angles of each Member, on AV, the true perſpective proportions of them are carried round as many Breaks, in the Object, as are required; by means of the Vaniſhing Point C (the Center) for one return, the other parallel; the Originals of them being, in this Example, parallel to the Picture, and the Object right-angled.

The Mouldings around the Mutules are determined, at their projectures, from the Plan, above (at *O*, *P*, &c.) and the width, in Front, from the Section of the Picture, MS (at m) by means of which, and the Vaniſhing Point C, they are very eaſily continued around the Mutules, from one Angle to the next, adjoining. Firſt, drawing parallel Lines, from m, till they cut Perpendiculars from *O*, *P*, &c. at *e*, *f*, &c. and *f*C, *h*C, &c. cuting Perpendiculars from J, at *i* and *j*; through which they are drawn, parallel to the Angle at *k*, and from *k* they are continued, after the ſame manner, around the whole.

Every thing being correct, and very particular in the Figure, makes it unneceſſary to give a further deſcription of it. To deſcribe every Line in the whole Proceſs, would take ſeveral Pages, be extremely tedious and apparently prolix.

For if, as I have before obſerved, the Student be not acquainted with Architecture, nor underſtands the ſeveral Parts of the Order, it will be impoſſible for him to ſucceed in this Example; but if he is, it will be found ſufficiently intelligible.

The Architrave, of this Order, it is needleſs to ſay any thing about; ſeeing it is compoſed of plane Facias, only; projecting, one over another, like Steps, of different heights and projectures, and is managed in the ſame manner; which, from inſpection of the Figure, may readily be deſcribed.

The Frize and Triglyphs have nothing difficult in them, eſpecially in this Poſition. In the parallel Faces they are geometrically proportioned and ſpaced (Theo. 9. Cor. 5) and in the returning Side, their places are determined from the Plan above, or by the 10th Problem; dividing the upper Line of the Frize, st, in the ſame Ratio, perſpectively, as the front Line, ns, is divided, at o, p, q. (Prob. 8.)

At No. 4, I have given two Triglyphs, more at large; in order, that the Parts may be more diſtinctly made out; in which, after dividing the front one, as in the Profile, geometrically, for the flutings (which are right angled) the indented Angles may be obtained, as in the Plan ABD, above; by drawing a Line from each Angle *a* and *b*, to the two Diſtance Points, if they are within reach, cuting at *c*.

Or,

Or, if one Diſtance (*E*) only; biſect the width of the Flute, at *d*, and draw d c, to the Center, cuting *a E* at *c*, as before; *e f*, &c. drawn, to *E*, give the reſt.

For the Triglyph in the returning Side; having drawn the front Line, a d, to its vaniſhing Point, c; from each angle, *a*, *b*, &c. in the front Triglyph, draw lines to *E*, cuting a d, at b, c, and d.

Or, if it be convenient to have the Diſtance of the Eye on the other Side, it would be better; draw a h parallel to AB, and take the diviſions, from a, at e f, g and h, equal to the geometrical meaſures; and, from them, draw Lines to the other Diſtance, as in the Figure (ſuppoſing it within reach) which give the ſame Points, more accurately defined. (See Prob. 8.)

From the ſeveral diviſions, on each Side, draw Lines perpendicular, as in the Figure; by which means, the Triglyphs, X and Y, are compleated. The Perpendiculars, from *c* and *b*, (in the Plan, above) give the indented Angles in the front Triglyph; in the other they are not ſeen.

This operation is ſuppoſed either at the top or bottom of the flutings.

The inclined Lines, at the Top, are determined, truly, by drawing a Perpendicular at the middle point, *d*, making *de* equal to half its width, and drawing a line to c, the Center, cuting the Perpendicular from *c*, at *f*; join *af* and *bf*.

It is unneceſſary to be more particular, or their vaniſhing Points may be eaſily determined; by making the ſame Angle at the Eye, as the inclined Lines make, either with the horizontal or vertical Line (Prob. 4.) and where it cuts a Line, paſſing through *E*, perpendicular (the Vaniſhing Line of an inclined Face), is the Vaniſhing Point of *af*. Either being found, determines the other; and, the inclined Planes of Y being parallel to thoſe of X, have the ſame Vaniſhing Lines, reſpectively, paſſing through the Eye in the Points of Diſtance.

The Vaniſhing Points of the inclined Lines, in the front Triglyph, vaniſh below the Horizon, and thoſe in the returning one (Y) which are ſeen (at f, g, h) vaniſh at the ſame Diſtance above it, at ⊙.

Although this method will ſeldom be practiſed, yet I affirm it to be the beſt, being by much the readieſt and the moſt correct. Nor, is there any occaſion to be at the trouble of forming the Plan, to get the indented Angle; for, the Flutes being proportioned at *ab* (as above) and the Vaniſhing Points determined, the Lines, *af* and *bf*, being drawn to their reſpective vaniſhing Points, determine at the ſame time, the internal Angle.

The Tenia (V and Z) both above and below, breaks regularly around; as it may be ſeen, and would be needleſs to explain; ſimilar Subjects having ſo frequently been repeated. Such minutias are beſt deſcribed by the Figure, only, being accurately drawn, and well defined.

The Abacus (W) which may be ſuppoſed over a Column, is a Square; and the entire Capitals, over the Pilaſters, Y and Z, at the internal and external Angles, although they are detached Mouldings, are managed (from the Profile) after the ſame manner as if they were continued; their perſpective Plans being delineated, as above; from which, the doted Lines, correſponding with the Profile Section, (MS) determine their ſeveral Angles.

METHOD 2nd. EXAMPLE XVII.

How to repreſent the ſame thing without having the Profile drawn, or a perſpective Plan.

Let AB be the given height, which divide into two Modules, and into Minutes, as before (the reſt being ſuppoſed not drawn,) The Scale (A M) of the projecture of the Mouldings, &c. at the top, is alſo neceſſary.

From a, the determined Angle, draw a k perpendicular, and transfer all the meaſures of the ſeveral Mouldings, from the Scale, to a k; at b, c, d, &c.

Make a *a*, *ab*, &c. equal to the ſeveral projectures M N, N O, &c. and proceed as in the Plan, above; drawing a *E* for the diagonal Line of the top; and, from the ſeveral diviſions *a*, *b*, *c*, &c. draw Lines to the Center (C) cuting the Diagonal, at *e*, *f*, *g*, &c. from which, draw perpendicular Lines; and, from the meaſures on a k, draw diagonal Lines, to *E*; which give the ſame Angles c, h, &c. as before; from which they are carried around the ſeveral Faces.

Plate XIX.

The perſpective Plan, in this, is ſuppoſed to be formed on the Top of the Cornice, the ſame as above; and ſince a k is the Interſection of a Plane, paſſing thro' the Diagonal or mitre Angle of the Cornice (one Side, in this Caſe, being parallel to the Picture) *E*, the Diſtance Point, is the vaniſhing Point of that Diagonal, conſequently, all horizontal Lines, in that Plane (being parallel) vaniſh in *E*.

Wherefore, a *E*, b *E*, &c. are each, the indefinite Repreſentation of a Diagonal of a Square, in different horizontal Planes, whoſe Interſecting Points are a, b, &c. and, the diſtance of each Angle from the Picture, is a *a*, a *b*, &c. which being transfered, perſpectively, to the Diagonal a *g*, each Angle, in its proper place, will be perpendicularly oppoſite to the Points *e*, *f*, *g*, &c. as in the Example.

And thus may the whole be compleated, when we are a little verſed in Mouldings, by transfering the meaſures, perſpectively, from one Diagonal to another, which will be further illuſtrated in the following Example.

EXAMPLE XVIII.

How to repreſent a Cornice when it is inclined to the Picture, on both Sides.

If the laſt Example be tolerably well underſtood, in the laſt Proceſs, this will be found eaſy, being performed by the ſame means. It muſt be obſerved, that the firſt Method, reſpecting the Profile, cannot be applied here, nor in any Caſe; but when the Mouldings are parallel to the Picture. The extra Plan may be uſed in all Poſitions; but I ſhall in the following Examples, do without; as it is only ſuppoſing the Plan formed at the Top, from which the Perpendiculars are drawn, and the ſeveral Angles of the Facias and Fillets determined.

Fig. 87.

Let AEB be a Profile of the Cornice, to be drawn, and *AC* the Side of one Plane, on which it is to be projected.

ML is the Horizontal Line, S is the Center of the Picture, and L the Vaniſhing Point of *AC*; which is ſuppoſed the upper edge of the Plane from which the Cornice is to project. Let the Top be ſuppoſed a Square.

S being the Center of the Picture, and L the Vaniſhing Point of one Side of a Right Angle (the Diſtance of the Picture being known) find M the Vaniſhing Point of the other Side; by Prob. 12. SM is a third Proportional to SL and the Diſtance of the Picture (found by Prob. 32, Geo.)

Alſo, find N the Vaniſhing Point of the Diagonal; by making MN to NL as one Side of the Triangle (whoſe Perpendicular, at S, is the Diſtance of the Picture) to the other. As N, or O (Fig. 51.) GN, or EO biſecting the Angle FGH, or AEB.

Draw *A*M and *C*M, and *A*N cuting *C*M at *K*; draw L*K*, till it cuts *A*M at *D*.

ACKD is the Top, of a ſquare Priſm, from which the Mouldings project.

From each Angle, *A*, *C*, and *D*, draw perpendicular Lines, which repreſent the Corners of the Priſm; the Profile being proportioned to the neareſt, *AB*.

From *B* (*AB* being equal to AB) draw *B*L and *B*M, cuting the Perpendiculars, from *C* and *D*, at *G* and *H*. *ACGB* and *ADHB* are the Grounds or Seats of the Cornice on the two Planes, which are ſeen.

The Projecture of the Cornice, around the Priſm, is next to be determined.

Draw the Diagonal *DC*, and produce it both ways; alſo, produce N *A* indefinite, and draw *A*k parallel to the Horizon; on which, take the diagonal projectures of the Cornice, as on AK, in the Profile.

EK being made equal and perpendicular to AE; conſequently, AK, the Diagonal of the Square AJKE, of the projecture AE, is the mitre Angle of a Right Angle, for the Cornice AEB.

From the ſeveral Projectures, C, D, &c. draw CG and DF, parallel to AB, and produce them to the Diagonal, cuting it in H and I.

Make *A*h, hi, and ik reſpectively equal to AH, HI and IK; and having made NO equal to the Diſtance of the Vaniſhing Point N, of the Diagonal, draw Oh, Oi, and Ok, till they cut N *A*, produced, at *b*, *i*, and *E*; from all which, draw

draw Lines to both Vaniſhing Points, L and M, cuting the other Diagonal, *CD*, produced both ways, at *F*, *I*, *k*, *l*, &c. which form the perſpective Plan of the principal Mouldings. *EB*, *FG*, and *HI*, being drawn, are diagonal Lines of each Angle, from the lower edge of the Fillet, at the top, to the bottom of the Cornice.

Let it be obſerved, that N, being the Vaniſhing Point of the Diagonal of a Square, i. e. of a Line biſecting the Right Angle, is the ſame whether the Top be a Square or other Rectangle; but, if it be not a Square, *CD*, produced, would not be the Mitre of the other Angles; becauſe it would not biſect them; neither would *EN* paſs through the oppoſite Angle, or Mitre *KL*; yet N would be the Vaniſhing Point of both, becauſe they would ſtill be parallel. The others are alſo parallel, and conſequently, they have the ſame Vaniſhing Point; which, being much inclined, is at a great diſtance.

Having ſhewn how the Mouldings are projected forward, on the Diagonal *AE*, and transfered to the other Diagonal; ſuppoſe No. 2 the ſame thing, prepared in the ſame manner; being diveſted of ſeveral preparatory Lines, which, altogether, render the Work confuſed, and are ſuppoſed to be rubed out. Fig. 87. No. 2.

On *AB*, take *a*, *b*, and *c*, equal to the ſeveral heights of the Mouldings (as at a, b, and c, in the Profile) and, from N, project them (as N *a*, &c.) till they cut Perpendiculars, from *i* and *b*, in *d*, *e*, *f*, and *g*; and, from them, draw Lines to both Vaniſhing Points, L and M, cuting Perpendiculars, from *k*, *l*, and *m*, *n*, &c. at o, p, q, &c. which compleat the Facias and ſmall Fillets; and there remains only, to join them by curved Lines, of the ſame kind as in the Profile.

If the Mouldings are very large, as many Points may be found, in the Curves, as are neceſſary to deſcribe them with accuracy; by Perpendiculars from *AE*, and meaſures transfered to *AB*, from correſponding parts in the Profile. But, they may be as well performed by a careful Hand; for after all attempts at exactneſs, in ſuch minutias, a judicious Perſon, in Mouldings, and Perſpective, would deſcribe them as perfectly, regarding the Poſition in which they are ſeen.

Let it be obſerved, that the Curves are always flatter, in mitre Angles, than in the Profile, and more ſo, the leſs the Angle; likewiſe, in the diagonal Section, *AEB*, they are flatter than in *CFG*; becauſe, the Plane of that Diagonal is nearer to a coincidence with the Eye; for if it be in the Plane (however ſituated) they are Right Lines.

The Mouldings, in the diagonal Plane *AEB*, ſeeing that, the full meaſures are applied, on *AB*, and projected forward to *E*d, are larger than the Originals; that is, than the given meaſure; as *E*abcd, than *A*abc, on *AB*; equal Aab, &c. in the Profile. The Picture cuts the plane of the top in e f.

In which Caſe, if *AB* be ſuppoſed in the Picture, *A*ec*B* on one Side, and *A*fgi*B* on the other, are ſections of the Mouldings by the Picture; and, the whole, efg*B*, of that Section, being on this ſide of the Picture, is projected, to the Picture. *AEeB*, *CFpG*, and *HI*c, are diagonal Sections through the mitre Angles; and, jklm is a Section perpendicular to the Plane *ACGB*.

If what I have advanced be well underſtood, all that follows, reſpecting right-lined Mouldings, will be eaſy and intelligible, almoſt from inſpection of the Figures.

EXAMPLE XIX.

How to repreſent an Entablature with Modilions in the Cornice, obliquely ſituated to the Picture, having internal and external Angles.

ABC is the geometrical Profile of the Cornice; F is the front of a Modilion, and FG is its Profile. CD is the Frize, and DE the Architrave; with two Facias. Fig. 88.

In this Example, I ſhall ſuppoſe the whole Cornice to be beyond the Picture, and touching it at the Angle *A*.

AFG is the Interſection of the plane of the top, HL is the horizontal Vaniſhing Line, C is its Center, and CE the Diſtance of the Picture. H and L are the Vaniſhing Points of the Sides; i. e. of the horizontal Lines of the Mouldings; M and N are the Diſtances of thoſe Vaniſhing Points, for proportioning Lines which vaniſh in them, reſpectively; and O is the Vaniſhing Point of the Diagonal, biſecting the Angle, HEL, made by the Radials of the Sides.

Draw the vertical Interſection, *AE*, of the diagonal Plane, paſſing through the mitre Angle; and, transfer all the meaſures of the heights of the Mouldings, from the Profile, to *c*, *d*, &c. equal Bc, cd, &c. alſo, their projectures, to *Aa*, *ab*, *b*B. equal Aa, ab, and bB.

Plate XIX. Draw *A* L, indefinite; and from all the Points *a*, *b*, &c. draw Lines to N, cuting *A* L at 1, 2, 3; from which, draw Lines to H, cuting a Diagonal, A O, at *f*, g, &c. the perſpective Seats of the projectures; i. e. of the mitre Angles of the Corona, K, and the Facia at F, in the Profile.

From the ſeveral Angles, k, l, &c. obtained, as in the foregoing Examples, draw Lines to both Vaniſhing Points, H and L, indefinite; and proportion them to their reſpective lengths, by means of the Diſtances, M, and N, reſpectively, of thoſe Vaniſhing Points.

The length being determined, according to the number of Modi ions, contained, or otherwiſe; make *AD*, equal to the firſt break, and draw *D* N, cuting *A* L at f, the internal Angle; and draw fO, the Diagonal Line of that Angle, in the plane of the Top; indefinite.

Draw b L, cuting that Diagonal, at g; from which, draw a Perpendicular; and c L cuting it at e, and join e f; the Diagonal of the Projecture.

Draw H e, H f, and H g, indefinite; and, make f *F* to repreſent a Line in the proportion of the Original, of f *F* to *A* f; (by Prob. 10) or, draw M f, and produce it to the Interſection, at *B*; make *B F* to *A D* in that proportion, and draw *F* M, cuting H f produced, at *F*. (Prob. 17.)

Draw *F* O, cuting H g, produced, at h; from which, draw a Perpendicular, cuting H e, produced, at i; and draw *F* L, h L, and i L, indefinite.

Produce N *F* to the Interſection, and make *FG* equal to the length of the front Line of the Cornice; draw *G* N, cuting *F* L at m; and, *G* 6 being made equal to the projecture of the Cornice (A B, in the Profile) draw 6 N, cuting *F* L at n; and n H, cuting h L, at p; from which, draw a Perpendicular, cuting i L at q.

Fig. 88. Thus, are all the Angles at the top and bottom of the Cornice obtained; and, having ſet off the meaſure of the upper Fillet, at *A*, let it be continued around, by means of the Vaniſhing Points H and L; then draw diagonal Lines, *F* i, and m q, by which the Mouldings are all determined (as at A C in the Profile) having, before, obtained their places on the diagonal Line, *A* c, at the firſt Angle.

The perpendicular Lines of the Corona fall below it, from the Point where they are cut by the upper Line of the ſame, at k.

On the returning Side, on the left hand, the Cornice is ſuppoſed to fall againſt, or is cut off by a Plane, parallel to the front Planes, and conſequently has not a mitre Angle.

Draw *A* H, indefinite; make *A* a repreſent a length equal to what the whole projects from the Plane (by means of *A* f, Prob. 10; or, by its meaſure, Pr. 17.)

Draw a L and b H cuting at r; from which, draw a Perpendicular.

Draw c H cuting it at d, and draw a d, which will terminate all the Mouldings; as the diagonal Lines determine the mitre Angles; by drawing Lines from every Angle, on *A* c, to the Vaniſhing Point H.

The Architrave, *E*, *H*, *I*, *K*, being compoſed of plane Facias, with one Moulding, only, on each Face, needs no particular deſcription.

The Modilions are determined, in the ſame manner, as the Mutules, in Ex. 16.

Make *A* g, on the Interſection, A F, equal to A g, in the Profile, and draw g N, cuting *AD*, at o, and draw o H cuting the Diagonal *A* O, at *f*; and, from *f*, draw a Perpendicular, giving the mitre Angle, S, of the front Planes, of the Modilions. Let it be continued around the ſeveral Breaks (as the Corona) in Pencil Lines.

Draw *f* L, cuting the next Diagonal, at *l*. *f l* is the Seat, on the top, of the front of the Modilions on that Side; which may be continued around, from one Diagonal to the other, by means of the Vaniſhing Points H and L.

Make *A b* equal to A b, in the Profile, and draw *b* N, cuting *AD* at 2; and draw 2 H, cuting *f l* at *b*. From N, project the Point *b* to the Interſection, at *i*; and make *i j*, *j i*, *j k*, &c. reſpectively equal to the width of the Modilions, and to the ſpace between them, alternately, as often as is requiſite; from all which, draw Lines to the Point N, cuting the Seat, *f l*, in their perſpective widths, at *m*, *n*, &c. from which, draw Perpendiculars, giving the ſeveral Fronts, x, x, as in the Figure.

The Modilions yy and zz, in the returning Side and Front, are determined after the ſame manner, *viz.* thoſe in the Side, y, y, and alſo v, v, by means of the Point M, and the front ones, z, z, by the Point N; their meaſures being ſet off at 1, 2, 3, &c. on the Interſection of the Top, as thoſe at x.

Having thus obtained the Fronts, draw Lines to the reſpective Vaniſhing Points H and L, till they cut the Plane from which they project; and, having drawn Right Lines for the edge underneath, firſt, the curved Lines may be drawn by hand, as in the Figure, regarding the different appearance of each, as they recede.

If the Parts are large, and require to be accurately projected, the Moulding, around the Modilions, may be managed the ſame as in the laſt Example, by a perſpective Plan, above the Cornice. But, a judicious Perſon will ſave that unneceſſary trouble; for, having obtained their mitre Angles, as at s, t, u, &c. the front Moulding being drawn, as any other continued Moulding, they may be returned at the ſides, and meet the inner Moulding, ſufficiently correct without the trouble of planing them.

In this Example, there is all the variety that is requiſite for ſuch Cornices; but, as the Corinthian Modilion may, to ſome, appear more difficult than the Ionic, I ſhall give a ſpecimen how it may be delineated, as briefly as poſſible.

In the following Leſſon, I ſhall not deſcribe, over again, how the Mouldings are to be drawn; and, I have (in order to have the Modilions larger) omited the upper Mouldings of the Cornice, and the Corona; having retained only the Planceer; which I ſhall conſider as the Plane, on which the Modilions are ſeated, and AB the Interſecting Line of that Plane.

The Vaniſhing Points, H and L, are alſo the Vaniſhing Points of this Figure; the Center or Point of view is at S, and the Diſtance is ES. Fig. 88. No. 2.

Draw the vertical Interſection of the Diagonal, AF; and, by a geometrical Scale of the Proportions, ſet off the heights of the Modillions and Mouldings at *a*, *b*, &c. alſo, make ab, bc, &c. equal to the width of the Modilions and ſpaces between; and transfer them, by means of the Point P (the diſtance of the Vaniſhing Point H) to AB (the firſt doted Line) which is the Seat of the fronts of the Modillions; and finiſh the ſquare Blocks *efgh*, &c. which encloſe the Modillions, as in the former Example; by which means, they are truly proportioned, perſpectively, as in the Figure, which would otherwiſe, be ſomewhat difficult.

In the ſide, of each Block, muſt be drawn, by hand, the Profile of the Modillion, perſpectively, as it is repreſented at X (the geometrical Profile) according as they are contracted, or obliquely ſituated to the Eye.

In the Front, of each, is alſo deſcribed the end of the Scroll, and of the Leaf. In ſhort, having firſt obtained the Blocks or Caſes which contain them, the reſt muſt be delineated by a judicious hand and Eye; for ſuch Figures will baffle all Rules, nor is it poſſible to ſubject them to any, by which they may with greater certainty be deſcribed; except the Profile, in the ſide of each, as the Truſs in the next Example (No. 2) which is a ſimilar kind of Figure.

The Moulding breaking around the Modillions, is firſt deſcribed as other continued Mouldings, at the front, on which their mitre Angles are obtained, from the Interſection AB; the meaſures, on which, not being equal, on both ſides, is owing to the front Lines of the Fillets, being in another Plane, ſtanding forwarder than the front of the Modillions.

The Dentils have nothing particular in them; the proportion being known, of their width and ſpaces between; a Right Line drawn through either Corner, as fg, parallel to the Interſection, being divided in the Ratio, and Lines drawn to any Point, G, in the Vaniſhing Line, according as the meaſures are taken, greater or leſs, will give their meaſures perſpectively (as by Prob. 8) but, their true meaſures can only be applied, at the Interſection of the Plane they are in, with the Picture (drawn through C) and projected, by means of Viſual Rays, to the Eye, at P.

Plate XIX.

EXAMPLE XX.

How to repreſent a Door Head, with a Pediment, ſupported by Truſſes or Conſoles.

Fig. 89. Let *A* be the determined Angle of the Cornice of the Object, in the Picture.

At No. 1. is the true geometrical proportion of the whole, in Front.

Through *A*, draw *A*B perpendicular, the vertical Section of a diagonal Plane, paſſing through the mitre Angle, whoſe Vaniſhing Point is G †. H and L are the Vaniſhing Points of the front and ſide Lines, which are horizontal; S is the Center of the Picture, and SE its Diſtance; I and K are the Diſtance Points of each, reſpectively, for proportioning Lines which vaniſh in them ‡; EG biſects the Angle HEL; conſ. G is the Vaniſhing Point of horizontal, diagonal Lines. (Pr. 21.)

† Prob. 3.

‡ Prob. 17.

Theſe things, here premiſed or given, are ſuppoſed to be found and determined, from the known poſition and ſituation of the Object, as in the former Examples. (See Prob. 12.)

All the meaſures, from the Profile, are produced, or applied to the Interſection, *A*B; and, by means of the Diagonal *AG*, the mitre Angle of each Moulding is obtained, as in the foregoing Examples.

To determine the breakings of the Mouldings around the Truſſes, draw a *D*, the Interſection of the Plane they are in, above the Cimma reverſa; and, by means of the Point I, project the Point *a* to that Interſection, at a.

Make a b, b c, and c *D*, reſpectively, equal to the Breaks, and to the opening between them, geometrical (as at v u, No. 1.) and draw b I, c I, &c. cuting *a* L, at *b*, *c*, and *d*, their perſpective Proportions.

Draw *c* H, indefinite; and by means of the Point K (the Diſtance Point of H) project the Point *c* to the Interſection, at d.

Make d e equal to the returning Side, and draw e K, cuting *c* H, at *e*; and, through *e*, draw L *e*, cuting *b* H; which determine all the mitre Angles.

The Mouldings are deſcribed, or delineated, as in the laſt Examples.

To determine the proportion of the Truſs, or Conſole; of which X is the Profile, and Z the Front (No. 1.) Through D, draw a horizontal Line, D F, and produce H *f* (from H) to that Interſection, cuting it, at h.

Draw h o, perpendicular; and, from h, ſet off, geometrically, all the proportions of the Truſs, at i, k, &c. in reſpect of its height (as in the Profile at X) from which, draw Lines to H, cuting *f n*, at *k*, *l*, *m*, &c. from which they are transfered to the other ſide, by the Vaniſhing Point L, at *v*, *u*, *x*; and, by the Point H, projected forward; which proportions the heights in the other Truſs.

The geometrical projectures are perſpectively proportioned, in the ſame manner as the returning Moulding, at *c e*, by projecting the Point *f* (from K) to the Interſection DF, at 1, and making 1, 2, 3, equal to the projectures in the Profile; from which, draw Lines to K, cuting h H at *g*, *h*, &c. and from them, draw perpendicular Lines; by which means, finding as many Points as are neceſſary, the true perſpective form may be determined.

At No. 2, is ſhewn (more at large) how the Truſs is deſcribed, having all the ſame Letters of reference; and, in proportion to the other, as 3 to 2.

If the width be not already determined, by the Mouldings, draw I *h*, till it cuts the Interſection, D F, at f. H, I, K, and L (No. 2) are ſuppoſed to be the Vaniſhing Points, in the Horizontal Line, anſwering to thoſe below.

Make f h equal to the width of the Truſs, and draw h I, cuting *b* L at *i*.

In the Profile (No. 1) take as many Points, in the Curve (1, 2, 3, 4) as are neceſſary; from which, draw lines parallel to the Horizon, cuting h o, at i, k, l, and m; and alſo perpendicular. Let thoſe meaſures be transfered to h o (No. 2)

from which draw lines to the Vaniſhing Point H; and, having proportioned *fh*, perſpectively, as f h (No. 1) by the Interſection DF (where their geometrical proportions are transfered, at 1, 2, 3) draw perpendicular lines from the diviſions on *fh*, cuting the Lines from i, k, &c. reſpectively, at 1, 2, 3, and 4, through which, a Curve being deſcribed will repreſent the out-line of the Conſole.

From 1, 2, &c. thus obtained, draw Lines to L; by which means, the exterior Curve may be deſcribed, as in the Figure; and, alſo, transfered to the other Conſole, which muſt be deſcribed after the ſame manner.

To determine the Pediment. Having obtained the Angle *B*, its perſpective length (Prob. 17) equal twice A*A*, geometrical; at I (the diſtance of the Eye from the Vaniſhing Point L) make the Angle, LIV, equal to the Angle of the Pediment, cuting VL (the Vaniſhing Line of the front Planes, Prob. 3) at V, and ſet off an equal Diſtance on the other ſide L; from which Points, Lines drawn through *A* and *B*, determine the middle, at *C*; all the Mouldings vaniſh in thoſe Points.

The height and true pitch, or inclination, may be thus determined.

Make B*E* equal *A*B; i. e. make *AE* equal to twice the height of the Pediment. Draw *BF* alſo perpendicular, and draw *E* L, cuting it at *F*.

Draw *A*F and *BE*, interſecting at *C*; *ACB* is the true perſpective outline.

Or, having biſected *AB*, perſpectively at *G* (by Prob. 8) draw a Perpendicular; and, from B, draw BL, cuting it, at *C* as before; and draw A*C* and *BC*.

From all the Mouldings at B C, draw Lines to the Vaniſhing Point L, cuting *CG*; and the projectures of the Mouldings being alſo perſpectively proportioned, on *C*H, at r, s, t; perpendicular Lines, from them, will cut others correſponding, from *CG*, drawn to H, in the true mitre Angles; and, from thoſe Angles, draw Lines to both Vaniſhing Points (V) of the Pediment, which will give all the Mouldings, with accuracy, beyond any other method whatever.

The horizontal width of the Frize and Architrave, are proportioned to the Truſs and Mouldings, on *fn*; by drawing Lines to L; the perpendicular widths of the Architrave (being parallel to the Picture) and the opening of the Door, by the Vaniſhing Points of the Diagonals of a Square (as at Y) in the Vaniſhing Line VL of that Plane; regard being had to the projecture of the Mouldings (See Pr. 26.)

The receſs of the Door, pq, is determined (by Prob. 10) by its width, m p.

Produce the Radial LE; make EM equal to the width, and EN to the receſs; or, make EM and EN, in the ratio of one to the other.

Join MN, and draw EO parallel to MN. O is the Vaniſhing Point of a Diagonal of the Soffit, or head of the Door Caſe; and mO being drawn, cuts pH at q, as required. The reſt is obvious, from the Figure.

This Method, when it is applicable, is preferable to any other; for, having obtained the true proportion of one Line (mp) in any Plane, any other Line (pq) in that Plane is eaſily determined, by the 10th Problem. Indeed, it may always be applicable (provided, the Diagonal Line mq (i. e. MN) be not very oblique to the Picture; and conſequently, its Vaniſhing Point (O) very remote; for, if the whole Diſtance (SE) cannot be uſed, half, or any other portion, may be taken, with equal propriety; and the Point, O, aſcertained the very ſame (by Prob. 12.)

Otherwiſe; if no Line, in the ſame Plane, be found or determined, there is but one general Method; which was applied, to proportion the return of the Moulding *ce*, or the front Line at *b*, *c*, and *d*, *viz.* by the true meaſure applied to the Interſection of the Plane it is in (Prob. 17) which being frequently uſed, would be needleſs to repeat; particular regard being had to the true Interſection of either Plane, mpq, of the Soffit (which is horizontal) or of the vertical Plane, W, of the Door Jamb.

For, ſince pq is the common ſection of both Planes, it is conſequently in both; and therefore, either Interſection will anſwer the ſame purpoſe.

PQ is the Interſection of the horizontal Plane mpq; its true Diſtance, from any other, being known, equal DQ; a Line drawn through Q, parallel to the Horizon, is its Interſection with the Picture; for, all parallel Planes have parallel Interſections (8. 7. El.) And, if Hp be produced, till it cuts PQ; a Line drawn through P, perpendicular, is the Interſection of the vertical Plane W.

For, pq cuts the Picture at B; therefore, P is its interſecting Point (Def. K) conſequently, PR is the Interſection of the vertical Plane, W, that Line is in (Prob. 3.) and, PQ of the horizontal Plane, mq.

EXAMPLE

Plate XIX.
Fig. 90.

EXAMPLE XXI.

How to delineate a Block-Cornice, and to break the ſame, or any other, around a Bow Window; which is half a hexagonal Priſm.

HL is the Horizontal vaniſhing Line, C the Center of the Picture, and CE is its Diſtance; H is the vaniſhing Point of the Front of the Bow, and L of the Side of the Building, at right angles with it.

Draw EH and EL making a Right Angle (HEL) and, with any Radius, on *E*, deſcribe an Ark, *x*, *y*, *z*, cuting EH, at *x*.

Make *xy* and *yz* each equal to E*x*; and draw *Ey*, cuting the Horizontal Line, at M; and E*z*, which, would cut it, if produced; M, and N (the ſuppoſed Point where E*z* would cut HL, produced) are the vaniſhing Points of the Sides and Diagonals of the Hexagon.

Biſect the Angle *v* E*x*, (i. e. HEL) at *u*, and draw E*u*, to P; which is the Vaniſhing Point of the Diagonal of the Right Angle.

Let AD be the Interſection, of the neareſt Angle of the Object, with the Picture; and A the determined height of that Corner; let A a be the height of a Plinth, above the Cornice; and a b the height of the Cornice, of which, X is the Profile.

The Dimenſions of the Bow window, and other parts of the Building, being known, it would almoſt be ſuperfluous to deſcribe how it is to be determined in Perſpective; ſufficient inſtructions for that purpoſe are contained in the 4th and 5th Examples. However, as the application of them to real Objects, may not, to ſome, be familiar, I ſhall, briefly, deſcribe it.

At No. 1. is a Plan of the Window, to a Scale of half the meaſures applied in delineating; and about one ſixtieth part of the real Object.

a k is the poſition of the Picture, applied cloſe to the Angle a, and cuting the Building at the Angle c; wherefore, great part of the Cornice is projected to the Picture, ſeeing it is on this ſide, a k.

Draw A F the Interſection of the Plane of the Top, parallel to the Horizontal Line; make H D equal to E H (the Diſtance of the Vaniſhing Point H) and draw A H and A L indefinite.

Make A B equal to the ſhort returning Plane (V) which is at right angles with W, the long ſide of the Building (a h, continued) i. e. make A B equal twice a b in the Plan, and draw B D, cuting A H, at *B*.

Draw N*B* indefinite (Prob. 13) and having found the point P, the diſtance of N (by Pr. 12) draw P*B* to the Interſection A F, cuting it at *G*.

Make *Ge*, on the Interſection A F, equal to twice b c (No. 1) and draw P*e*, cuting N*B* at *C**. Draw *C*H, indefinite; and draw *C*D cuting A F at *d*; make *dF* equal to *Ge* (i. e. to the Front) and draw F D, cuting *C*H at *D*.

From all the Angles, *B*, *C*, and *D*, draw Perpendiculars, which give all the Faces of the Priſm, that can be ſeen (Y, and Z) i. e. b c, and c d (No. 1.)

As there is not room, on the Interſection, to ſet off the whole meaſure of the Plane W (equal four times a h, No. 1) take A G a third part of it, and L*E* one third of E L, and draw G*E*, cuting A L at *H*; from which draw a Perpendicular.

Thus, having determined the Building, we now proceed to the Cornice.

From X, the Profile, transfer all the meaſures, to A D, of the heights of the Mouldings, at a b. Draw the Diagonal P a; and determine the mitre Angle of the Mouldings (as in Example 17th) at a*a*b*b*.

* Becauſe the point *C* (in *B C*) is beyond the Interſection, A F, it is conſidered as projected to the Picture, its place being on this ſide, in the Original; as a k (No. 1) the Picture, cuts the Angle c.

Draw aH and bH, cuting *B*h, at c and h. Draw Nc and Nh, till they cut *C*g, at d and g; and draw dH and gH, cuting *D*f, at e and f. Alſo, from i, draw iL, repreſenting a Facia, below the Cornice; and iH, cuting *B*h, and continue it around each Face of the Hexagon.

The mitre Angles of the Hexagon are thus obtained.

In every regular Poligon, Right Lines drawn through the Center, biſect the Angles of the Poligon; and in a Hexagon, they divide it into ſix equilateral Triangles, each Diagonal being parallel to two oppoſite Sides. (See No. 1.)

*x*E*y* and *y*E*z* are two ſuch Triangles; conſequently, E*x*, E*y*, and E*z*, are parallel to all the Sides and Diagonals of the Hexagon †, producing the Vaniſhing Points, H, M, and N, of the Diagonals or mitre Angles. † Cor. 1. Th. 6.

But, the Angle, at b (No. 1.) where the Hexagon joins with the Building is internal, and its Mitre is g b, produced; conſequently parallel to f c; therefore, they have the ſame Vaniſhing Point (M)

a b is the Angle of the Building (which is a Right one) where the Mouldings are firſt projected; c h is the internal, and d g, e f, two external Angles of the Hexagon.

Produce Se, Sd, and Mc, indefinite; and draw H*a*, cuting Mc, produced, at *c*.

Draw N*c*, cuting Sd at *e*, and *e*H cuting Se, produced, at *f*; *a*, *c*, *e*, and *f*, are the extreme Angles of the Cornice; which being obtained, the reſt is managed as in the former Examples; by transfering all the meaſures, of the heights of the Mouldings, from a b to c h, and from c h to d g, &c. alſo, the projectures of the Mouldings, from the Diagonal a*a* to c*c*, from c*c* to d*e*, and from d*e* to e*f*; by means of the vaniſhing Points H and N; from which Diagonals, perpendiculars being drawn, give the Corona, at *b*, which is carried around in the ſame manner; and alſo the ſmall Moulding at b, h, g, f.

The Blocks, are obtained after the ſame manner as the Modilions, in Ex. 19th.

The internal Angle at q being obtained, as above, and the hither face of the firſt Block deſcribed (by means of the Vaniſhing Point L) through the outer Angle draw pq parallel to the Horizon, and ſet off geometrically, the number of Blocks and ſpaces between them, at 1, 2, 3, &c. from which, draw Lines to ſome point (P) in the Horizontal Line, cuting p*q*, at *g*, *h*, &c. the perſpective proportions of the fronts of the Blocks, on the Face Z.

On the other Sides, they may be determined after the ſame manner, or otherways; as on the Face Y, by their Seat, *BC*, at the top, projected to it from the Interſection AF; how they are finiſhed is beſt explained by inſpection of the Figure.

On the Side W, they are determined from their true meaſures, by drawing m n, through n, parallel to AF; m n is the Interſection of the Plane they are in with the Picture (which p q is not) therefore, having (from the point Q) projected the firſt to m n, at r, ſet off the true geometrical meaſures, from r towards m, at r, s, &c. and, by means of the ſame point Q, project them to rL; i. e. draw rQ, sQ, &c. cuting r L, which give their true places, on that Side. Fig. 90.

When the full meaſures, on n m, exceed the bounds of the Picture, make uſe of the half meaſures and diſtance, &c. as in former Examples.

The Blocks, in the returning Side, V, are determined, after the ſame manner, from the true meaſures; by means of the point D, the Diſtance Point of the vaniſhing Point, H, of that Side.

The Windows, and the Mouldings around them, are determined as the Door, &c. in the foregoing Example; by ſetting off the true ſpace and meaſure of the Moulding from I to k, and transfering them from one Plane to the other, as in the Figure.

G*e* and *d*F, on the Interſection AF, being the true geometrical meaſure of each Plane; divide them in the true meaſures of Piers, Mouldings, and Saſh Squares, at *i*, *k*, *l*, and *m*; from which draw Lines to the Points D and O, reſpectively, cuting *BC* and *CD*, at *n*, *o*, *p*, *q*; and, from them, Perpendiculars being drawn, determine their perſpective proportions on thoſe Planes, at *r*, *s*, &c. and the proportions of the Saſh Squares are projected to their true places, from *t* and *u*, by means of the Vaniſhing Points L and Q, reſpectively.

I cannot help remarking, here, that the apparent intricacy of repreſenting Mouldings, in Perſpective, with accuracy, has led ſome Perſons to imagine that the whole Work is intricate, and the Diagrams complicated; imagining they muſt neceſſarily go through the whole, to be acquainted with it. No ſuch thing is abſolutely neceſſary. If a Perſon be well acquainted with the Principles of the Theory, and the Elements of Practice, contained in the 3d, 4th, and 5th Sections of this third Book, he will find no myſtery in their application to any Object whatever; the different Examples, here given, ſhew how they may be applied, with ſucceſs, in various caſes, and under various circumſtances, which might not readily occur, to a Perſon not thoroughly converſant in it. They muſt not imagine, becauſe there are a multiplicity of Lines in a Plate, in which there are ſuch variety of Parts in the Object, that the whole is incomprehenſible; let them firſt make themſelves Maſters of the ſimple Leſſons and Examples, before they attempt a complex one, or it will be impoſſible for them to ſucceed in it. Can they imagine, that in ſuch a Subject, as the 86th Figure, where every eſſential part is minutely deſcribed, it can be done without Lines, by contemplating the Figure only? Or, more particularly, the three laſt Figures, in which, every horizontal Line is inclined to the Picture; every one of which muſt be perſpectively proportioned, to its place and poſition. Although the deſcription is full and explicit (the preceding Leſſons being underſtood) yet, to give a minute inveſtigation of the 89th Figure, would fill, at leaſt, half a dozen Pages, and would appear the moſt prolix and tedious deſcription imaginable.

Notwithſtanding the operative Lines in them neceſſarily croſs each other, yet, if they are traced to their proper References, that intricacy (which ſome object to) will no longer exiſt. It is obvious, that, to have avoided it, wholly, the Figures muſt have been much ſmaller, the Plates much larger, or their number greatly augmented, which would have enhanced the price, unneceſſarily. If they imagine that others have managed thoſe matters more ſimply, let them compare, and ſee what they can make of Pozzo, the only Author, amongſt the Italians, who has attempted it; nor has he attempted the deſcription, but the delineation only, and that, in the moſt eaſy poſitions; wherefore, his elegant Deſigns, convey no better inſtructions, than any other good Print, by inſpection. Mr. Kirby, in his firſt Eſſay, has run away from the Subject before he has well begun it; and, in his pompous Work, what has he done? nothing, to any purpoſe. Mr. Highmore, in his 37th Plate, has delineated a Cornice, at No. 3. tolerably, parallel to the Picture; but No. 4, being inclined, is intolerable, and his deſcription of it more ſo. In ſhort, a complicated Subject cannot be deſcribed without Words, nor can the operation be performed without Lines; although, as I have obſerved, in the Preface, not half the Lines, which appear in theſe Diagrams, are neceſſary to be drawn at all, in the operation.

SECTION VIII.

OF CURVILINEAR OBJECTS.

THE Perſpective of curve-lined Objects, the Subject of this eighth Section, is the moſt difficult, of all other; ſeeing that, Curve Lines cannot be projected as Right Lines, by means of interſecting and vaniſhing Points, indefinite; neither can any portion be taken, or cut off, perſpectively, otherwiſe than by drawing a Chord Line from one Point to another in the Original, and finding its Vaniſhing Point; or, by any means, finding the repreſentation of the extreme Points, from their correſponding Points in the Original.

There are various ways of projecting the repreſentation of a Circle, in Perſpective; all which, do no more than find the repreſentations of various Points in the Circumference. For, by the Theory of curvilinear Perſpective, it is ſuppoſed, that the deſcription or delineation of a Circle, or Sphere, in Perſpective, is ſome one or other of the Sections of a Cone; as in Fig. 28, Plate 7; it is obvious, that if Right Lines (EA, EB, &c.) from the Eye to the ſeveral Points A, B, C, &c. in the Curve (which is ſuppoſed to be a Circle) in the Plane Z; be cut by another Plane (X or Y) the Points a, b, c, &c. being joined, carefully, by a ſteady hand will generate a Curve (adg) which, to the Eye at E, will (as it is obvious it muſt) exactly coincide with the original Curve; ſeeing that, it is in the ſurface of the ſame Cone, of which, the Eye is its Vertex.

Hence it is manifeſt, that, the more Points there are found in the Repreſentation, the more exactly may the Curve be deſcribed; but after all, it depends greatly on the Hand and Eye; inſomuch that, without great nicety in both, the repreſentations

ſentations of curve-lined Objects will have a lame, and very diſagreeable appearance. It is, therefore, no wonder to ſee ſuch bad Repreſentations of round Objects, as are to be met with; but it is a matter of ſurprize, that any Perſon, who attempts it, ſhould have ſo little judgement as to turn the Curves the contrary way; or to make them flatter in thoſe parts, where, it is obvious that, they would have a greater curviture.

Without having the leaſt notion how to project Curves, perſpectively, is it poſſible for a Perſon, who has been uſed to ſketch at all, to place a cylindrical Object, of any kind, or a Vaſe, &c. before him, and not ſee, immediately, how the curves are to be deſcribed? or, what part of the Object appears more or leſs curved than others? is is not obvious to a common Eye? yet may we frequently ſee Vaſes repreſented, whoſe greateſt ſwell, at the top, is a Right Line, whilſt the Curves of the lower part take a contrary direction, to each other; indicating, that the Eye is between them; in which Caſe, the top would be the moſt curved. Would not ſuch attempts at Perſpective be better let alone? and content themſelves with a perfectly geometrical Repreſentation? in which they would all be Right Lines; ſave only, the external Figure of a vertical Section through the middle.

But, what can be ſaid for the performances of thoſe Artiſts, whoſe Works are an honour to their Country and to the Age they lived in, to ſee them, often, greatly deficient in thoſe particulars. I could wiſh to ſee the Works of the preſent Age more perfect; which, in other reſpects, ſeem to vie with the moſt celebrated amongſt the Antients; yet do not pay ſufficient attention to thoſe neceſſary Appendages, which are eſſentially requiſite, to a perfect Picture.

As a Circle is the firſt and principal of curve Lines, ſo it is the only one that can be reduced to any certain Rules, in delineating it perſpectively. And, of all the various ways to project the repreſentation of a Circle, in Perſpective, the beſt, and moſt practicable, is to ſuppoſe a regular Octagon to be inſcribed, or the Circumference divided into eight equal Parts; or, if very large, into ſixteen.

Irregular curved Objects are not Subjects for Perſpective; all attempts at a Spiral, or twiſted Column, &c. by Perſpective Rules, would be in vain. Various other Objects, as Rocks, Mountains, Rivers, Trees, &c. are not fit Subjects for Perſpective; and conſequently Landſcape Views cannot be taken or delineated by its Rules; becauſe it is impoſſible to have the true geometrical Figures and Proportions of ſuch Objects, as before mentioned. I ſhall however, in an Appendix, deſcribe the uſe and application of an Apparatus for taking Views, with eaſe and great exactneſs; the beſt calculated, for the purpoſe, of any I know or have ever heard of For, notwithſtanding what many imagine and affirm, of the poſſibility of taking Landſcape Views with accuracy, by Sight only, I know it is impoſſible to be done; and cannot conceive it to be any way derogatory to the abilities of the moſt eminent Artiſt, to make uſe of any expedient; by means of which, he may be enabled to make a more correct Portrait, and Picture. I do not mean that he ſhould, rigidly, deſcribe every minutia of the Objects, as in Trees, &c. by it; but I muſt affirm, that he would take the apparent Magnitudes of the ſeveral Objects, their Figures, and their Bearings in reſpect of each other, with much greater accuracy than it is poſſible to do by Sight.

PROBLEM I.

To find the Repreſentation of a Circle; the Original being given, in any Plane, whoſe Vaniſhing Line, its Center and Diſtance are given; according to Brook Taylor.

Firſt; by means of the Vaniſhing Line, and one Vaniſhing Point only.

Let AFG be a Circle, in the Geometrical Plane; of which *GK* is the Interſection, and V*E* the Vaniſhing Line; *C* is its Center, and *CE* its Diſtance. Fig. 91. No. 1

Plate XX. Fig. 91. Draw AB, at pleaſure, cuting the Interſection at a; and through C, D, G, &c. (Points aſſumed at pleaſure, in the Circumference) draw Lines parallel to AB, cuting the Interſection, at a, c, d, &c.

†Cor. Th. 3. Draw EV parallel to AB, producing the Vaniſhing Point, V, of thoſe Lines†.

Draw aV, cV, &c. and, to E, draw Viſual Rays from every Point, A, D, G, &c. cuting the indefinite Repreſentations of the Lines paſſing through thoſe Points, reſpectively, at *a*, *d*, *g*, &c.

Having thus obtained as many Points as are neceſſary, a Curve (*adifb*) deſcribed carefully through thoſe Points, will be an Ellipſis, and it is the true Repreſentation of the Circle AFG.

Or, having drawn the parallel Lines AB, &c. and their indefinite Repreſentations, as before; make V*E*. equal to VE; alſo, make ab equal to aB, ge equal to gH, &c. and draw b*E*, e*E*, &c. which will give the ſame Points as before.

This needs no Demonſtration; ſeeing that, the Points *a*, *b*, *c*, *d*, &c. are projected the ſame, as in Prob. 6th, where it is fully demonſtrated. And, that the Curve is an Ellipſis, is demonſtrated in Th. 2nd. Sect. 5; of Curvilinear Perſpective; for it is evident, and manifeſt, that the Viſual Rays EA, ED, &c. from the Eye to every Point in the Circumference, would cut the Picture in the correſponding Points *a*, *c*, *d*, &c. therefore the Curve *adf*, deſcribed through thoſe Points, is the true ſection of the Cone of Rays, and conſequently it is an Ellipſis.

Secondly; by the Directing Line and one Director. No. 2.

Fig. 91. No. 2. Let No. 2. be a Circle, nearly in the ſame Poſition to the Eye (at E) as before; having the ſame Interſection, *GKI*, and the ſame diſtance of the Eye (SE.)

Draw CD parallel to the Interſection, and diſtant from the Eye equal to the diſtance of the Vaniſhing Line (VL) from the Interſection (KI.) CD is the Directing Line of the plane of the Circle. (Def. 10.)

Draw AD, at pleaſure, cuting the Directing Line at D; and, from ſeveral Points, B, F, &c. in the Circumference, draw Right Lines to D, cuting the Interſection at c, d, e, &c. the Interſecting Points of thoſe Lines.

Draw ED; and, from the Interſecting Points, K, c, &c. draw Right Lines parallel to ED, which are the indefinite Repreſentations of thoſe Lines, reſpectively.

Draw the Viſual Rays EA, EB, &c. as before, cuting thoſe Lines at *a*, *b*, *f*, &c. and deſcribe a Curve through them, which is the true repreſentation of the Circle AFH. Draw VL, the Vaniſhing Line of the plane of the Circle.

Dem. Becauſe the Lines AD, BD, &c. have the ſame Directing Point (D) their Repreſentations are parallel between themſelves (Cor. 1. Theo. 14.)

Conſequently, ſeeing that Ka, cb, &c. paſs through the Interſecting Points, K, c, d, &c. parallel to ED, they are the indefinite Repreſentations of thoſe Lines; and conſequently, the Viſual Rays EA, &c. will cut them, in the ſame Points as before, which is manifeſt; for, if Ea, Eb, &c. be drawn, parallel to AD, BD, &c. reſpectively, their Vaniſhing Points, a, b, &c. are produced; by which, the affinity between the different Methods of producing the ſame thing is accounted for.

PROBLEM II.

To deſcribe the Repreſentation of a Circle, having the Repreſentation of one Diameter given.

Fig. 92. Let AB be the given Diameter; let *ECE* be the Vaniſhing Line of the Plane it is in; *C* is its Center, and CE its Diſtance.

Produce AB to its Vaniſhing Point, V; and biſect AB, perſpectively (Prob. 8) e. g. Draw A*bc* parallel to the Vaniſhing Line, and take two equal Diviſions A*b*, *bc*, at pleaſure. Through B, draw *c*B, and produce it to the Vaniſhing Line at G, and draw *b*G cuting AB, at C, the Center of the Circle; through which, draw DF, parallel to the Vaniſhing Line.

Make VG equal to VE, and draw AG, cuting DF at D; and, make CF equal to CD; or, through B, draw GF. DF is a Diameter of the Circle.

Make *CE*, on both Sides, equal to *CE*; draw *CC* indefinite; and, through D or F, draw *ED* or *EF*, cuting *CC* produced, at H; through which, draw ab parallel to DF; and, through D and F, draw *CD*, *CF*, cuting it at a and b.

Draw the Diagonals a*E* and b*E*; which will pass through C, the Center of the Circle, and cut a*C*, and b*C*, at d and c; draw cd, cuting *CH* at I; IH is a Diameter, and abcd is the representation of a Square, circumscribing the Circle.

Make EE^1, and EE^3, each equal to *EE*; from both which Points, draw Lines through D and F, cuting the Diagonals at e, f, g, and h; which Points are also in the Circumference.

Through the Points, A, H, f, F, g, I, h, D, and e, if a Curve be described, it will be the true Representation of a Circle, whose Diameter is equal to ab.

DEM. For, suppose ab the Intersection of the Plane the Original Circle is in; and *ECE* its Vanishing Line. *E*, *E* are the Vanishing Points of the Diagonals of a Square †, and the two Diagonals, ac and bd, cut each other in its Center, C ‡; which is, consequently, the Center of a Circle, inscribed. † Prob. 19. ‡ 16. 1. El.

And, because HI passes through C the Center of the Circle, HI represents a Diameter perpendicular to ab, seeing it vanishes in *C*; and DF, passing through C, is also a Diameter, parallel to the Picture.

Also, because CD is equal to CF, and EE^2 is equal to *EE*; E^2 and E^3 are the Distance Points of the Diagonals; and consequently, seeing CD is equal to CF, the diagonal Diameters, eg and fh, represent Lines equal to DF. (See Prob. 8th, and 10th, Case 3rd, for a further Demonstration.)

PROBLEM III.

How to describe the Representation of a Circle, in any Plane whose Vanishing Line, Center and Distance are given; and Intersection.

Let AB be the Intersection of a vertical Plane, and *ECE* its Vanishing Line; its Distance is *CE*. Let AB be the Diameter given. Fig. 93.

Bisect AB, at D; and draw CD perpendicular to AB.

Make DC equal to AD, and describe the Ark DEF, a fourth part of the Circumference. Draw AC, cuting the Circumference, at E; from which, draw a Perpendicular, Ee, to the Intersection.

Make Bc equal to Ae, and draw A*C*, B*C*, and D*C*; also c*C* and e*C*; and draw the Diagonals A*E* and B*E*, cuting A*C* and B*C*, at I and K; draw IK; and *FH*, through S, parallel to AB.

Through the Points, D, *F* and *H*, also through *a*, *e*, *b*, *c*, and *d*, where the Diagonals, and IK, are cut by c*C*, e*C*, and D*C*, if a Curve be described, accurately, it will be an Ellipsis, and the true representation of a Circle, viewed oblique.

If the Circle be large, and eight Points are not sufficient; bisect the Arks, DE and EF, at f and g; from which, draw Perpendiculars to AB; and make b and d, equally distant, as a and f, from D; or from A and B.

Draw a*C*, f*C*, d*C*, and b*C*. From *A* and *G*, &c. where the Diagonals are cut, draw *Af* and *Gg*, &c. parallel to AB, cuting f*C* and a*C* at *f* and *g*; and the others at *h*, *i*, *k*, *l*, *m*, and *n*, through which Points, the Representation will also pass, and may be described with more accuracy.

The affinity between the Original and the Representation, which the corresponding Characters particularize, is sufficient Demonstration. S represents the Center of the Circle, C.

This Method of projecting the Representation of a Circle is, of all others, the best and most practicable. It is nearly the same thing as Prob. 25th; for, the eight Points *a*, D, *e*, *F*, *b*, *c*, *d*, and *H* being joined by Right Lines, will be the Representation of an Octagon, which is circumscribed by the Circle; as, in the other, the Circle is inscribed, i. e. touches every Side, as this passes through its Angles

Plate XX. Fig. 93. Notwithſtanding, the Methods, in Prob. 1ſt, by Brook Taylor, are facile and ſimple, yet I believe, they are ſcarce ever uſed in Practice. If the Circle be large, and Diſtance adequate thereto, they are utterly impracticable; becauſe there is a neceſſity for having the whole Circle and Diſtance, at once, in their true places. Whereas, by the laſt, the Diſtance is applied on either, or on both ſides of the Center, as in all other Caſes whatever. Nor is AB neceſſarily the Interſection of the Plane of the Original Circle. For, if the place of the Circle be determined, on the Picture (either its Center, or the neareſt part of the Circumference, at D) a Line drawn through S or D, parallel to the Interſection, or Vaniſhing Line, anſwers the ſame purpoſe. AB or *FH* being made equal to the known Diameter, in that place, and a quarter of a Circle deſcribed, as DEF, of that proportion (which, it muſt be obvious, is as ſufficient as the whole) the reſt is as already deſcribed. Other Points, if requiſite, may be taken, as f and g, and more, if neceſſary.

In common Practice (in a Circle not very large) eight Points being ſufficient, there is no real neceſſity for deſcribing an Ark, geometrically; for e (or c) the Point where a Perpendicular, from E, cuts the Diameter, is diſtant from A or B ſomewhat more than one ſeventh part of the whole Diameter, AB; ſo that, in all common Caſes, it may be aſcertained near enough.

The 2nd Problem is uſeful in many Caſes. Having obtained a Line, AB, by any means, on the Picture; which being known to be the Diameter of a Circle, the whole Circle may be projected by the means there deſcribed; or by Prob. 10th, Caſe 3rd, with the greateſt exactneſs. Having obtained the parallel Diameter, DF, the affinity with this laſt Problem is diſcernable.

EXAMPLE XXII.

How to repreſent a plain, circular Arcade, caſually inclined to the Picture.

Fig. 94. Let VM be the Vaniſhing Line of the Horizon, C the Center of the Picture, CE its Diſtance, and A the Interſecting Point of the hither Angle of the Steps.

Through A, draw AJ parallel to the Horizon; which may be conſidered as the Ground Line; alſo, draw AG perpendicular, the vertical Interſection of a Diagonal Plane; on which, ſet off, from A, the ſeveral heights and proportions of the Object, at D, G, &c.

† Ex. 13. The near Angles of the Steps being firſt obtained, as at A, a, b†; their length may be acquired by Prob. 7th. If they exceed the bounds of the Picture, take any equal portion of their length, as AJ, one third part; alſo, V being the Vaniſhing Point of that Side (ſee Prob. 21, Meth. 3rd) and VE its Diſtance, make V*D* one third of VE, and draw J*D*, cuting the indefinite Repreſentation AV, at N. AN repreſents a length equal to thrice AJ. (Prob. 17.)

The Steps are finiſhed as in the 13th Example.

Next, the proportion of the Plinth, ec f, is determined by the ſame; and the others, gh, ik, and l, by Example 4; which, with the Piers, are ſo many Parallelopipeds, of equal magnitude and equally ſpaced. (See Example 4.)

Their meaſures are ſet off, at 1, 2, 3, on the Ground Line; and projected to their Seats, on the Ground, at 3, 4, 5, 6, &c. by means of the Point *E*, on one Side; the Diſtance, VE, of the other Vaniſhing Point (V) falls out of the Picture.

Their height, and the upper Border at H, are obtained, by their geometrical height, AD (as the Pedeſtal in Example 13th.) Thoſe above are perpendicularly over the Plinths below; the Band, or Caping on the Top, is determined from G; and, by drawing Lines to both Vaniſhing Points, V and Y.

The laſt is not in the Picture, and muſt be drawn by Prob. 13, or the Point muſt be aſcertained; where, EY, produced, would cut the Vaniſhing Line VM. *E* is the Diſtance Point, of the Eye, for the Vaniſhing Point (Y) of the Lines AY, &c.

To repreſent the Arches; which, in this Example, are inclined to the Picture (and conſequently, their Repreſentations are Ellipſes) the laſt Figure is adapted.

D, on the vertical Interſection AG, is the height of the ſpring of the Arch.

Make DE and EF equal to the meaſures on AB (Fig. 93) i. e. make DF equal to the height of the Arch (half the ſpace between the Piers, equal CF) and DE equal De (equal LE) and, by means of the Vaniſhing Point of the Diagonal (M) transfer them to the Angle of the Building, at H, I, K; or, if HK be the determinate height of the Arch, at that Angle, the reſt was unneceſſary.

From

From H, draw Lines to both Vaniſhing Points, V and Y, cuting the Piers, at *a*, *f*, m, n, &c. Biſect m n, perſpectively, at S ‡, the center of the Arch. ‡ Prob. 8.

Draw S p perpendicular, cuting a Line, from K to the Vaniſhing Point, Y, at p, the crown of the Arch; alſo, draw n o and m q perpendicular; m q o n is half a Square, circumſcribing the Arch.

Draw the Diagonals S o, and S q, cuting a Line from I, at r and s.

A Curve deſcribed through m, r, p, s, and n, is the front of the Arch, the Repreſentation of a Semicircle. If more Points are requiſite, ſee the laſt Problem.

The inner Curve is deſcribed as the outer; n *n* being the thickneſs of the Pier, obtained below; from C and d in the corner Pier.

From all the Points in the outer Curve, draw Lines to the Vaniſhing Point V; and, *m n*, the inner Diameter, anſwering to m n, being drawn, S V cuts it at *ſ* the Center; from which, the Diagonals *ſo*, *ſq*, being drawn, in the Rectangle *m q o n*, give the Points *r*, and *s*; and a Perpendicular from *ſ* gives *p*; then, a Curve drawn through *m*, *r*, *p*, *s*, and *n*, determines as much of the Soffit of the Arch as can be ſeen.

After the ſame manner, the Arches *a d f*, &c. are deſcribed, in the returning Side; which would be ſuperfluous to repeat over again; as the Lines themſelves ſhew their uſe and application the ſame.

If V E^2 be made equal to V E, on the Vaniſhing Line of the front Plane, E^2 is the Vaniſhing Point of one Diagonal (as *c e*) in each front Arch.

EXAMPLE XXIII.

To repreſent an upright Cylinder, of any given Dimenſions.

Let A B be its Diameter. The height of the Cylinder is ſuppoſed to be known. Fig. 95.
Let V be the Center, and V C the Diſtance of the Picture.

Deſcribe a Square A B D g (Prob. 19) and inſcribe a Circle a c e g (by Pr. 2 or 3.)

At the two Extremes of the Ellipſis, a and d, draw Perpendiculars, which repreſent Sides of the Cylinder. Their height muſt be determined by a Perpendicular either from A or b, or any Point in the Circumference.

Make A *A* and b *b*, each equal to the determined height of the Cylinder; the Point *b* is in the Circumference at the Top; and, if A C be drawn, it will cut Perpendiculars from a and e, at *a* and *e*, which are in the ſame Circumference. *b* V will cut a Perpendicular from f at *f*; and thus, as many Points as you pleaſe may be obtained, as *a*, *b*, *c*, *d*, *e*, *f*, *g*, and *h*, correſponding with a, b, &c. below; through which, the upper Curve may be deſcribed.

Or, having obtained any three Points, the Curve *a*, *b*, *c*, *d*, may be determined, by Prob. 11, with the greateſt accuracy.

The Plinth, A B D is parallel to the Picture; and the Abacus, *B* F G, is equally inclined on both ſides, each of which repreſents a Square (ſee Prob. 29 and 30) the Curves are ſtill the ſame; and if they are equally diſtant from the Vaniſhing Line (V C) they are equal and ſimilar Ellipſis.

It is obvious that, as the Diameter of the Cylinder depends on the Ellipſis, at either Baſe, it is very liable to error; as the Ellipſis is contracted or lengthened, it will be ſmaller or larger in Diameter. To obtain the true Diameters of ſeveral Columns, with accuracy, they muſt be drawn in their true geometrical proportions, as in the next Figure.

Let A B, D F, and G H be the Sections of three Columns by a horizontal Plane, Fig. 96.
in which the Eye may be ſuppoſed to be, at E; and, let *A H* be a Section of the Picture, i. e. ſuppoſe a Plane ſtanding upright on *A H* parallel to the Columns; E C perpendicular to *A H* gives its Center, and Diſtance.

The

Plate XXI. The Viſual Rays EA, EB, &c. being drawn, are in the ſame Plane; and the Points *A*, *B*, *D*, &c. where they cut the Picture, it is evident, are the places, where the apparent edges of the Columns will appear, and conſequently, *AB* is the apparent width of the Column AB, *DF* of DF, and *GH* of GH; which, on account of the Rays, EG, EH, interſecting the Picture more oblique than thoſe from DF, has a larger Repreſentation; notwithſtanding it is really further from the Eye, and from the Center of the Picture, at C.

If the Eye be moved to *E*, the doted Lines ſhew the difference in their Proportions, from the two Stations or Points of View; where, the difference between the Repreſentations of DF and GH are much larger than at E.

If the Picture be at a h, parallel to *AH* (the Eye being at E) the Proportions of the Repreſentations are in the ſame Ratio, as their Diſtances, E c to E C.

The true poſition of the Picture from that Station, is *a h*, whoſe Center is at *c*, E *c* perpendicular to *a h*, biſects the Optic Angle, *a* E *h*; which Optic Angle, of the other Pictures, is i E h; c i being equal to c h.

And thus the Diameters may be obtained on any Picture, ſituated to the Columns in any Angle, at pleaſure as (*HAK*) the apparent Diameters on *AK*, are where the Viſual Rays cut it, at *A*, *i*, *k*, *l*, *m*, *K*.

See this matter more fully treated, in the ſixth Section, of the Theory, Fig. 34.

EXAMPLE XXIV.

How to repreſent ſeveral Cylinders, in the ſame Right Line; inclined to the Picture, at pleaſure.

Fig. 97. Let X, Y, and Z, be the Seats of three Columns, on the Ground Plane, between the parallel Lines, AB and CD; inclined to the Picture in the Angle BAF, at diſcretion. AF is the Interſection; and HL, the Horizontal Vaniſhing Line; C is the Center, and CE the Diſtance of the Picture.

Find the Vaniſhing Point, H, of the inclination of the Line of the Columns, AB (Prob. 17.) and make HEL a Right Angle; which biſect by the Right Line EO. L is the Vaniſhing Point of the other Sides of the Squares; in which the Columns, i. e. their Plans are inſcribed; and O of the Diagonals.

It would be ſuperfluous, after ſo many Examples of the ſame kind given, to ſhew how the Repreſentations of thoſe Squares are obtained; either by applying their ſeveral Diſtances from A to a, b, c, &c. (as in Ex. 4) or, as the Square *ABFD* is found (Prob. 19) ſeverally, at diſcretion.

The doted Lines ſhew how the laſt is managed; and it is the beſt, when there are Circles to be inſcribed; as they are repreſented at *af*, *cg*, and *eh*.

Which being obtained, ſeverally, by the laſt Example, draw AK, perpendicular, and equal to the height of the Columns.

Find the correſponding Squares of the tops of the Cylinders, in which inſcribe Circles perſpectively †, and draw perpendicular Lines, *ai*, *cl*, *em*, &c. repreſenting the apparent edges of the Cylinders.

† Prob. 3.

If they were to repreſent Columns, diminiſhing at the upper ends, the Squares containing the Circles muſt be repreſented leſs, in proportion to their reſpective Diameters; and the Lines, which are, in this Caſe, perpendicular, will be gently curved; having obtained the Diameters above and below, a curved Ruler is the beſt expedient, for drawing the apparent Lines of the ſides of the Columns.

Now, although this method of obtaining the apparent Diameters, if it be accurately performed, is ſtrictly true; for, the repreſentations of their Baſes and Tops being true Ellipſes, there cannot poſſibly be any error; yet, the performance of it is liable to great error, becauſe it is impoſſible to deſcribe the Ellipſis accurately; and therefore, the Diameters cannot, by this means, be truly obtained; eſpecially at the bottom, being ſo near the Vaniſhing Line; but, much more accurately, by the laſt Example.

Then, ſince EC is the Diſtance of the Picture, produce it to the Interſection, at G; make GS equal to EC, and conceive the Picture to ſtand erect, on its Interſection AF, and S to be the Station Point, at the foot of the Spectator; for SG the Diſtance of the Picture is equal to EC, from the Eye to its Center; the ſpace between the Interſection and the Horizontal Line not being a part of the Diſtance, ſeeing they are both on the Picture. (See Fig. 37, No. 1.)

It is equally the ſame, if S be conſidered as the Eye, and AF the Horizontal Line; and ſuppoſe X, Y, and Z, ſections of the Columns by the Horizontal Plane.

Right Lines being drawn to S, as the Station Point; or being, conſidered as the Eye, in the Horizontal Plane, are Viſual Rays to the extreme edges of the Columns; which, by their Interſections with the Picture, at 1, 2, 3, 4, 5, and 6, give the apparent proportions of the Columns, on the Picture; from which, perpendiculars being drawn, give their true places, and repreſentative Diameters, beyond any other means whatever.

EXAMPLE XXV.

How to repreſent the Tuſcan Baſe, its Plinth parallel to the Picture, and ſeen obliquely.

C is the Center, nearly as in the laſt; but the Diſtance is greater, equal CH: the Horizontal Line is the ſame; and the Interſection, or Ground Line, is AB. Fig. 98.

At X is a Profile of the Baſe, conſiſting of one Torus and Plinth.

Being parallel to the Picture, the Plinth, AabB, in Front, is geometrical; compleat the ſquare of the Baſe, perſpectively†; and, within that, another Square (abcd) equal, in width, to the Circle at the bottom of the Torus, in which deſcribe an Ellipſis‡; the nearer part, efg, only, will be ſeen. † Prob. 19. ‡ Prob. 3.

Produce the corner of the Plinth, from A; AJ is conſidered as the Interſection of the Diagonal Plane. Make AJ equal to the height of the Fillet above the Torus; and having made a Perſpective Plan below (the geometrical proportions, from the Profile, being ſet off, from A and B, as A*a*, B*b*) from the Angles *A*, *B*, and *D*, draw Perpendiculars *A*I, *B*K, and *D*L.

From J, on the vertical Interſection, draw JH, cuting *A*I at I; alſo draw IK parallel to the Ground Line (AB) and compleat the Square KL, of the Fillet; in which, deſcribe the Ellipſis *def*, and another parallel to it, perſpectively, equal to its width.

Make AG equal to BE; or, through E, the middle of the Torus, draw EG, and compleat the Square EF; in which deſcribe an Ellipſis, as at *ghik*,, which gives the greateſt ſwell of the Torus; and, if greater accuracy be required, deſcribe another Ellipſis, above that; and thus, as many repreſentations of Circles as you pleaſe being deſcribed, the Curves, *dg*e, and *fk*g, deſcribed over their extremes, falling into the Ellipſis on the Plinth, at e and g, will be the true Contours of the Torus, in Perſpective.

Although I can ſcarce ſuppoſe that any Perſon would go through this proceſs, who would delineate a Baſe; yet is there no better or readier way of doing it, w.. certainty. Nevertheleſs, this diſſection of it will greatly aſſiſt the imagination; and is the beſt method of acquiring a true knowledge of the nature of curvilinear practical Perſpective.

hik, at the bottom of the Plinth, is an Ellipſis, which is the perſpective Seat of the Shaft of the Column; from which may be obtained the edges of the Column, at MN; but it is of no other uſe, ſeeing the Curve of the Ellipſis, at MN, is very different from that below, on account of its being nearer to the Vaniſhing Line; in which, the Curve would be loſt totally.

On acount of the oblique ſituation of the Eye, in reſpect of this Object, it muſt neceſſarily appear diſtorted, when the Eye is oppoſite to it; but, let the Eye be placed oppoſite to the Center, at C, and at the Diſtance CH, the Eye being then in the true Point of view, it will appear perfectly round, and without any diſtortion.

Plate XXI.
Fig. 99.

EXAMPLE XXVI.

How to repreſent a Doric Capital, inclined to the Picture, caſually, or at diſcretion.

S is the Center of the Picture, and C is the Vaniſhing Point of one ſide of the Abacus; the other is out of the Picture; the Diſtance is equal to S L.

E is the Eye, i. e. the Diſtance of the Vaniſhing Point of the other Side (found by Prob. 12) and F is the Vaniſhing Point of a Diagonal, biſecting the Angle.

Let A be the place of the neareſt Angle, in the Picture; and let Z be the geometrical Profile. AB is the Interſection of the plane of the top; and it is the meaſure of the ſquare of the Abacus.

A *B* being drawn indefinite, draw BE cuting it at *B*, and draw AC, and *B*C, and a Diagonal AF, cuting *B*C; compleat the ſquare of the Abacus (Prob. 21.)

Draw AD the Interſection of the mitre Diagonal, and transfer all the meaſures from the Profile to AD; from which, draw Lines to F; and having got the Points *c* and *e*, on the Diagonal AF, the Seats of the Projectures of the Facia, Z, and the Fillet at x (obtained as uſual, by ſeting off the meaſures at *a* and *b*) from *c* and *e* draw Perpendiculars, giving the Points a and e.

Compleat the Squares cd and fg, in which deſcribe Ellipſes; ab being drawn parallel to AB, may be conſidered as the Section of the under Face of the Abacus.

From E, or any other Point in the Horizontal Line, draw Ec to b; biſect ab, at *d*, and make a 1, b 2, each ſomewhat more than one ſeventh part of ab; or with the Radius a *d* deſcribe an Ark of a Circle, and get the Points 1 and 2, as by Prob. 3rd, by which means, the Ellipſis, i. e. the Points *e*, *f*, *g*, *h*, *i*, by which it is deſcribed, are obtained.

After the ſame manner, in the Square, efg, of the fillet at x, deſcribe the Ellipſis j, *k*, *l*, and another parallel to it, equal to the width of the fillet; and alſo, another above that, for the thickneſs of the ſmall Bead, at x; which, being ſo little diſtant from each other, will not vary much in their Curves; otherwiſe, every different repreſentation of a Circle muſt be ſeverally deſcribed, as the foregoing.

As it is ſomewhat more difficult to get the neceſſary Points, when the Square is inclined than parallel; let the remainder of the Curves be thus obtained.

Through y draw a Line parallel to AB; and (becauſe this Object is viewed centrally) where it cuts the Vertical Line, at i, ſet off if and ik equal; making fk equal to the Diameter of the fillet, at y, geometrical; draw fS and kS.

Prom the Point j, where fd cuts AD, draw jF, cuting iS at o, the repreſentation of the Center of that Circle; and, through o, draw Lo, cuting fS and kS at h and m; through which, draw gh and lm parallel to fd; ghlm is the Square of that Circle, in which an Ellipſis muſt be deſcribed, as before; with the Fillet and Aſtragal, as was done above; particular care being had, to make them lead true, at the extremes, as if they were continued around.

The Square of the ſection of the Shaft (pq) is obtained in the ſame manner; and, an Ellipſis being deſcribed in it, from the Extremes of which, perpendicular Lines are drawn, for the Shaft or apparent edges of the Column. How they fall into the Cavetto is impoſſible to deſcribe, ſeeing, the Line is loſt inſenſibly.

The large Ovolo can only be deſcribed, truly, as the Torus, by means of various repreſentations of Circles, and deſcribing, carefully, a Curve over the extremes of them all; for it muſt be obvious, that the contour of ſuch Mouldings, though circular, is not in a Plane, but is continually changing its place, from the upper Circle to the lower.

Let it be obſerved, that the Curves are the ſame, whether the Squares, in which they are deſcribed, be parallel to the Picture or oblique, in any inclination whatever.

EXAMPLE XXVII.

How to repreſent a large Crane, or Water Wheel, vertical.

Let ABCD (No. 1.) be the geometrical figure, and proportion of the Wheel; and S its Center, or Axis. Its Poſition is accidental, as the vertical Line, bg paſſing through the Center, indicates. Fig. 100.

LetC be the Center of the Picture, and CE its Diſtance, in the Horizontal Line.

Let AB be the Interſection, of the Plane of the Wheel, with the Picture; which, the Wheel is ſuppoſed cloſe to; and let BD be the Ground Line.

Transfer the diameter of the Wheel to the Interſection, AB, at *FG*.

Draw CV perpendicular to the Ground Line, which is the Vaniſhing Line of the Face of the Wheel. Make C*E* equal to CE; draw *FC* and *GC*; and *FE*, cuting *GC* at *H*, and draw *HI*, parallel to AB.

FGHI repreſents a Square, containing the Wheel; in which, deſcribe the Ellipſis, abcd; and within it another, repreſenting the Rim of the Wheel.

Make B*K* equal to its determined width, and draw *KL* parallel to AB, by which deſcribe the Curve *OPQ*, repreſenting the cylindrical Surface of its convexity; and there may be two other Curves deſcribed, equal in width, repreſenting the breadth of the Rims, and another, ſhewing the thickneſs of one.

The Levers, or Vanes, a, b, c, d, &c. (No. 1.) may be thus obtained.

Make A*G* and B*F* each equal to their width; and draw AC and BC; produce *EF*, cuting CB, at *C*, and draw *CD* parallel to AB, cuting CA, at *D*.

On *CD* deſcribe, perſpectively, the exterior Square, *CDHI*, in which deſcribe an Ellipſis, by a Pencil Line; which, with the interior Curves, repreſent concentric Circles (as ABCD, abcg, No. 1) this laſt will limit all the extremes of the Levers. Their places are found, by drawing perpendiculars, from each, to the Ground Line; as ip, hq, &c.

Draw a Perpendicular from the Rim, at J (No. 1) cuting the Ground Line at P; from which transfer all the meaſures Pp, Pq, &c. (where perpendiculars from the Levers, at i, h, &c. cut it) to B*p*, B*q*, B*g*, &c. from which, draw Lines to E, cuting BC at 1, 2, 3, &c. from which draw Perpendiculars, cuting the outer Ellipſis at i, h, &c. as in the Figure, where the correſponding Characters particularize the ſeveral Levers.

Any one (as k) being obtained, a Line drawn through S, which repreſents the Center of the Wheel, gives its oppoſite, at *d*.

By the ſame Method the croſs Timbers, &c. may be obtained.

This Method, though the ſame as the old Authors practiſed, is not repugnant to the new Principles, by Brook Taylor; and may do, accurately enough, in common Caſes, and where room is wanting; but, as a contraſt, I will alſo ſhew, how much more ſimple, maſterly, and correct, they may be obtained, by means of their reſpective Vaniſhing Points; and, when they are not very remote, Interſecting Points.

AB was conſidered as the Interſection of one Plane of the Wheel, with the Picture. A Right Line, CV, drawn through C, the Center of the Picture, parallel to AB, is the Vaniſhing Line of them both, and C is its Center.

The Horizontal Line is of no uſe, in this operation; no regard being had to any other Vaniſhing Line or Interſection, whatever, but that of the Plane of the Figure.

CE, perpendicular to CV, the Diſtance of the Picture, is alſo the Diſtance of this Vaniſhing Line, which, is the Vertical Line of the Picture; and E is the true Place of the Eye, for that Plane. (Theo. 4.)

Find the Vaniſhing Points, D and V, of the Inclination of the Timbers, parallel to AC, and BD (No. 1) i. e. make the Angles CED, CEV, reſpectively equal to the known or determined Inclination of AC, and BD, with the Horizon †; cuting the Vaniſhing Line at D and V, the Vaniſhing Points of thoſe parallel Timbers. † Prob. 2. Sect. 3.

Or,

Plate XXII.

Or, having found either, make DEV a Right Angle, it will give the other; becauſe the Timbers are at right angles with each other. (Prob. 4, Sect. 3.)

Biſect the Angle DEV by the Right Line EF; F is the Vaniſhing Point of the Timbers, E, G, &c. (No. 1.) which make half right angles with the other. In the ſame manner, i. e. by biſecting the Angles all the other Vaniſhing Points, G and H, &c. of the Levers are acquired; which, ſeeing they tend to the Center of the Wheel, may be drawn without more preparation; as GS and HS, &c. which gives the Levers, at i, *i*, and a, *k*, &c. and FS gives the Timbers at E and G.

The Timbers at I are parallel to them, and have the ſame Vaniſhing Point; the others, at O, F, and H, are at right angles with them. Make FEY a Right Angle, and they will tend to that Point, where EY would cut the Vaniſhing Line.

As the croſs Timbers, do not paſs through the center of the Wheel, they may be thus obtained; with the greateſt accuracy.

Let PJ, at the farther extreme of the Wheel (No. 1) be produced, and produce the Timbers, &c. till they cut it, at M, and N, &c. PQ may be conſidered at the Interſection of the Picture, coinciding with AB, or K*L*.

Transfer the interſecting Points M, N, &c. to AB and *KL*, at v, x, y, and z.

Draw v D and x D, giving the repreſentations of the Timbers, parallel to AC; whoſe thickneſs are obtained at v 1, x 2. y D and z D give thoſe on the other ſide of the Wheel, as they are ſeen between the hither Timbers.

From A draw a Line through the Center S, which gives the two Vanes, at n and e, whoſe Vaniſhing Point is not in the Picture, nor neceſſary.

The Interſecting Points of the other croſs Timbers, parallel to BD, are below the Interſection, BD, which, if they were remote, could not be acquired; wherefore, from or through either of the extremes of thoſe which are obtained, draw *r* F and *s* F cuting the inner Curve of the Rim at *t* and *u*; and, through them, draw V *t*, and V *u*, which give thoſe Timbers.

The Levers, it is obvious, are all between the two Ellipſes, as their Originals are between the Circles, at No. 1.

The Points a, k, i, &c. in the hither Front, being obtained, draw a *a*, i *i*, &c. parallel to the Horizon; for the Front is, at right angles with the Picture.

The Supporters, or Stays behind them, are beſt deſcribed by the Figure; to particularize every minutia, would take above another Page to little purpoſe. The ſquare of the Axle being obtained, at S, the Sides are parallel; and the Timbers at I and O form a Square, whoſe Vaniſhing Points are F and Y.

Or, thoſe at I may be produced to the Interſection KL, cuting it at J and K; and one of the other, at Q. The reſt is obvious, on inſpection.

EXAMPLE XXVIII.

How to repreſent circular Steps, in Perſpective; with a round Pedeſtal on the Step.

Fig. 101. Let CE be the Horizontal Vaniſhing Line; C the Center, and CE the Diſtance; and let AB be the meaſure of the firſt Step.

Deſcribe a Square ABDF (Prob. 19th) in which inſcribe the Ellipſis acbdefgh. Make c G equal to the height of a Step, as at X (No. 1) and by drawing Perpendiculars, a 1, b 2, &c. and, from *A* and *B*, Diagonals cuting them, the lower Curve may be deſcribed.

Draw AI or GH, perpendicular; and on them, from A or c, ſet off all the meaſures of as many Steps as are required, *viz.* *A*A, A*C*, or Gc, c i, equal (as in the Profile, for two Steps) and from them draw to E, or C, as in the Figure.

Find the Square of the next Step JKLM (Ex. 11.) in which deſcribe the Ellipſis *abcd*, &c. as before; and the lower Curve, *ghikl*, may be obtained, likewiſe, in the ſame manner. And thus, as many Steps may be repreſented as are required.

N. B. AB is not neceſſarily the Interſection of the Picture, but parallel to the Interſection; and AB the known or determined meaſure of the firſt Step, in that place.

To repreſent a circular Pedeſtal, on the Steps.

At No. 1 is the Profile of the Pedeſtal; from which, tranſfer all the meaſures of the Plinth, Mouldings, &c. to the vertical Interſection, GH or AI, or both; the one conſidered as the ſection of a vertical Plane perpendicular to the Picture, thro' the Axis of the Pedeſtal, and the other a diagonal Section. Fig. 101.

Make *CD* equal to the Plinth of the Baſe; CI to the height of the Cornice, and at 1, 2, 3, &c. ſet off the proportions of the Mouldings.

The ſquare of the Plinth being obtained, at NO, and an equal one at PQ, in both which deſcribe Ellipſes; as, for the Steps below.

The other Curves muſt all be obtained after the ſame manner, by inſcribing Ellipſes in Squares; as pq is a Side of the ſquare of the Facia, rs of the bottom line of the Cornice, and t u, of the upper Line of the Baſe Moulding.

It is obvious, that, as the plane of the Curve falls nearer to the Vaniſhing Line, the Curve is more excentric, i. e. flatter; as in right lined Objects, every ſection, parallel to the Baſe, becomes more obtuſe and acute angled, the Figure being more contracted, as it approaches nearer to its Vaniſhing Line; in which it is wholly loſt to ſight, ſeeing that, the Eye is in the Plane of the Figure, and conſequently, its whole repreſentation is in the Vaniſhing Line. Yet, there is no poſſibility of deſcribing them in another Plane either above or below, ſo as to be of any ſervice, in reſpect of the Curve, as every Curve, the farther it is removed from the Vaniſhing Line of the Plane it is in, becomes more curved, i. e. it is more convex; or, as workmen phraſe it, quicker, or more acute; which implies, that it falls off from a Tangent, more ſuddenly.

From what has been deſcribed above, in delineating round Objects, it appears nothing more than drawing a ſquare one, of the ſame dimenſions, and inſcribing an Ellipſis in every repreſentation of a Square.

It muſt be obvious to every Perſon of diſcernment, that, whereever the Eye is ſituated, in reſpect of a convex Object, as a Vaſe, &c. (Fig. 102) the Vaniſhing Line of the Plane of the Circles (CE) cuting the Object, the Curves abc, def, &c. or *abc*, *def*, &c. on either ſide of the Vaniſhing Line, are always concave towards it; and conſequently, if a concave Object be repreſented (as the Abacus, of the Ionic or Corinthian Capital, Ex. 30 and 32) the Curves will be convex, towards the Vaniſhing Line. It is alſo manifeſt, that if the Curves, that is, if the Planes they are in, be equally diſtant from a Plane paſſing through the Eye, parallel to them, being equal and ſimilar Curves, of any kind, and perpendicularly one oppoſite the other, their Repreſentations will alſo be equal and ſimilar. Fig. 102.

How is it poſſible then, for any Perſon, who has any pretenſions to the Art of delineating, to run into ſuch groſs abſurdities as are exhibited in No. 2. I have ſeen a Vaſe ſo repreſented, on the Pannel of a Coach Door, which was very well painted, and, in other reſpects paſſable.

The Plinth, AB, was parallel, and the Side, BCD, was ſeen; but, inſtead of the Lines, in that Face, tending to a Vaniſhing Point, they inclined to each other towards the Eye; and conſequently, gave the Plane, ABC, the appearance of inclining, or aſcending. The Mouldings, at E, where curved contrary to the Baſe, which plainly indicates the Horizontal Line, HI, to paſs between them; and, the Curves, being regular, alſo evinces the Eye to be at E, and conſequently the Face, BCD, of the Plinth, could not be ſeen. No. 2.

The Lines at FG, the greateſt ſwell of the Vaſe, were Right Lines, which alſo evinces the Eye to be on a level with them; whilſt the nulled, or fluted part, abc, was a parabolic Curve, riſing as in the Figure; and the Top, at K, gently curved.

Is it not ſurprizing, that any Artiſt ſhould be ſo regardleſs of his Reputation, as to ſuffer ſuch a medley of inconſiſtencies to go out of his hands; and to palliate it by ſaying, in ſuch things, it is not worth the while to ſtand conſidering about it. I grant it would not be worth the while, to delineate the whole by the Rules of

Plate XXII.

Perſpective; but ſurely, the Lines in the Plinth are as eaſily made to incline the right way, as wrong; and if, by curving the horizontal Lines of the Vaſe, it was intended to give an appearance of roundneſs; would not that be better effected by curving them properly? it would take no more time; and, if the Delineator was not inclined to think much about it, I would adviſe all ſuch, to place an Object before them, and their Eye in the Point of View, as they intend the Object to appear; they will then aſſuredly ſee, how unpardonable ſuch miſrepreſentations are, becauſe they are as readily, without loſs of time, done right as wrong.

I have ſeen full as groſs abſurdities on the Stage; which ought to be regulated by the ſtricteſt Rules. In Theatres, Perſpective might be diſplayed to great advantage; and certainly, its Rules ought not to be diſpenſed with, on any account whatever. I do not mean in every minutia, but in the whole Deſign; for, if liberties be taken, in order to appear better in one Point of View, it will appear worſe in another; particularly, in Scenes repreſenting a grand Palace, &c. or other regular piece of Architecture, there is no poſſibility of departing from the Rules; in detached Planes, as in Scenery, they cannot repreſent one entire Building, in any Point of View, without adhering ſtrictly to Perſpective.

To the beſt of my remembrance, in that Maſter-piece, by Servandoni, in the Pandemonium (though I think, the groteſque Stile, of that Deſign, is not altogether conſonant to the grand deſcription of it, in Milton's Paradiſe loſt) the Eye, in that Scene, is on a level with the Baſes; and conſequently, the Ruſtics, on the Columns, appear curved. It is a long time ſince I ſaw it; but I believe, the firſt, from the Baſe, is as much curved as the upper one. The whole has a grand and ſtriking effect, chiefly occaſioned by the magnificent diſplay of Lights and Colours; which being left out, I am perſuaded that this Deſign would loſe much of the applauſe it has obtained.

EXAMPLE XXIX.

Fig. 103.

How to repreſent an Ionic Capital, perſpectively, having one Face parallel to the Picture.

This Example, and more particularly the 31ſt, of the Corinthian Capital, is a matter of real difficulty. To repreſent ſuch Objects as the Volutes (and other decorative parts, of which the latter is compoſed) will indeed baffle all Rules. I ſhall nevertheleſs attempt to lay down ſome, which will give their true places, with certainty; and, if the Practitioner will carefully obſerve how ſuch Objects appear, according to the poſition in which they are ſeen, and intended to be delineated, he will not be much at a loſs, if he has any hand at Ornament, in Foliage, &c. and carefully adheres to the Rules I ſhall preſcribe.

Fig. 103. At Fig. 103, No. 1, is the true geometrical Plan and Elevation of the Ionic Capital; not according to the Antique which is not ſo difficult, ſeeing that, the Volutes or Scrolls are in one Plane. On the left (at X) is the Plan of the Volute ſhewing its poſition in reſpect of the Abacus and Ovolo, &c. On the right (at Y) is the Elevation; which appears elliptical, on account of its Poſition, which is known from the Plan.

If ab be the Line of its direction (which is ſomewhat curved) to the front, A B; it is obvious, that it makes a conſiderable Angle with it; to which, perpendiculars being drawn give e c, its apparent width in front; which meaſures, being tranſpoſed to the other ſide, at d and g, and perpendiculars drawn, give its apparent geometrical Front; of which Z is the true.

Having deſcribed the geometrical Plan and Elevation, I now proceed to ſhew how it may be delineated, perſpectively.

Let A B be conſidered as the Interſection of the Picture, with the Plane of the top of the Abacus. Let V L be the Horizontal vaniſhing Line, and C the Center of the Picture. The Diſtance is greater than can be applied on the Horizontal Line, take C *E* half the Diſtance, and deſcribe the Square A B D G (Prob. 19) which contains the whole Abacus.

The Curve e f g, which is the ſixth part of the Circumference of a Circle, may be eaſily obtained; as thus.

From c, F, and d, any Points at pleaſure, in AB, draw Perpendiculars, c 1, d 2, &c. to the Curve; and draw cC, FC, and dC, which repreſent thoſe Lines, indefinite. Make c c, F f, &c. repreſent the finite Parts (Prob. 15) through the Points e, c, f, d, and g, deſcribe a Curve, which will be a portion of an Ellipſis, and repreſents the Curve e f g.

The returning Side, BD, is obtained by means of Ordinates, at 1, 2, 3, after the ſame manner, which are parallel on that Side. The interior Curves may be projected by the ſame means, and the Plans *X*, *Y*, and *Z* of the Scrolls, as any other geometrical Figure (in Sect. 5) from which, their Repreſentations are projected, as below; the returning Angles at A and B, being perpendicular and parallel to the Diagonal AC, tend to the Diſtance Points.

This preparation is ſufficient for an extra Plan; I ſhall next ſhew how far it is uſeful in finiſhing the Repreſentation below. This Plan may be ſuppoſed to be the top of the Abacus, as a Plane ſimply; but I ſhall ſuppoſe it to be above, as in Example 12th, and that the Repreſentation is below it.

Let *AB*, No. 3, be now conſidered as the Interſection of the top, inſtead of AB; to which Line transfer all the meaſures from AB; and form the out Line of the Plane *ABD*, as before; by means of the ſame Vaniſhing Points.

To compleat the Abacus; biſect *AB*, at *F*, and draw a Perpendicular, *FG*.

From *F*, ſet off all the meaſures of the heights of the Mouldings, and draw Lines from them to C; ſuppoſing *FG* a vertical Section of a Plane through the middle of the Capital.

The Center (S) being obtained, draw a perpendicular, from S, cuting thoſe Lines; through which, draw the parallel Lines *ce*, *df*, &c. till they cut Perpendiculars from the correſponding Mouldings, in the Plan above. Or, for greater accuracy, make *BC* equal to F f, the receding of the Curve from the Picture; where, make a true Profile of the Moulding, of the Abacus; from which, draw Lines to the Center of the Picture, cuting *ce*, *df*, &c. from the Axis, at *e* and *f*.

Having obtained the returning Moulding, at the Corner *B*, by means of the perſpective Plan, above, and a perſpective Section, at *FG* (obſerving, that it is a mitre Angle, touching the Picture, and conſequently, recedes more than the true Moulding, as may be ſeen by the geometrical Plan, No. 1, at A or B) from which, Lines tending to the Point of Diſtance, on the right hand, cuting Perpendiculars from the Plan, above, give the other Mitre, *gi*.

From *g*, *h*, and *i*, draw Lines to the Center (C) cuting Perpendiculars from the Angle D, in the perſpective Plan; by which means, the ſeveral Angles at *D* are obtained; and the curve Lines may be drawn by a careful hand, as in the Figure, nearly parallel amongſt themſelves; otherwiſe, if great accuracy be required, each Curve muſt be got, ſeverally, in different Planes.

The Abacus being compleated, proceed to the Scrolls or Volutes.

Having carefully obtained the perſpective Plans (*X*, *Y*, and *Z*) of the Scrolls, which are regular Trapezia (as at X, No. 1) not regarding the hollowing.

In this Diagram, it is too near the Vaniſhing Line; but may, for greater exactneſs, be at a greater Diſtance from it (as in Ex. 13.)

Draw a b parallel to *AB*, the Interſection of a Plane paſſing through the middle of the Scrolls (as at *ab*, No. 1) its Diſtance from *AB*, is equal to the diſtance of *ab* from the top of the Capital, geometrical.

By means of the Interſection ab, the repreſentations of thoſe Trapezia, *x*, *y*, and *z*, are eaſily obtained; transfering the geometrical meaſures to that Interſection, firſt; Perpendiculars, from No. 2, will cut Lines tending to C, the Center of the Picture, and give all their Angles, ſufficiently correct; but, if greater accuracy be required, their Vaniſhing and Interſecting Points may be had, as in other Figures, which are in the Horizontal Line; but, on account of their Inclination to the Picture, they are beyond its limits.

Now,

Plate XXII.

Now, the uſe of theſe Figures is very obvious; by which means, the ſeveral apparent Faces, with their ſeveral diviſions, are determined, paſſing through the Eye of the Volute, &c. But, if an extra Plan be drawn, as above, Perpendiculars from *X*, *Y*, and *Z*, give the ſame, in reſpect of their widths.

Hence may be ſeen the unavoidable diſtortion of the Volute on the left; which, on account of its oblique poſition to the Picture, and the Eye, is dragged out beyond its geometrical proportion; and conſequently, cannot pleaſe the Eye but in the true Point of View. It is true, the Diſtance it too ſhort; nevertheleſs, every oblique parallel Repreſentation, of ſuch Objects, muſt be more or leſs diſtorted, according to the diſtance, and oblique ſituation of the Eye.

Having got their dimenſions, of height, by means of a perpendicular, h i, from the middle of their geometrical Plan (X, No. 1) and the place where they fall on the Ovolo (which, muſt be drawn as in the Doric Capital, without its Abacus, Ex. 26) the reſt muſt be delineated by a ſteady hand, guided by a nice Eye; for, without great nicety in both, it is not poſſible, by the Rules of Perſpective, to project ſuch Objects; ſeeing that, the Curve is not only continually varying from the Center to its utmoſt extent, but it is alſo continually growing forward, from the utmoſt extreme to its Center, and is not deſcribed on the ſame Plane, or other Surface, which circumſtance renders it utterly impoſſible to be deſcribed perſpectively, by rule; nevertheleſs, by the Rules preſcribed and judgment in drawing, it may be projected ſo as to pleaſe the moſt critical Eye,

Below this outline is a finiſhed Capital, diveſted of the neceſſary Lines for projecting it. It is ſomewhat farther removed from the Horizontal vaniſhing Line, but, in other reſpects, it is the ſame; excepting the decorative parts, of foliage, &c. which, according to the poſition in which the Capital is viewed, will differ in Figure; they were omitted, above, for the ſake of continuing the Lines of the Abacus, which they hide, in this. No Rule can poſſibly be of any uſe in delineating ſuch Ornaments; a real Object, before the Eye, is the beſt and only means to effect it.

EXAMPLE XXX.

Is an Ionic Capital, according to the Antients, which is inclined to the Picture.

Fig. 104.

This Capital is delineated with much more eaſe than the other, for ſeveral reaſons; becauſe its Abacus is a Square, with only a Cima reverſa; and, the Volutes are both in one Plane, which is vertical.

The Abacus, is ſuppoſed to cut the Picture, at A, through which, draw AB, parallel to the Horizon; V is the Vaniſhing Point of the front Lines, its diſtance is VD; the other Vaniſhing Point is out of the Picture.

Make A B equal to a Side of the ſquare of the Abacus, and project the Square *ABC*, perſpectively (Prob. 21) and finiſh the Abacus, as in Ex. 13. of the Pedeſtal.

The hither Scroll projects through the Picture; becauſe the corner of the Abacus, at A, cuts the Picture, and the Scroll projects beyond it, in this Capital.

The Interſection of the Plane of the Volutes may be determined, truly, from a geometrical Plan; as at X, an Angle of the Abacus, inclined to *AB* as the Object is ſuppoſed inclined to the Picture.

The whole Moulding projects before the plane of the Scrolls; therefore, produce *cb* to *a*, cuting *AB*. Make A a equal *Aa*, draw a d perpendicular, and produce *ab*, cuting a d at c; all that part of the Scroll, on the right hand of a d, projects through the Picture.

Make c d equal to the height of the Scrolls, or a d of the Capital, and draw d V.

Deſcribe a Rectangle, incloſing the Scroll, at each extreme, as c d e f, *cdef*; and, within theſe, others may be deſcribed, as in the Figure (dividing c d geometrically, at 1, 2, 3, 4, and drawing Lines to V; alſo, dividing b f, perſpectively, at 1, 2, 3, and drawing perpendiculars) or, ſeveral points may be determined, by means of which, the Scrolls may be very accurately delineated, perſpectively, but muſt own it is a laborious proceſs, and may be drawn accurately enough, within the large Rectangles.

The

The other Front, or rather the End of this Capital, is different, and is not eaſily delineated, truly. Having obtained the outline of the oppoſite Scroll; at *g b1*, biſect the bottom Line of the Abacus, at *k*; where, deſcribe the repreſentation of a Semicircle, *kl*, in a Plane parallel to the Front, but different in dimenſions to the Scrolls. The proportion, and figure of it, may be had in Books of Architecture.

This Face repreſents a kind of Balluſter, ſpreading, at its extremes, to the width of the Scroll; but the Axis of it does not paſs through their Centers.

The Ovolo is the ſame in this as in the modern Capital; ſave only, that it is not ſeen on the ſides, at all, being wholly hid by the Balluſters.

EXAMPLE XXXI.

To repreſent a Corinthian Capital inclined to the Picture.

Fig. 105.

This Object is ſuppoſed to be direct before the Eye; and conſequently, the Axis of the Capital paſſes through the Center of the Picture, at C.

CE (perpendicular) is the Diſtance, VL the Horizontal Line, and V the Vaniſhing Point of one ſide of the Capital, or rather of its Poſition; for, the Object not being right lined, ſave the ſhort returning Angles, it has indeed no Vaniſhing Point; yet they are neceſſary, in the Delineation.

Let a be the Point in which a Square, incloſing the Abacus, touches the Picture; through which, the Interſection of the top may be drawn; but it will be more eligible to take another, above, at BD, parallel to the Horizon.

The Abacus of this Capital is the ſame, in proportion, and every other reſpect, as the modern Ionic; therefore, the meaſures may be taken from Fig. 103, No. 1, being delineated by the ſame Scale; and excepting its ſuppoſed poſition, being different, the delineation of it would be the ſame.

Take A perpendicularly over a, and draw AV; and AΥ to the Point in which EY would cut the Horizontal Line (Prob. 13) indefinite.

Biſect the Angle VEY, by the Line EL, L is the Vaniſhing Point of a Diagonal.

Find F, the diſtance of the Vaniſhing Point Y (Prob. 12) and, becauſe the diſtance of V cannot be in the Picture, take VG three fourths of VE (or any other part, at diſcretion.)

Take AB three fourths of AB (No. 1) which biſect at *c*, and take all the other meaſures, at *a*, *b*, &c. on that ſide, each three fourths of the real meaſures on AB (No. 1) but, on the other, take *d*, *e*, *f*, &c. the full meaſures.

From all which Points, draw Lines to F and G, reſpectively, as in the Figure; cuting AV, and AY, at *g*, *h*, *i*, &c. from which, draw Lines to both Vaniſhing Points, V and Y, indefinite.

Take A 1, and A 2, equal c 1, Ff, &c. (No. 1) and draw 1 F, 2 F, cuting AY; from which Points draw Lines to V, cuting the Lines from *g*, *h*, &c. to Y, at *n*, *o*, *p*; and from *j*, where 1 V, 2 V, cut the Diagonal AL, draw Lines to Y, cuting *k* V, &c at *r*, *s*, *t*; through which Points, on both ſides, draw the Curves of the Abacus.

At the Corner A, deſcribe an Equilateral Triangle (x) perſpectively, and the ſame at each Corner, *B* and *C*; giving the mitre Angles of the Moulding, as at A and B; (No. 1.) which, in this caſe, are not biſected, having a greater projecture at the Corners, than in front.

This operation done, draw a V, and a Y indefinite; to which, draw Perpendiculars from *g*, *h*, *i*, *k*, &c. alſo, from *n*, *o*, *p*, &c. and draw g Y, h Y, &c. cuting them at n, o, p, &c. through which Points, deſcribe the Curves b n o p q, and a r s t u.

In reſpect of theſe Curves, the operation above is not abſolutely neceſſary.

After the ſame manner, the Abacus may be compleated; *viz.* by projecting the curves of the Fillet, in the middle, and at the bottom, in their reſpective Planes.

The places of the Leaves may be obtained by deſcribing an Ellipſis in the repreſentation of a Square, inclined to the Picture, as the Abacus.

Plate XXII.

This may alſo be done above, at v, u, x, y, z. The Diagonals and the two Diameters, whoſe Originals are perpendicular to the Fronts, give the apparent width of each Leaf, in the firſt row, allowing a little ſpace between them, and they alſo give the middle of the upper Leaves; between each, is a kind of Balluſter, from which the Caulicola and Scrolls ſpring.

The true places and middle of the Heads may be obtained, by deſcribing repreſentations of larger Circles, concentric with the other. At Z is a Leaf, in front.

Deſcribe two Ellipſes (cef and ghi) giving the greateſt projecture of the Leaves, in their true places; their heights above the Aſtragal (*FG*) are taken, from the Profiles (at X) to the greateſt projecture of the Heads (at *a* and *b*) which projectures (*ac*, and *bd*) reſpectively, are added to the diameter of the Column at the Aſtragal, for the diameters of thoſe Circles, repreſented by cf and gi.

Theſe Ellipſes being obtained, in their true places, inſcribed in Squares (as *Hl*) inclined to the Picture as the Abacus, for the upper one; the Diagonals and parallel Diameters, give the true middle of each head of a Leaf, at c, d, e, &c. Thoſe of the lower row fall between them; as deſcribed, in the Plan above.

The Volutes, at the corners of the Abacus, are, except in dimenſions, the ſame as the Ionic, and muſt be obtained the ſame, by perſpective Plans, at *x*, *y*, *z*.

How they fall into the Caulicola, with the other decorative parts of the Capital, the Figure only can deſcribe; for, I fairly own that it is not in words to deſcribe it, ſo as to be of any ſervice in delineating. Being well acquainted with the ſeveral parts of the Capital, the Rules I have given are ſufficient; the reſt is beſt deſcribed by a careful and attentive peruſal of the Figure.

Thoſe who are not verſed in it, would do well to draw from a real Capital, firſt, in all poſitions; after which, they will be able to delineate it by Rule.

SECTION IX.

Shewing the application of the whole to entire BUILDINGS, and regular pieces of ARCHITECTURE.

IN this Section, I have applied the whole of what has been already done to compleat Objects, and particularly to Architecture; as being, of all other, the fitteſt Subject, and gives the greateſt luſtre to Perſpective; on account of the regular diſpoſition, and arrangement of the ſeveral parts of a Building, and being compoſed, chiefly, of plane Surfaces, which generate Right Lines.

It is, to me, ſurprizing, that Artiſts, in general, have no better notions of Perſpective; if they would give themſelves time to think about it, ſeriouſly, they could not commit ſuch palpable miſtakes and groſs abſurdities; even ſuppoſe they were not acquainted with the Rules, having a true Idea of the meaning and intent of Perſpective; which cannot but be obtained, from the Apparatus, to this Work.

Whatever we are about to delineate, it is ſuppoſed that the Object, itſelf, is on the other ſide of the Picture, generally (it may be on this ſide.) Is it not, then, rational to conſider, and thence to conclude, that the Picture we are delineating ſhould be directly between the Eye and the Object; or what reaſon can poſſibly be aſſigned that it ſhould not? and yet it is, almoſt generally, the caſe. Some, not conſidering it properly, would tell us, that it is in order to ſee the End or Side of an Object, as well as the Front; as if that could not be effected otherwiſe; true, it cannot, on their Hypotheſis, that one Face muſt neceſſarily be parallel to the Picture.

I ſhall illuſtrate the whole of this matter in few words.

Figure 106 exhibits the Plan and Elevation of a plain Building, geometrical, which is to be delineated; nearly the ſame as the Object, in the Apparatus; in the ſame Poſition, and at the ſame Diſtance, nearly, by a Scale of the proportion.

In the first place, I shall shew how to determine the true Position of the Picture, the Station being previously determined. Plate XXIII.

Let S be the Station fixed on, from which it is intended to delineate the Building; X is the geometrical Elevation of the Front, Z is the Plan, or Seat of the Building on the Ground; in the Position, and at the Distance, the Building is to be represented, as seen from the Station S. Fig. 106.

The Station, and consequently the Point of View being determined, it is evident, that the Building cannot vary in its appearance at that Station; but it is nevertheless manifest, that there may be a great variety of Pictures, or Representations of the Object; seeing that, every different position of the Picture, or section of the Pyramid of Rays (forming a solid Angle, under which the Object is supposed to be seen) will produce a different Picture. All which, Representations, will affect the Eye alike, in the true Point of View; nor is it possible it should be otherwise, seeing that, when the Eye is in the true Point of View, every Line, on the Picture, is seen under the same plane Angle, and consequently, the whole Representation under the same solid Angle, as the Original.

Hence, it is easy to account for the many distorted and preposterous Representations which are frequently to be seen; and which, is mostly owing to the absurd position of the Picture, in respect of the Object and of the Eye; or frequently to the distance of the Object, itself; a circumstance which ought to be particularly considered; for, if it be too near, it is impossible, in such case, to produce an agreeable Picture.

Then, since every different section of the Pyramid of Rays produces a different Picture (for all parallel Sections produce similar Representations†) certainly, that Section which is perpendicular to the Axe of the Eye, or the Station Line, SL, will be the most natural and agreeable Representation; seeing that, the parts of the Representation will differ but very little (if the Optic Angle ASC be not large) from the true Appearance, i.e. from a Section made by the surface of a Sphere, of which S is the Center. † 1. 8. El.

Consequently, then, if the Station be fixed, from which it is determined to delineate an Object, the Position of the Picture is also determined, as follows.

ABC is the place of the Object, its Seat on the Ground Plane; and S is the Station determined on. Draw the Right Lines AS and CS, which determine the Optic Angle (ASC) of the Object, AC; for, S may as well be the Eye, and AS, CS Visual Rays, the Eye being perpendicularly over S.

Bisect the Angle ASC, by the Right Line SL; SL is the Station Line for that Object; to which, the Picture must (if placed properly) be perpendicular; all such Sections are similar, and are the most just Representations.

VY, perpendicular to SL, is the true Position for a Picture of that Object, delineated from the Station S; and if Right Lines are drawn, from S, to the several parts of the Plan, as a, b, c, &c. their sections with the Picture, *a*, *b*, *c*, &c. determine the apparent width of each part on the Picture, perhaps with greater accuracy than by any other means.

Here, the Picture is supposed close to the nearest Angle of the Object, at B; and consequently, the perspective section of the Rays, by VY, is the largest that can be made on this side the Object. Now, if an Ark (*ABC*) be drawn, on the Center S (Radius SB*) it is obvious that the section of the Rays by that Ark (or a Sphere of that Radius) differs but little from the section by the Plane, on VY; because the Optic Angle, ASC, is but small, and therefore the extreme Rays, AS and CS, cut the Plane not very oblique; which, being increased, occasions Distortion.

SV, parallel to AB, and SY, to BC, determine the Vanishing Points of horizontal Lines, in the Building. 1, 2, 3, &c. may be considered as Intersecting Points of several Lines in the Object.

* BS perpendicular to the Picture does not necessarily fall on the Angle B (in the Apparatus it is somewhat on the right hand) for, if the Station was ever so little on either side, it would fall differently, as must be obvious, from the method given of determining it.

Having

Plate XXIII.

Having thus determined the true position of the Picture from the Station S, answering to the Picture, MNOP, in the Apparatus; which is directly between the Eye and the Object; I shall, next, shew the great absurdity of placing it otherwise.

Let every thing remain in the same Position; S is the determined Station for the same Object; and consequently it must appear the same, whatever Position the Picture is supposed to have, in respect of the Object. But let it be observed, again, that the Representation may differ very widely; as will be shewn.

It is usual with all (who know a little of common Perspective only) to make one Face of the Object parallel to the Picture; in which case (the Object being right angled) the Center of the Picture is, necessarily, the Vanishing Point of horizontal Lines in the other Side. Consequently, if two Faces of the Object are seen, the Picture must necessarily be obliquely situated, in respect of the Object and the Eye.

Now, can any Person conceive it rational, or eligible, to place the Picture in the Position ED, parallel to the End of the Building, to be seen from the Station S; and yet, this is the very supposition, in the Representation exhibited by Fig. 107; whose Center is C, and Distance CE (equal twice SE, 106) which Position answers to the Picture MN*O*P, in the Apparatus,

How egregiously absurd is the Idea of viewing a Picture in that Position; and yet, in any other Point of View it cannot appear like the Object represented. The distortion of the Front of the Building is obvious, but much more so, is the Face X, of the Bow Window; whilst the other (Z) is geometrical, as well as the end of the Building.

Fig. 107.

The Position being determined, and the Center (C) fixed; and also, the Intersection BG, of the front Plane (its distance from the Center, equal twice EF, 106) the Representation is delineated by Prob. 19 (as by Ex. 1 and 2) in which case, C, the Center of the Picture, is the only Vanishing Point.

Now, those Artists, who make a kind of necessity of delineating their Objects after this method, cannot, I conceive, give a reason, why, in a general View, such Objects must necessarily be so situated to the Picture, as to have some Face parallel to it; or, why the Picture should not be placed parallel to the Front (AB) if it must be so to either; excepting that, they rather choose the one to be parallel than the other; for there is as much propriety in one, as in the other, but the Representation will be much more distorted; because the Distance (SG) of the Picture standing on AG, is less than SE in the former; and, the much greater length of the Front (AB) occasions a greater obliquity of the Rays, with that Picture. For, SG being the Direct Radial of that Picture, ASG is but half the Optic Angle under which it is seen; but it is manifest, that the Eye cannot take in as much on the right-hand, in which case, the whole Angle will be very obtuse.

CASE THE SECOND.

Fig. 108.

Fig. 108 exhibits a Representation of the same Object and from the same Station, in that position of the Picture; in which, the Front is geometrical; but the projectures of the Cornice, Steps, &c. are dragged out preposterously; and the Face X, of the Bow Window, is worse distorted than before.

C is the Center of the Picture, and CE its Distance, equal to twice SG (106.) The whole of the Object ought by no means to exceed the Point E, from the Center; whereas, it extends above twice CE, which is preposterous.

KL is the Vertical Line; and the Angles, CEK, CEL, being made each equal to the Inclination of the Roof, determine the Vanishing Points of the Lines FG, and F*D* in the Roof; K is also the Vanishing Point of HI parallel to FG; the rest is determined as the former, in which, those Lines are parallel to the Picture.

In both these Pictures, the horizontal Lines, in the Face, X, of the Bow Window, vanish in the Point of Distance, in the Horizontal Line; equal CE, on the other Side of C, because the Figure of it is a regular Octagon. (See Prob. 25.)

CASE THE THIRD. EXAMPLE XXXII.

Fig. 109 exhibits a true and natural Repreſentation of the ſame Object from the ſame Station, in the moſt judicious poſition of the Picture; and, in order to ſhew the affinity between this and the Picture MNOP, in the Apparatus, I have made uſe of the ſame Letters for Reference, where they can be properly applied. Fig. 109.

VY is the Horizontal vaniſhing Line, and C the Center of the Picture.

Draw CE perpendicular to VY, and equal to the Diſtance (equal twice SB, 106) and make the Angles CEV, CEY equal, reſpectively, to the Angles BSV, BSY (106) or, make CV equal twice BV, and CY twice BY; V and Y are the Vaniſhing Points of horizontal Lines in the Front and End of the Building †. † Th. 10. and 11.

Then, becauſe the Angle B touches the Picture, BG is the Interſection of both Planes (in the Apparatus, the Object being at ſome Diſtance from the Picture, each Plane has a ſeparate Interſection, *BG* and *BG*) on which all the meaſures, of the heights of the Windows, &c. are ſet off, geometrically (as in the Elevation, X, 106; each being double) as Bm, mn, &c. from which, draw Lines to both Vaniſhing Points, V and Y, indefinite.

Make V*E* equal to VE, and Y*E*2 equal YE. From B, ſet off Ba, ab, &c. as in the Plan (106) and draw a*E*, b*E*, &c. cuting BV, in the Points *a*, *b*, &c. the perſpective proportions of the Windows, &c. from which, draw Perpendiculars, cuting Lines drawn from the ſeveral heights, on BG to V, in the repreſentations of the Windows, &c. AB the length of the Front, gives B*A* its perſpective length.

Or, if B 1, 2, 3, &c. be ſet off, as from B to V (106) from which, Lines drawn to Y determine certain Points; on BV, or aV.

By which means, they are projected in the Apparatus, *viz.* by the Interſecting Points; and the Steps by the ſame, which may be finiſhed by Example the 13th. Or, the place of each may be taken, from VY, 106, where the Lines, AS, &c. cut the Picture; which for the Bow Window, will be the readieſt; obſerving to take them double, becauſe it is but half the proportion.

Otherwiſe; the Bow Window may be projected, by Example 5th, obſerving the difference in Figure; and, being too near the Vaniſhing Line, another Interſection may be taken, at pleaſure, at a ſufficient Diſtance (Ex. 12.) as it has been exemplified in various Caſes.

The Cornice, being plain, is managed by Example 18th, and the Pediment, by the 20th. The Vaniſhing Points are determined by making the Angle Y*E*W equal to the inclination of the Pediment; cuting the Vaniſhing Line YW, of the Plane of the End, at W; the other will fall as much below the Horizon.

W is therefore the Vaniſhing Point of *GF* and *HI* (as in the Apparatus) and *FD* tends down to the other, in WY produced.

The ſemicircular Window, and the Circle in the Pediment, are determined on BY, in reſpect of their widths; the heights on BG; as the Arches in Ex. 22; for, the Plane being inclined to the Picture, they are conſequently Ellipſes; as alſo in the former poſition; in the other they are Circles. (Theo. 9, Cor. 5.)

Thus, are delineated three different Pictures, or Repreſentations of the ſame Object, from the ſame Station and Point of View; and, notwithſtanding they are ſo very different, yet they are all true Perſpective. It is obvious, which is the neareſt to the true Appearance, but, when the Eye is in the Point of View, of each, they will appear the ſame.

In Fig. 107; let the Eye be placed oppoſite to the Center, at C, at the Diſtance CE; and having attentively obſerved the Appearance, then let the Eye be moved to 108, viewing it, at its proper Diſtance, CE; and laſtly, to 109, at its reſpective Diſtance. I ſay, that, each being ſeen in the true Point of View, exactly (which can only be done perfectly, by fixing a ſmall Pin hole at the true place of the Eye) every part of each being ſeen under the ſame Angle as the Original, muſt neceſſarily appear the ſame.

Now, becauſe the Eye is perpendicularly on the Picture, at 109, and at the proper Diſtance, the Angle it ſubtends is equal to ASC (106) and, becauſe the Angle does not exceed 25 Degrees, the portion of a Sphere, of that Radius, will not differ much from the Plane of the Picture; and conſequently, this Repreſentation is the neareſt to the true Appearance.

SCHOL. This Leſſon, properly digeſted, will give the Student a clear Idea, of the cauſe of Diſtortion, in the Repreſentation, and teach him how to avoid it; for notwithſtanding the Diſtance may be ſufficient for the Object (the Picture being judiciouſly placed) yet if the Picture be placed according to the two firſt, it muſt neceſſarily be diſtorted. Although the real Optic Angle, reſpecting that ſingle Object, is not varied; yet the Angle under which the whole Picture is ſeen, may be varied and increaſed infinitely, from 25 Degrees (the true Angle) to any quantity, leſs than 180, two Right Angles.

A brief and curſory retroſpect of the Theory; ſhewing the immediate and abſolute dependance of Practice thereon.

Obſerve, that No. 15 alſo refers to the Apparatus.

The firſt Theorem is exemplified in the Picture MNOP, which is parallel to the Plane BGFDC. This is applied to Practice, by the 9th Theorem.

The ſecond demonſtrates, that the Vaniſhing Line, and Parallel of the Eye, &c. are always parallel to the Interſection of the Original Plane.

This has been exemplified, in practice, throughout the whole; but chiefly in horizontal and vertical Planes. It is illuſtrated generally, by the Diagrams.

Fig. 15 (No. 2 and 3) illuſtrates the Theory for all Planes perpendicular to the Picture; or inclined in a certain poſition, reſpecting the Horizon and the Picture; and if a Plane paſſes through the Spectator's Eye (at E) parallel to the Original Plane HIFG, its Vaniſhing Line, VW, is parallel to the Interſection, *FH*. (8. 7. El.)

The great uſe of this knowledge is obvious, for, if either be determined or found, as *FH*, and any one Point, as V) the Vaniſhing Point of AB, &c.) the Vaniſhing Line, VW, is determined; and conſequently, the Parallel of the Eye is alſo determined, the Diſtance of the Vaniſhing Line being known; by Theo. 7.

Theorem 3rd has been frequently exemplified, and beſpeaks its uſe and application ſufficiently; particularly in Example 12. The Corollaries deduced from it, eſpecially the firſt, is derived from the ſecond general Theorem, Book the firſt.

The 4th is a generally uſeful Theorem; in common Practice, as by the old writers on Perſpective, more Planes are perpendicular to the Picture than in any other poſition. The Horizontal Line (the Vaniſhing Line of the Ground, and of all other horizontal Planes) alſo the Vertical Line (of all which are perpendicular to the Picture and to the Horizon) both paſs through the Center of the Picture.

The Planes FIHG and FIKD, are perpendicular to the Picture MNOP; their Vaniſhing Lines alſo, paſs through the Center (C) of that Picture.

The 5th, that Planes, whoſe common Interſection is parallel to the Picture, have parallel Interſections and Vaniſhing Lines, is exemplified in the Apparatus.

The Line BG, in the Object, is parallel to both Pictures, and it is the common Interſection of the two Planes, AHGB and BFC.

If thoſe Planes are produced they will cut the Picture in *BG* and *BG*; which are their Interſections; the Planes being vertical and the Picture vertical, their Interſections are perpendicular to the Horizon, and conſequently parallel. For the ſame reaſon, their Vaniſhing Lines (OP and RW) are parallel; and, becauſe they are parallel to their Interſections, reſpectively, by Theorem 2nd.

The uſe of this knowledge is evident; having obtained, by any means, one Interſection or Vaniſhing Line; and alſo, by any means, the Interſecting or Vaniſhing Point of a Line in any other Plane (whoſe common Section, with it, is parallel to the Picture) the whole Interſection or Vaniſhing Line is conſequently determined.

In the laſt Example the common Interſection (BG) of the contiguous Planes is in the Picture; conſequently, it is the Interſection of both.

By the 6th Theorem, Vaniſhing Lines, of contiguous Faces of Objects, are determined according to the Inclination of the Original Planes, to one another.

In the laſt Example they were at right angles; but, being inclined in any Angle whatever, their Vaniſhing Planes being parallel to them, conſequently they make the ſame Angle with each other, at the Eye; and their common Interſection, which paſſes through the Eye, is parallel to the Interſection of the Originals; and, when it is not parallel to the Picture, produces its Vaniſhing Point (Def. 22) which, Vaniſhing Point, is the Interſection of the Vaniſhing Lines.

How the Vaniſhing Lines are found, in particular Caſes, is determined by Fig. 15, No. 2 and 3; *viz.* when the common Interſection is either parallel or perpendicular to the Picture. When it is perpendicular by Problem the firſt; and when parallel, by the third; and, when it is inclined to the Picture, in general, by the fifth. In all which Caſes, it is obſervable that the Interſection of the Vaniſhing Lines, is the Vaniſhing Point, and the Interſection, of the Interſecting Lines, is the Interſecting Point, of the common Interſection of the Original Planes; which is an eſſential thing to be well underſtood; for Practice.

Theorem 7th. A Line drawn through the Center of the Picture and the Center of a Vaniſhing Line is perpendicular to the Vaniſhing Line.

The uſe of this Theorem is to find the Center and Diſtance of every Vaniſhing Line, the Center of the Picture being determined, and Diſtance known.

To do which, there is only to draw a Perpendicular, as CS (109) from the Center of the Picture, to the Vaniſhing Line; and, forming a right angled Triangle, of the Direct Radial, or Diſtance of the Picture, CE, and the Diſtance between the two Centers; the Hypothenuſe is the Diſtance of the Vaniſhing Line. Def. 20.

In Theorem the 8th it is aſſerted, and proved, that the Repreſentation of every Right Line, in Perſpective, is a Right Line.

Any two Viſual Rays, EF, EI, or EF, ED, together with the Original Line, form a Triangle, which are all in the ſame Plane, paſſing through the Original Line and the Eye; which, by its Interſection with the Picture, generates the Right Line fi, or fd, its Repreſentation; and if the Original Line was extended, infinitely, its repreſentation will ſtill be a Right Line. (1. 7. El.)

The 9th Theorem contains the neceſſary knowledge for Practice, relative to Planes which are parallel to the Picture.

Firſt; all Lines in ſuch Planes, are parallel to the Picture; conſequently they have no Vaniſhing Point (Cor. 1. Th. 1.) but their Repreſentations are (by Theorem) parallel to the Originals; that is, to the Vaniſhing Line and Interſection of whatever Plane they are in; and their proportions to the Originals are alſo there determined; *viz.* as the Diſtance of the Picture, to that, of the Plane they are in.

The Corollaries, of this Theorem, are moſt uſeful Leſſons; which need only to be read over, with attention, to apply them to Practice. The fifth ſums up the whole; that the Repreſentation of every Figure, in ſuch poſition of the original Planes, is ſimilar to its Original; that is, it has the ſame geometrical Figure.

Theorem the 10th is ſelf-evident, being read attentively; and the Corollaries are uſeful practical Leſſons deduced from it, which are fully explicit in themſelves.

Theorem 11th is of the greateſt uſe; inſomuch that, without it, we cannot draw the repreſentation of one Line, inclined to the Picture, except by means of expedients; making uſe of Lines, otherwiſe, unneceſſary in themſelves.

Notwithſtanding this is the 11th Theorem, yet its application is prior to ſeveral others, reſpecting Vaniſhing Lines; ſeeing by its means, only, they can be determined; as in Prob. 2nd. It being impoſſible to determine any one Vaniſhing Line, before the Vaniſhing Point of ſome Line in the Plane be determined, through which the Vaniſhing Line muſt neceſſarily paſs.

For firſt, the Center of the Picture muſt be determined, before the horizontal and vertical Vaniſhing Lines can be drawn; or that of any other Plane, perpendicular to the Picture; or indeed of any other whatever; becauſe it is by means of Lines,

Lines, in horizontal Planes, others are determined; whoſe Vaniſhing Line is always determinable from its known poſition, the Center of the Picture being fixed; or, which is the ſame thing, the height of the Eye being determined. Notwithſtanding the Center of the Picture (being the Vaniſhing Point of Lines perpendicular to the Picture) is fixed arbitrarily, yet it is as much ſubject to the 11th Theorem as any other; and is only determinable from the fixed and invariable poſition of the Lines, which vaniſh in it, to the Picture.

As this Theorem has been applied in almoſt every Example, or Problem, where Vaniſhing Points of Lines, inclined to the Picture, are required; I ſhall only illuſtrate it by an Example or two, from the Apparatus.

The Parallel of the Eye muſt be imagined, paſſing through the Eye of the Spectator parallel to the Vaniſhing Line. Def. 9.

For the parallel Picture; EC cuts them both perpendicularily; therefore it makes equal Angles (i. e. Right Angles) with the Parallel of the Eye and the Horizontal Line; as the Original Lines, AB, HG, &c. make with the Interſection of a horizontal Plane paſſing through them; or any other, whatever.

In the direct Picture; EV, producing the Vaniſhing Point of the ſame Lines, alſo makes equal Angles with the Parallel of the Eye and the Vaniſhing Line, of either Plane, AHGB, or FGHI, as the Original Lines, make with the Interſection.

And laſtly; EW*, makes equal Angles with the Parallel of the Eye of the Plane FGHI, and the Vaniſhing Line, VW, of that Plane, as FG, or HI, makes with *FH*, the Interſection of that Plane with the Picture, or, with WY, of the Pane BFC, as FG makes with BG, its Interſection.

This is practically exemplified, in Problems 5, and 21, Method 3; 23rd, &c.

By Theorem 12th, the Indefinite Repreſentation, of a Right Line, is a Line drawn through its Interſecting and Vaniſhing Points.

This Theorem has been exemplified frequently; but I ſhall in this place illuſtrate it further, it being moſt eſſential in Practice.

Whatever Plane any Right Line is in, if it be not parallel to the Picture, it will cut it, ſomewhere, in the Interſection of the Plane it is in. IF, HG, and FG, are all in one Plane; which being produced, the two firſt cut both Pictures, at *F* and *G*; the other (FG) is parallel to one Picture, and therefore cannot cut it; but it will cut MNOP at *H*; *F*, *G*, and *H* are, therefore, the Interſecting Points of thoſe Lines; and conſequently, the Plane FIHG, being produced, would cut that Picture, in the Line *FGH*; and the other in *FG*; as by the 10th.

But, EV and EW being parallel, reſpectively, to thoſe Lines, the Points V and W are their Vaniſhing Points; wherefore, the Lines *F*V and *G*V, alſo *H*W, are the indefinite Repreſentations of thoſe Lines; by this Theorem

The ſame thing is applicable in all Planes whatever; as *B*V, *G*V, &c. in the Front and End, which are vertical; and a V, *B*V, MV, &c. on the Ground Plane.

By changing V for C, it is all applicable to the parallel Picture (MN*O*P) ſave only, FG is parallel to it, and therefore has no Vaniſhing Point; for the Indefinite Repreſentations of all Lines, in ſuch caſe, are infinite, ſeeing they have neither Interſecting nor Vaniſhing Point; and are, conſequently, parallel to the Vaniſhing Line of the Plane they are in.

In Theorem 13th, is contained the whole knowledge of proportioning Right Lines, indefinitely drawn on the Picture (by the 12th.) It may, to ſome, appear too mathematical for Practice; but, notwithſtanding that Theorem is more ſtrictly

* Let the triangular Piece be fixed, by the two Pins, to the Picture MNOP, of which it is a continuation; and bring the piece, which folds back, to the ſame Plane; the continuation of the Vaniſhing Lines, &c. on them, ſhow their affinity with Figure 109. If the thread, at W, be drawn through the Eye of the Spectator, EW will be parallel to the Lines, FG and HI, in the Roof of the Object; and conſequently, W is their Vaniſhing Point. (Def. 22.)

By the ſame, the thread EV determines the Vaniſhing Point of AB, GH, &c. on both Pictures; to the firſt, it is perpendicular; therefore C is its Center, or Point of View; whilſt another, EC, perpendicular to the other Picture, gives its Center (Def. 17) directly in the middle of the Repreſentation, as it ought always, when there is but one Object.

and

and rigidly demonstrated than any other, it is nevertheless very applicable to practice. For, whether it be considered mathematically or not, 'tis certain that it is put in practice, by every one, in proportioning Lines; even without its ever entering into his Head what Proportion means; on the contrary, if he really understands what is there demonstrated, with how much more pleasure, satisfaction, and certainty, would he proceed in the application of it to Practice.

The Theorem tells us, that the Distance of the Representation of any Point (in the indefinite Representation) from the Intersecting Point (of the Original Line) is in that proportion to the whole Indefinite Representation; as the Distance, between the Original Point, and the Intersecting Point, is to its Distance from the Directing Point of the same Line.

Now when the Premises of this Theorem are clearly understood (for which purpose, the words are adapted, for brevity and perspicuity, as much as may be) what is there of mystery in it? For the first, in respect of the Proportion, simply. If the Distance of the Original Point, from its Intersecting Point, be to its Distance from the Directing Point, as 2 to 3, 5, &c. or as 3, to 5, 7, 8, &c. in any Ratio, whatever; then, the Distance, of the Representation of that Point from the Intersecting Point, will be in the same ratio to the whole Indefinite Representation *.

Can any thing be more simple and easy? nor does it depend on the real measures, but on the ratio of one to the other. Wherefore, since the Distance of some Points, in original Lines, cannot easily be obtained, from their Intersecting and Directing Points; I have shewn, further (which Dr. Brook Taylor has not) that the Distance of the Original Point, either from the Intersection of the Plane it is in, or from the Picture, is in the same ratio to its Distance from the Directing Line or Plane, a circumstance of great utility; because we may, with ease, have the measures of one, when we cannot, without applying Geometry, or by Calculation, have the other. For, the Distance of any Point in the Object, from the Picture, may be had; and the Distance of the Directing Plane, from the Picture, is nothing more than the Distance of the Eye from it; hence it is, with the greatest facility imaginable, reduced to Practice, in any position of the Lines whatever. It is most frequently done by the last; and commonly, by its Distance from the Intersection of the Plane the Original Line is in.

And further; the representation of any Point in a Line being obtained, the representation of any other Point in the same Line, or any other, cuting the former in that Point, may be obtained by the same Theorem; when the Intersecting Point cannot be had, nor is in the least degree necessary.

The 14th Theorem, which is more theoretic than practical, or really useful, is fully explicit in itself; and the Corollaries, deduced from it, need no further Comment, in order to a clearer elucidation.

Thus I have, briefly, taken a transient retrospect of the whole Theory: and I think it cannot fail of answering the End I aimed at, which is most obvious. As the Reader, who has advanced thus far, must now be perfectly acquainted with every Rule given; yet, being eager in his pursuit of what he deemed really and only useful, he may not have given that attention to the Theory (which is frequently refered to) as I could wish, for his more perfect knowledge in it.

He cannot now be insensible of the great advantage resulting from a well-founded Theory, the very essence of all useful Knowledge; for, without it, like a blundering Workman in any mechanic Art, he may continually be runing into error; which, for want of a just knowledge of the Theory, he must, unavoidably; nor can he, with ease and certainty, extricate himself from the difficulties he will frequently be immersed in.

* See this fully and practically illustrated in Prob. 6. Sect. 4. and indeed in almost every Problem and Example, throughout the whole Work.

Plate XXIV.

EXAMPLE XXXIII.

Is a Tuſcan Arcade, with Pilaſters; having one Face parallel to the Picture.

Fig. 110. The ſcale of Proportion being determined, and a Profile, of the Order, proportioned to that Scale (at AD) as in the previous Examples, let AB, the Interſection of the upper face of the Step, be conſidered as the Ground Line; and, equal to the height of the Eye (in proportion to the Object) let CE be the Horizontal vaniſhing Line, C its Center, and CE the Diſtance.

Then, the Object being right angled, the horizontal Lines in the returning Side vaniſh in the Center of the Picture†; in the other Face, every part of it, which are in Planes parallel to the Picture, are repreſented by Figures ſimilar to their Originals, according to the 9th Theorem, and its Corollaries.

† Cor. to Theo. 4.

The place of the Sub-plinth, at X, being determined, by its diſtance from A, the Step (*AB*) is firſt repreſented; then, the Plinths, X, Y, and Z, are equal, and have equal ſpaces between them; their places are obtained by ſeting off their true meaſures from *A* towards B, as a, b, c, &c. on the firſt Step; and after the ſame manner, the Piers are obtained, and the Plane of the Arches, whoſe Centers are got by biſecting bc and de, at *a* and *b*; and *ae* being made equal to the Diſtance of the Plane they are in, from the Picture. Draw *c*E, cuting *a*C at f; and a Line fg (parallel to AB) cuts *b*C, at g; f and g are the Seats of the Centers of the front Arches, on the Plane of the firſt Step.

Draw FG parallel to AB, and equal to the height of the foot of the Arches; from *a* and *b* draw perpendiculars, *a*F and *b*G, and from F and G, draw to the Center, C; then, Perpendiculars, from f and g, cuting FC and GC, at h and i, give the Centers of thoſe Arches; which, being parallel to the Picture, are Semicircles. (Cor. 5, Theo. 9.) The Centers of the inner Curves, are in the ſame Lines, FC, and GC, at k and l, and determined as the other.

The Mouldings in the Cornice, Pilaſters, &c. may be proportioned by the general method (Ex. 15) as the Doric Entablature in the 16th.

The true Profile of the whole Order being deſcribed geometrical, at AD, and the Picture being ſuppoſed cloſe to the Plinth, at A, draw AD perpendicular; which is conſidered as a vertical Section of the Picture, and CE its Diſtance.

Then, E being the Eye, and EA, ED, &c. Viſual Rays from every Part to the Eye, it is obvious, that the parallel Mouldings have their places and proportions transfered from the Profile, to AD (as in Ex. 15 and 16) and, becauſe the Cornice projects beyond the Picture, as Db, it is projected to the Picture, at D; the difference is only in the proportion of the whole; for if the Picture was at the greateſt extreme of the Cornice, as ab; it is manifeſt, that the proportion of all the Mouldings, &c. would be leſs than thoſe on AD. (See Example 15th.)

The returning Face (*A*HIK) being perpendicular to the Picture, the Vertical Line (AD) is the Vaniſhing Line of all Planes in that Front. (Theo. 5.)

The meaſures of the Piers, Arches, &c. being ſet off, at *d*, *e*, &c. are proportioned, by drawing Viſual Rays *d*E, *e*E, &c. as in the foregoing Examples; and the Arches, are managed as in the 22nd Example, their Repreſentations are ſemi-Ellipſes; to particularize the whole would be ſuperfluous.

The Mouldings, in all ſuch Caſes, *viz.* when a true Section of them is parallel to the Picture, may be proportioned, by making a geometrical Section or Profile at S, where the Picture is ſuppoſed to cut the returning Mouldings, if they were continued beyond the Picture.

In this Example, the Diſtance is conſiderably too little, for taking in ſo much as is repreſented; in this poſition of the Picture. The deſign, of which, is to ſhew how diſtorted the returning Mouldings of the Pilaſters, &c. are, beyond the firſt Arch; which is neceſſarily the caſe in all ſuch Views, being parallel, except a ſufficient Diſtance be taken; in which Caſe, the returning Front would be greatly contracted. On the contrary, being both inclined, they may be ſhewn to much greater advantage, at a leſs Diſtance, without any ſenſible diſtortion; as may be ſeen in the next Example.

EXAMPLE

EXAMPLE XXXIV.

Is the Repreſentation of a Doric Arcade, with Columns, inclined to the Picture.

If the Problems and preceding Examples are underſtood, there is nothing more requiſite to the delineation of this Object; the Situation, Diſtance, and Poſition to the Picture being previouſly determined. Fig. 111.

In Prob. 21, the Rectangle may be conſidered as the Plan of ſuch an Object, promiſcuouſly ſituated; or the Plan of the Building in Fig. 106, inclined to the Picture, on VY. The foregoing Example ſuppoſes the Picture in the poſition DE; the Repreſentation, of which, is as Fig. 107.

The height of the Eye being determined, and the Center of the Picture (C) draw the Horizontal Line; and, at a convenient Diſtance, take AB for the Interſection of the Ground Plane, on which ſet off the ſeveral meaſures, at a, b, c, &c. and, having obtained the Vaniſhing Points of the Sides (both which are, in this Caſe, off the Picture) and their Diſtances, D and E, being fixed (by Prob. 12) compleat the perſpective Plan; as in Example 16, of the Doric Entablature.

The direction of the Lines, in the Figure, ſhew how it is effected, a repetition of it is almoſt unneceſſary; ſeeing that, the rectilinear part is managed as inclined Lines, in general; and, the plans of the Columns are repreſentations of Circles inſcribed in Squares, by Prob. 3rd.

Let the Reader obſerve, once for all, that, either a true geometrical Plan of the Building, muſt be drawn out, or its meaſures known; which are applied to the Ground Line, or other Interſection, as there is found Occaſion. The Profile, and meaſures of Elevation, muſt alſo be known; for it muſt be obvious, that every Object delineated by the rules of Perſpective, are projected by the true and real proportion of one part to another, which muſt be known, or conceived.

Aa is the width of the Step; ab, bc, &c. being made equal to the Sub-plinths of two Columns, draw aD, bD, &c. cuting AL (drawn to the Vaniſhing Point of the front Lines, indefinite) at H, K, L, &c. bh and kc are each equal to the break at the Pier, and hk to the opening between the Piers. Being firſt projected to AL, by means of the point D; from which they are drawn to the Vaniſhing Point of the Lines in the End, till they meet the returning Lines of the Front (as at I) which are obtained on FQ, drawn to the Vaniſhing Point. Ad is the meaſure of the break at the corner of the Step; and, by drawing dE, the internal Angles of the Step and Sub-plinth are obtained, at G. From *a* and *b* (making A*a* equal to the receding of the Arches, and *ab* to the thickneſs of the Wall) draw Lines to D, cuting AL, and from thence to the Vaniſhing Point of the End; where they cut the Diagonal GE, they are returned at the Front; the reſt is obvious, on inſpection. The Columns at MN recede from the Front, being in a Line with the firſt returning Column at the End, at G.

The perſpective Plan being compleated, or ſo much as is neceſſary (for thoſe Parts which cannot be ſeen, it would be uſeleſs to particularize, except for the affinity of thoſe which are) take *A*, at the proper Diſtance from the Vaniſhing Line (perpendicular over A) equal to the determined height of the Eye; and from it draw indefinite Lines to the Vaniſhing Points (as *AB*) and, from the Plan below, cut off ſuch portions as are the repreſentations of the ſeveral Parts, and proceed as in the foregoing Examples.

Here, the neceſſity of a Plan being formed, or, which is the ſame thing, the Lines proportioned below, by an imaginary interſection, AB, is evident; on account of the great Diſtance, and the Eye not being far out of the Ground Plane; otherwiſe, that operation would be unneceſſary; for, if the height of the Eye was equal to the Diſtance of AB, from the Vaniſhing Line, then, the Plan formed, by means of that Interſection, would be the real Baſe, or Seat of the repreſentation, of the Object, and no other would be wanted; in which, the expediency of the 12th Example is remarkably obvious. Being farther removed from the Vaniſhing Line, the ſeveral parts are more diſtinct, and may be done more correct.

Plate XXV.

The Step being firſt drawn, the Sub-Plinths, next, are but ſo many equal Parallelopipeds of a certain proportion and diſtance from each other (as in Example 4.) The places and proportions of the Columns are determined from their Plans below, by drawing Perpendiculars from them (or by Example 22) the Baſes and Capitals, being inclined, by Example 26; and the Entablature, by Ex. 17, 18, and 19.

The Balluſtrade, may be planed on the top of the Cornice, or at ſome diſtance from it; the Pedeſtals, or plane parts, are directly over the Columns, and equal to their Diameters at the top; and the places of the Balluſters are got by dividing the Line *AC*, perſpectively (by Prob. 8) into the number of Balluſters and ſpaces between them (equal or otherwiſe.) The Plinths, at their tops, are perpendicular over the Baſes; and central Lines (hi) being drawn, through each, the Balluſters are eaſily formed, from the known geometrical figure of them.

The Doric Entablature is deſcribed at large in Example 16 (but parallel to the Picture) how it may be inclined is ſhewn in the 18th and 19th Examples.

The Plane of the Arches being obtained, the Arches are repreſented by Example 22, and the Conſoles by the 20th; the reſt, it would be needleſs to deſcribe, as the previous Examples contain every neceſſary Leſſon for complex Objects.

EXAMPLE XXXV.

Is the Repreſentation of a compleat Building, with a Portico in the Ionic Order; having a ſpherical Dome, and Cupola.

Fig. 112.

At No. 1 is half the geometrical Plan, and, at No. 2 is the Elevation; in proportion to the Repreſentation, as one to four.

The Deſign is intended as an exterior Building, adapted to St. Stephen's, Wallbrook, but of a different Order; having a Front towards the Manſion Houſe.

This Example ſums up all the foregoing, in one entire Building.

The Horizontal Line being drawn, at diſcretion, and the Center of the Picture fixed, as uſual, let AB be the Ground Line, and D the Place of the neareſt Angle of the Building. The Diſtance of the Picture is 15 Inches; C is its Center.

The Inclination of the Front and End, to the Picture, is determined by the Line ab (No. 1) on which the Picture is ſuppoſed to ſtand.

The Diſtance of the real Ground Line from the Horizontal, being equal to the height of the Eye, which is elevated above the common height, yet is too low to form a correct perſpective Plan, in its true Place; therefore, at any convenient Diſtance take another Line *AB*, parallel to AB, and proceed as in the laſt Example.

Having laid down the diſtance of each Vaniſhing Point, at E and C, as uſual; take the known meaſures of the ſeveral parts of the Building and apply them on each ſide, from the Angle at *D*. Compleat the exterior Plan, *CDEF*, as in the laſt Example; from which, the ſeveral parts of the Building, above, are proportioned; as the perpendicular doted Lines ſhew. F is the Vaniſhing Point of Diagonals; i. e. of a Line biſecting the Angle of the Building.

It muſt be obvious, that any Line (AB or *AB*) being conſidered as the Ground Line, will give the ſame proportions; which, being properly conſidered, there is nothing more in it, than a ſuppoſition of the Eye being more elevated above the Ground Plane; and conſequently it is more ſeen and better defined.

The exterior Plan being compleated, from which the corners of the Steps are determined, and drawn in their true places, by Example 13. Alſo the Sub-plinths of the Columns, v, u, x, &c. from their correſponding Plans below; the Diagonals of which, being drawn, give a central Line, the Axis of each Column; the heights of the ſeveral parts are determined on the diagonal Interſection, at D, F being the Vaniſhing Point.

The Columns with their Baſes are drawn and finiſhed as in the laſt Example; and the Capital, being the antique Ionic, by Ex. 31ſt. The interior part of the Portico has a Pilaſter oppoſite each Column, which are alſo determined from the Plan, below; ſome of which are hid behind the Columns; perpendicular Lines from the Plan, ſhew how much, and where they are ſeen entire.

The Plane of the Front, of the Pediment, over the Portico, being much inclined to the Picture, its Vaniſhing Line is very remote; the Diſtance of the Vaniſhing Point of horizontal Lines, in that Face, may nevertheleſs be aſcertained, (as E) by Ex. 12, and the Lines drawn by the Expedients, in the 13th.

As the Diſtance of the Vaniſhing Point of horizontal Lines on the left Hand, would fall off the Picture, it is taken half, at C; and conſequently, the meaſures applied on the Interſection or Ground Line (*AB*) for that ſide, are alſo, each half of the real meaſure, agreeable to the 13th Theorem, and Prob. 6 and 7.

The front Pediment is thus determined. If the Cornice projected equal to the Steps, which the Picture is ſuppoſed to touch, a Right Line drawn through D, perpendicular to the Ground Line, would be the Interſection of the Plane of the Pediment; but, as its projecture is leſs, it will fall to the left hand of it; as the Line *ab* (No. 1) which is a continuation of the front Line of the Cornice, cuting ab, at *d*, indicates ſufficiently.

Or, having determined the perſpective Plan of the Cornice, at a, b, c, produce ab, till it cuts the Ground Line at d, and draw de perpendicular, which is the Interſection, of the Plane of the Cornice in the Pediment.

Then, having made Je equal to the known height, and determined the front Line of the upper Moulding, as fg, ſet up the height of the Pediment, from e to i, and draw ik, tending to the Vaniſhing Point (by Prob. 13.)

Biſect fg (perſpectively) at h, and draw hl perpendicular, cuting ik, at l, the true pitch, or middle of the Pediment, which may be compleated, by Ex. 20.

The Pediment in the other Front, on the left hand, may be projected by means of its Vaniſhing Points, by the ſame Example.

To this Front is added a Pedeſtal which is continued around the Building; more for the ſake of diverſifying the Leſſon, than propriety in the Building; the heights of the Pedeſtal, Windows, &c. are proportioned on the Interſection, DK, of that Front, at *a*, *b*, *c*, &c. for the Niches and Windows over them.

The Body of the Building being compleated, the ſeveral parts being determined from the Plan below, with the breakings or returnings of the Cornice, and other Mouldings (as in former Examples) let the Circle of the Dome be planed, at GH, perpendicular Lines from the extremes of that Ellipſis, give the extreme apparent edges of the cylindrical part, IK, below the Dome; of which Cylinder, the Ellipſis, on the Ground Plane, may be ſuppoſed its Baſe.

The curve Lines, in the Cornice, around the Dome, are deſcribed in their reſpective Planes; which are thus determined.

From the Vaniſhing Point on the left, draw through the Center, S, of the Plan below, cuting the Ground Line at J; which repreſents a Line paſſing through the middle of the Plan (No. 1) ſuppoſing a Section made by a vertical Plane through the middle of the Building.

Draw JL, perpendicular, the Interſection of ſuch a Plane with the Picture (Prob. 3) on which, all the meaſures of the heights of the Dome, &c. are applied, in the ſame ratio as on de, in the Elevation (No. 2) at M, N, &c.

From M, the height of the Cornice, draw a Line parallel to the Ground Line, which is the Interſection of the Plane of the Cornice, with the Picture. A Right Line, drawn from M to the Vaniſhing Point on the left, will cut a perpendicular Line from S at ſ, the repreſentation of the Center of the Circle of the Cornice.

From C, the Center of the Picture, draw C*S*, through ſ; and through ſ, draw mn, parallel to the Interſection; make *S*R and *S*T each equal to half the Diameter of the extreme Circle of the Cornice; and draw RC and TC, cuting mn, at m and n. mn is a Diameter of the Repreſentation, parallel to the Picture; the whole Circle may be compleated, by Prob. 2, Sect. 8th.

Plate XXV.

After the ſame manner, all the other Circles, of the Corona, &c. may be projected, in their reſpective Planes; as the ſeveral Circles in parallel Planes, by Ex. 28.

The Cornice, &c. being compleated, ſet up, from N, the height of the Dome, and take ſeveral Diviſions *a*, *b*, &c. anſwering to the ſame ratio on de, in No. 2.

Through theſe diviſions, parallel Lines being drawn, as rs, each may be conſidered as the Interſection of a Plane, cuting the Dome (as *r s*, No. 2) in each of which, an Ellipſis being deſcribed, repreſenting a Circle of the true diameter, made by that Section; a Curve deſcribed over all their extremes (as in Ex. 25 and 26) is the true exterior contour of the Dome; which may be conſidered as an Ovolo, inverted; as in the Dorie Capital.

By any other means (the Dome being plain) I cannot conceive it poſſible to obtain the true perſpective Figure of a Dome; which is generally higher than a Hemiſphere; eſpecially in a lofty Building, which would otherwiſe appear flat, below; beſides, it is a more graceful Figure; as the magnificent Dome of St. Paul's ſufficiently evinces, to a judicious Eye. But, being a Hemiſphere, or other Segment of a Sphere, the Eye being below its Center, the Contour of the exterior Curve, on the Picture, will be elliptical. (Th. 3rd, of curvilinear Perſpective.)

By this method, it may be truly projected, let the Figure be what it may, whether a Hemiſphere or leſſer Segment. Nevertheleſs, a judicious Eye, and judgment, guided by a thorough knowledge of Perſpective, may direct a ſteady Hand to trace a Curve, ſufficiently correct, in common Caſes; the height and Vertex being truly aſcertained.

The Dome in the Frontiſpiece is octagonal; and, by means of ſeveral Octagons, repreſented perſpectively, in various parallel Planes, the true Figure of it may be projected. Or, by means of ſeveral Points, in an Angle; for, if the true Curve of an Angle be obtained (as at Y, No. 3) from the Curve, X, of a vertical Section through the Dome, perpendicular to two oppoſite Faces (by means of perpendicular and horizontal Ordinates, *ab*, *cd*, &c. having made h*A* equal to the angular projecture, and divided it at a, c, e, as *A*h is divided, at *a*, *c*, *e*; perpendiculars, a b, &c. being drawn, from the points *b*, *d*, &c. in the curve at X, draw lines, parallel to *AA*, cuting them at b, d, &c. through which the angular Curve A d g may be deſcribed) then, having drawn any one, perſpectively, they may be tranſfered from one to the other, by the Vaniſhing Points of horizontal Lines in the Cornice around it, at the foot of the Dome.

By the ſame means, the Ribs, in this Dome and Cupola, may be deſcribed.

After this deſcription of the Dome it would be loſs of time to deſcribe particularly the projection of the Cupola. From the Plan below, its place and dimenſions are truly aſcertained, together with the place of each Pilaſter and Window; from which perpendicular Lines are drawn; and, the height of each part ſet up, on the Vertical Interſection, H N, as the Dome, and other Parts, below; of which, the Cupola is a ſimilar Figure, nearly.

EXAMPLE XXXVI.

Is a Repreſentation of that much famed Building, by Inigo Jones, of St. Paul's, Covent Garden; and the Buildings adjacent.

Plate XXVI.

As the laſt Example was a Leſſon for an entire Building, ſo this is intended for a Leſſon in detached Buildings, or diſtinct and ſeparate Objects; which may be conſidered as a Street-View.

Having, in the laſt, given a compleat perſpective Plan of the whole Building, it would be multiplying Examples to little purpoſe to do the ſame thing here; let it ſuffice, therefore, to ſuppoſe, that the Plan of the whole, in this piece, is truly deſcribed on the Ground Plane, on which the Objects ſtand; their meaſures being applied to the Interſection of the Picture or Ground Line, as in the preceding Leſſons, which it would be quite ſuperfluous to repeat; I ſhall, therefore, only make ſome general Remarks.

It may be obſerved, that the Church is nearly in the ſame Poſition as the laſt Object, but a much ſimpler, and conſequently an eaſier Figure to repreſent.

The Portico is of a Tuſcan Order; which, for its ſimplicity and ſingular Stile, is much eſteemed by the admirers of the antique Grecian Temples, of which, this is eſteemed a perfect Model. The great projecture of its Cornice (if it may be ſo called) has a bold and ſtriking appearance; owing to the great effect of Shade it occaſions. The attached Parts, at Y and Z, which is the ſtaircaſe into the Gallery, ſeem not to belong to the Body, but are adapted merely for conveniency; I am of opinion that it would make a better Figure without them. The other, at V, is the Veſtry Room; which, with a ſimilar Object, on the other Side, adds to the appearance of the back Front.

How theſe ſeveral parts are projected, may be ſeen in the laſt Figure; having formed a Ground Plan of the whole; their ſeveral heights being ſet off, on the vertical Interſection of the front Plane, or proportioned to the Pilaſter, at the corner.

The two Gates, or entrances into the Church-Yard (X) are a great ornament to the Church, and graceful in themſelves. After what has been done, there is nothing ſingular in their Conſtruction, the geometrical Figure being known.

The Houſes, one on each ſide, are equally diſtant from the Church; 'tis pity they are not both in the ſame ſtile, as that on the Left, for the ſake of uniformity. The Hut, adjoining to the Houſe on the Left, at W, is the Pariſh, Watch-Houſe. I could wiſh ſome other place had been deſtined for it, as it blocks up an Avenue, and breaks the regular Order.

The Fronts of the Houſes and the Gates are in a Right Line, much inclined to the Picture; the Portico projects before them. There is no occaſion for a Ground Plan of the whole, for the meaſure of each Part being known, the Line may be proportioned, by the Problems in the 4th and 5th Sections; and, the heights are determined on the Line AB, which may be uſed as an Interſection of the whole, conſiderd as one Plane.

The Pediment has been frequently deſcribed; and, the Bell-Turret may be determined by the length of the Roof; or, more correctly, from a Plan formed below; as the Cupola in the laſt Example.

On the Left is ſeen the opening into Henrietta-Street, at P; and one of the Green-Sheds, at Q; the place, of which, is obtained by its known diſtance from the Church, and the line of Direction in which it ſtands, reſpecting the Church, projected if neceſſary. Or, the Ground Line may be ſuppoſed on this ſide of the Shed, parallel to the bottom edge of the Picture. The Horizon is about the natural height of the Eye; the Center of View is at C, in the middle of the Picture, and the Diſtance is about 16 Inches.

EXAMPLE XXXVII.

Is the Repreſentation of another well-known Building, viz. The Royal Hoſpital, of Invalids, at Chelſea.

Plate XXVII.

The Stile, of this Building, is ſomewhat ſimilar to that of St. Paul's, Covent-Garden, in reſpect of its ſimplicity. There is, in the Guſto of the whole, ſomething remarkingly pleaſing; the Symetry of the ſeveral parts, in reſpect of each other and of the whole, is finely preſerved, in a regular and gradual ſubordination, from the Principal to the ſeveral Offices; the agreeable arrangement, of which, is uniform, and perfectly harmonious. Here, the attention is not attracted by the richneſs of the dreſs, or maſterly execution; the mind is not captivated by grandeur and magnificence, but moſt agreeably entertained, with an elegance of Deſign; ſimple, yet majeſtic; and comely, though unattired; perfectly agreeable in its native ſimplicity, and almoſt naked Beauty, devoid of every luxuriant Ornament.

In the laſt Example, the Line of the Building was much inclined to the Picture; in this, it deviates little from a Perpendicular; which is far more agreeable than to be perfectly ſo.

In

Plate XXVI.

In this View, many, who pretend to know Perſpective, would immediately fix on the Point V, in which all the horizontal Lines, of the Front, converge, to be the Center of View; but it is not ſo; for, according to my general maxim, it is at C, in the middle of the length; and, notwithſtanding there is not one Line vaniſhes in it, yet, it governs the whole.

If V had been the Center, or Point of View, the Lines BD and HI, &c. would be parallel amongſt themſelves, and to the Horizon; but, they converge to a Point, at a great Diſtance, on the left Hand.

The Diſtance of the Picture is about 19 Inches; at which Diſtance (the Eye being perpendicularly oppoſite to C) the whole will appear as the Original, at the ſtation intended; and, I am perſuaded no Perſon will ſuppoſe that it would have a better effect, or ſo good, if V had been the Center of View; for, the Optic Angle would then be almoſt double of what it now is; and conſequently, the Repreſentation would differ more from the true Appearance; which, now, is inconſiderable, the Optic Angle not exceeding 34 Degrees.

The Inclination of the Building to the Picture, and the Diſtance, being determined, the Vaniſhing Point, V, is fixed by Prob. 12; and the indefinite Line, GV, being drawn, it is proportioned as uſual, by ſetting off the real meaſures, or their ratio, from G, at K, L, &c. and drawing Lines, which repreſent Viſual Rays, to E, in the Horizontal Line.

The meaſures are taken half, becauſe EV is but half the Diſtance of the Vaniſhing Point, V; as the whole Diſtance exceeds the bounds of the Picture.

The horizontal Lines on the returning Sides, whoſe Vaniſhing Point is at a great Diſtance, may be drawn by the Expedient, No. 5, Prob. 13. Or; having proportioned the Line of the Angle, at AB, into the ſeveral Diviſions, at a, b, c, &c. for the Windows, &c. and having drawn one Line to its Vaniſhing Point, as BD (by the 12th) the Diſtance of the Vaniſhing Point being determined*; produce BD, and, at a convenient Diſtance, draw FG parallel to AB, and proportion FA in the ſame Ratio as AB (by Prob. 33, Geom.) draw a*a*, b*b*, &c. which will tend to the ſame Point.

But, when the Inclination is not very great, the beſt Expedient is to fix a Lath to the Board and make uſe of a long Ruler; or, in large work, fix a ſmooth String to the Vaniſhing Point, and ſtrain it, in a Right Line, to the ſeveral Points a, b, c, &c. and then apply a Ruler to the direction of the Cord.

The Hip of the Roof, at H, may be determined from a Plan, on the Ground Plane; or by the known Inclination of BH to the Picture, and its Vaniſhing Point; and, the Line HI by the ſame, as in the 13th Problem.

The Window Frames, in the Roof, being drawn, as the other Windows, and the ſtreight Cornice, fh is obtained, by which, the Pediments over them may be completed, as in Example 20; by finding the Vaniſhing Points of fg and gh, in the Vaniſhing Line of the Front, paſſing through V, perpendicular; the beſt method for the front Pediments. The Vaniſhing Line of the ſide Planes is too remote to uſe for thoſe Pediments; which, in ſuch caſe, differ but very little from their geometrical form and proportion.

The Cupola is beſt done by a Ground Plan, as in the 35th Example.

The Columns and Pediment over them have nothing particular in the delineation; the middle Pediment is more contracted than the hither one, ſo that the Mouldings are ſeen on one ſide only; but is done after the ſame manner; and, if they have the ſame inclination to the Horizon, the Lines in them have the ſame Vaniſhing Points as thoſe over the Windows; becauſe the Pediments are all in parallel Planes, and conſequently they have the ſame Vaniſhing Line †.

† Theo. 5. and Cor. 1.

The Trees (on the right hand) excepting their diſtances from each other, are not ſubject to the Rules of Perſpective; becauſe no proportion of the Boughs can be taken. The Rails and Poſts have nothing ſingular in them; as the Rails are

* This Vaniſhing Point is about ſeven Feet Diſtance, from the Center of the Picture.

parallel to the front of the Building, they have conſequently the ſame Vaniſhing Point (V) as the horizontal Lines, in the Front; and the Poſts, being equally or otherwiſe ſpaced, are determined as in all other ſimilar Caſes.

The heights of the Figures, in the Walk, are thus determined.

Take any Point, L or M, in the Ground Line, and make MN equal to the height of a Figure (by the Scale of proportion, of the meaſures of the Building) and draw MO and NO, to any Point in the Horizontal Line; then, wherever you intend a Figure, as at *a*, *b*, or *c*, draw *a*m, *b*m, &c. parallel to the Ground Line cuting MO, at m, or *m*; and draw m n, or, *mn*, parallel to MN, which is the height of the Figure in that place, repreſenting an equal height to MN.

Or, wherever you intend a Figure of any kind, draw a Line through the place of the Feet, cuting the Ground Line and Vaniſhing Line, obliquely; and, where it cuts the Ground Line, draw a perpendicular, and ſet up the height of the intended Figure; draw another Line to the ſame Point in the Vaniſhing Line, between which, a perpendicular being drawn is the height of the Figure.

By the ſame means, the proportion of any other Object may be determined.

EXAMPLE XXXVIII.

Is a View of the Queen's Palace in St. James's Park, and adjacent Buildings.

This neat and elegant Building was formerly the Town Reſidence of the Dukes of Buckingham; which Title becoming extinct, it had for ſome Years been occupied by a diſtant branch of that ancient Family, without the Title, who made no Figure, ſuitable to ſuch a noble Manſion; inſomuch, that it was gone greatly out of repair when purchaſed by his Majeſty. It has, ſince, been much enlarged, repaired, and beautified, and made the Winter Reſidence of the Royal Family. The octagon Building, on the left hand, was added at the ſame time, and is the Library. Plate XXVIII.

Since which, there has been great improvements made; the ſemicircular Area, in front, has been enlarged; but I think it would have been more advantageous, had the Palliſadoes ſtood on a dwarf Wall; for want of which, they are not diſtinguiſhable, at ſome Diſtance from it. The Gate leading to Chelſea, and the Lodge are alſo new; and, ſeveral old Trees were cut down, to make a more ſpacious and open View to the Canal, &c. in the Park.

This Building would, with propriety, admit of a full Front View; but ſuch a one is by no means pictureſque; as, one Side, in ſuch caſe, is but a duplicate of the other, which is not the caſe in this View; the Center, of which, is in the middle of the Picture, as uſual.

The Front is inclined, and conſequently, the horizontal Lines vaniſh, but at a great diſtance, becauſe the Inclination is very great; the Diſtance of which Vaniſhing Point is determined by the 12th Prob. The Vaniſhing Point of horizontal Lines in the other Faces is at V, in the Pavilion on the left hand; the Horizon is high, for the conveniency of ſeeing the Area, more commodiouſly.

The Diſtance of the Picture being determined*, and the Vaniſhing Point V, the Diſtance of the other is a third Proportional, to CV and the Diſtance of the Picture, found by ſquaring the Diſtance, and dividing by CV. (Pr. 12.) (Pr. 31. Geo.)

As the octagon Building has two Faces parallel to the Fronts of the Houſe, the adjacent Faces vaniſh in the Vaniſhing Points of Diagonals; i. e. all the horizontal Lines, in one Face of the Octagon, vaniſh in that Point where a Line biſecting the Angle, made by the Radials of horizontal Lines, in the Front and End of the Building, cut the Horizontal Line; and the other, on the Left, where a Line, making a Right Angle with the Line of biſection, cuts it; as it has been exemplified in various caſes, in Mouldings, &c. both which fall out of the Picture.

* In this Picture it is about 12 Inches.

Plate XXIX.

On account of the great Diſtance, and the Horizon being high (between the Mouldings in the upper part) their inclination is not perceptible; as is frequently the caſe, in Views taken at a tolerable diſtance, and perhaps from an eminence, which is generally made choice of in Landſcape Views; otherwiſe, the Ground would be too much contracted, unleſs diverſified with riſing Grounds, &c. The diſtant Buildings, in ſuch caſe, are repreſented nearly geometrical; for, unleſs they are ſituated conſiderably above or below the Eye, in proportion to the Diſtance, the inclination of the Lines is not diſtinguiſhable; and, if they are obliquely ſituated to the Picture, their Faces are contracted, almoſt geometrically; ſo that, diſtant Objects, of all kinds, are not cognizable to the Rules of Perſpective, otherwiſe.

It is unneceſſary to ſay more, in reſpect of this Object, as Rules have been given for projecting all kinds of Figures of which it is compoſed; and finding the Vaniſhing Points, &c. at any given Diſtance and poſition of the Picture.

Figure 106 exhibits a general Method for contracting the Faces, &c. of an Object, and for obtaining the true place of each part, in reſpect of its bearing with others (being detached) from any determined Station, which, in many caſes, is the beſt method of proceeding, and the leaſt liable to error.

EXAMPLE XXXIX.

How to repreſent Doors and Window Shutters, open, in any given Angle; or at pleaſure.

Fig. 113. Let AV be the indefinite Repreſentation of the Side of a Building in any horizontal or vertical Plane; which, from the point A, is required to be perſpectively divided into certain finite parts, repreſenting Windows, Piers, &c.

By the 8th Problem, Ac and VE are drawn parallel, between themſelves (VE may be conſidered as the Vaniſhing Line, and Ac, the Interſection, of whatever Plane the Original Line is in) make VE equal to the Diſtance of the Vaniſhing Point (V) of the Line AV; and, on Ac, take Aa, ab, &c. equal to the known proportions of the Piers and Windows, or Apertures of any kind.

Draw aE, bE, &c. cuting AV in the Points *a*, *b*, *c*, &c. which are the repreſentations of the Original Points, a, b, &c. Wherefore, if ab be ſuppoſed the geometrical proportion of a Window, then, *ab* repreſents the ſame perſpectively; and ſo of any other diviſion, as *bc* of a Pier, &c.

N. B. Let it be obſerved, that it does not depend on the real meaſures being applied on Ac, but, that they are in the ſame ratio, reſpectively, as EV is of the diſtance of the Vaniſhing Point; agreeable to Theorem 13th, which is frequently exemplified in the preceding Work; and, whether AV repreſents a Line perpendicular or inclined to the Picture, there is no difference in the proceſs.

Fig. 114. Let AB repreſent the aperture of the opening of a Door, in the ſide of a Room, &c. which is required to be ſeen open, in any poſition, at pleaſure.

Now, AB, the width of the Door, is the radius of a Circle, which it would deſcribe if the Door revolved quite around; and conſequently, the Door itſelf (being a Rectangle) would deſcribe a right Cylinder, of which, BC is its Axis.

By Prob. 10th, Sect. 4th. or, Pr. 2, Sect. 8th, deſcribe the repreſentation of a Semicircle, A*a*F, whoſe Radius (given or found) is AB.

It is manifeſt, that AB the perſpective width of the Door, would, in its ſemi-revolution (from AB to BF) deſcribe the ſemi-Ellipſis A*a*F; that is, the point A (B being fixed) would deſcribe the ſemi-circumference of a Circle: and conſequently, in whatever poſition the Door is open, the point A will be ſomewhere in that Semicircumference, as at a or *a*; wherefore, aB, *a*B, &c. are alſo repreſentations of Radii, of the ſame Circle.

Take the Point a or *a*, in the Circumference, at pleaſure (according as you require the Door more or leſs open) and draw aB or *a*B, which produce to its Vaniſhing Point, H or *H*. Draw the Perpendicular ad, or *ad*, and, through C, draw HC, or *H*C till it cuts ad or *ad*, at d or *d*; then is aBCd, or *a*BC*d*, the repreſentation of the Door, open in the poſition required.

Or,

Or, if the Angle of the opening, on either ſide, be known or determined; produce A B to its Vaniſhing Point (V); S being the Center of the Picture, draw S E perpendicular, equal to its Diſtance, and join E V; make the Angle V E H, or *V E H*, equal to the Angle determined; which will give the Vaniſhing Point H, or *H*, of the top and bottom Lines of the Door, in the poſition required.

If the Door be required more open, as *A* B, making an acute Angle, *A* B F, on the other ſide; then, the Vaniſhing Point will be on the other ſide of V.

The proceſs is the ſame, in every Poſition.

After the very ſame manner, Window Shutters, being vertical, are determined; the Radius, of each, being half the width of the Window, A d or d D. Fig. 115.

A B C D is the aperture of a Window, to which there is required the repreſentations of folding Shutters, open in any poſition or angle, at pleaſure.

Produce A D and B C, both ways, and biſect either, perſpectively (as B C) and, make B *a* and C *g* each repreſent half B C. On *a d* and *d g* deſcribe repreſentations of Semicircles, *a b c d*, and *d e f g*; in which, according to the determined angle or opening of the Shutter, take *b* or *c*, *e*, *f*, or *g*; from which, draw perpendicular, and draw *b* B, or *e* C to its Vaniſhing Point, at S or V; from which points, draw S A, or V D, and produce them cuting the Perpendiculars, at b or e, which compleats the Shutter A B *b* b, or C *e* e D.

A B *c* c repreſents another, leſs open; and C *f* f D is one parallel to the Picture, conſequently, it has the true figure and proportion of half the Window.

A B *a* a, and C *g* g D repreſent others, quite open, againſt the Wall.

The Shutters Y and Z deſcribe Semicircles in vertical Planes, as a b c d, or *a b c d*; or, more properly, they deſcribe half horizontal Cylinders. Fig. 116.

The firſt, being parallel to the Picture, are Semicircles†; the others are perpendicular to the Picture; or they may be inclined, in any Angle whatever; the Vaniſhing Points of the deſcribing Lines are in the vertical Vaniſhing Line (V L) of the Plane of the Circle; as at V and L, &c. the front Lines are parallel. † Theo. 9, Cor. 5.

In the former they are parallel to the Picture; the front Lines vaniſh in the Center.

SECTION X.

OF INSIDE VIEWS, IN GENERAL.

THE application of the Rules of Perſpective, whether to the interior or exterior parts of Objects, is the ſame, for, Planes and Lines, of which Objects are compoſed, are ſtill the ſame however ſituated; whether they form internal or external Angles, of Objects ſeen externally; or, forming Rooms or Concavities of any kind. Nevertheleſs, at firſt Sight, there appears a difference; for I believe that, many who have practiſed Perſpective with tolerable ſucceſs, in exterior Objects, are ſomewhat puzled, at the firſt attempting an Inſide View; not knowing, rightly, where to begin or where to leave off; how to place their Picture, or determine its Diſtance.

I have ſeen an attempt to repreſent an Octagon Building, internally, in which Picture were introduced ſeven Sides or Faces, out of the eight; it was a misfortune that the other could not be ſeen alſo, quite around, and then it would have been a maſter-piece, indeed. However, from what was ſeen, and the tendency of the Lines, it was manifeſt, that the Diſtance of the Picture was not more than one fourth part of its width, or length; and conſequently, the Optic Angle was above 120 Degrees, which ought not on any account (except when the Eye is confined to the true Point of View, always) to exceed 60.

Plate XXIX.

'Tis the ſame thing, if, in order to ſee the whole of a Dome and Cupola, internally, we advance ſo far into the Building, that the repreſentations of ſuch Subjects are only fit for horizontal Pictures, or Cieling-Pieces; for, when they are repreſented on a vertical Picture, they appear (at a proper Diſtance to take in the whole) as if falling, or not upright. Yet are theſe things to be met with in Pictures and Prints, by Men of ſome diſtinction in the Arts.

That the Perſpective Repreſentation of the inſide of a fine Building is more difficult to manage than the exterior is certain, and is manifeſted in theatrical performances; which, being repreſented on ſeveral detached Planes, it is impoſſible, by the Rules of Art, to make them correſpond in every Point of View; but, they are very rarely connected in any one Point. There are, undoubtedly, many fine Performances of the kind; yet, without attempting to diſparage their Authors, I am confident, that, were they better acquainted with Perſpective, they would produce better and more natural Repreſentations; even that famed Scene in Cymon is a jumble of inconſiſtencies, both in point of Deſign and Execution; although it was a bold attempt out of the common mode of Repreſentations.

Few Artiſts have made Perſpective ſo much their Study, to know how to proportion one part to another, on detached Scenes, ſo, as to make the whole unite in the proper Point of View, whether the Repreſentation be internal or external; indeed, from the conſtruction of the Theatres, it is ſcarce poſſible to be done; nor are the Rules of Perſpective ſufficient, for the purpoſe, without a tolerable knowledge of Lines, geometrically. It is the leaſt qualification of a Scene Painter to be excellent in Landſcape, in which a ſmall knowledge of Perſpective is requiſite; but, in order to execute Deſigns in Architecture with correctneſs, and a juſt proportion of the ſeveral Parts, requires a thorough knowledge of Perſpective. Is it not ſurprizing, that all who are concerned, or any way engaged in Scene Painting, do not make Perſpective their immediate ſtudy; being the baſis, the very ſoul and exiſtence of their Profeſſion? yet, to my certain knowledge, ſeveral Artiſts, employed in it, are not only totally ignorant of it, in Theory, but they are, almoſt, wholly unacquainted with its Rules, which, to me, is moſt unaccountable.

In inſide Views, the bounds of the Picture limit the whole, every way, which renders the operation, in ſome caſes, more difficult than external Views.

In order to ſhew the inſide of a Room, Temple, &c. properly, a Section is ſuppoſed to be made; that is, the hither Wall (or incloſure of any kind) is ſuppoſed to be removed, and the whole Inſide laid open to View; ſo that, a proper Station and Diſtance may be taken, from which, the whole, or as much as is required, may be ſeen under an Angle not too large; for, to ſuppoſe that we can be within a Room and exhibit the whole, or nearly, is truly abſurd; yet it has been attempted, by thoſe who have not a juſt notion of Perſpective.

In reſpect of determining the places and proportions of Doors, Windows, &c. it muſt be obvious, there can be no difference whether they are interior or exterior; the whole of which is contained in Prob. 8th, and has been univerſally applied throughout the Work; particularly in Ex. 2nd and 4th, alſo in the 16th, 19th, and 21ſt, in finding the places of the Mutules, Modilions, Blocks, &c.

EXAMPLE XL.

How to repreſent the inſide of a plain Building; the Section, by the Picture, being parallel, to the Ends of the Buildings.

In repreſenting the Inſide of any Room, Church, &c. it is uſual to take the Station in a Line drawn through the middle of the Building, which indeed appears the moſt rational; but in ſuch caſe, in a regular Building (one ſide being a duplicate of the other) it is not ſo pictureſque, as when the Station is towards either ſide, as at S; from which, the inſide of the Room is to be viewed; by which means, either Side may be ſhewn to greater advantage, being more oppoſed to the Eye than the other.

Fig. 1.

No. 1. is the Plan of a Priſon; on the left hand are Doors into ſeveral Cells, &c. and on the right are Doors and a Window, into the Area, or open Yard; at the farther end is a paſſage, at R, leading to the Keeper's Apartments, and, at the End is a Stair-caſe; a few Steps of which lead, through a Door, to the Courts of Juſtice. V, U, &c. are Piers ſupporting the Roof. The Elevation, and Sections, are ſuppoſed to be underſtood. Plate XXX. Fig. 1.

Let AB be the place of the Picture; by which, a ſection of the Building is ſuppoſed to be made, and the hither Wall removed; ſo that, the whole of the Inſide, beyond it, being open to view, is intended to be repreſented on the Picture AB, from ſome determined Station, as at S; as ſeen through the Picture, being ſuppoſed a tranſparent Plane. SP, SQ, &c. may be ſuppoſed Viſual Rays cuting the Picture, which give the true place of each part of the Building thereon; indicating alſo, what Parts can be ſeen and which not, being obſtructed by the Piers.

Let AB be produced to b, a b is the Ground Line of the Picture; but for the conveniency of making a correct perſpective Plan, take AB at a proper diſtance, below; in which, let A B be taken equal AB (No. 1.) or in any other proportion to AB; it is, here, taken double, or twice.

Let the Horizontal Line (ECG) be drawn, at a proper diſtance from a b, equal to the height of the Eye above it; in which, take DC double AC (No. 1. where a perpendicular, from S, cuts AB) C is the Center of the Picture; alſo, make CE double SC (No. 1.) E is the Eye, or Point of Diſtance, by which the ſeveral parts of the Picture are proportioned; C D and C F are each half the Diſtance. HI, drawn through C, is the Vertical Line of the Picture; which determines the ſituation of the Eye, in reſpect of the Building.

Let the perſpective Plan be drawn, making *IA*, *IB*, &c. each double the meaſures in the Plan, for the thickneſs of the Piers, and their diſtance from the Station Line. How it is perfected, it would be impertinent to explain; the Point E being uſed, where the meaſures *Cb*, *Cc*, &c. are the full meaſures (that is, double thoſe at ab, bc, &c. in the Plan, No. 1.) and, when it is more convenient to apply the half meaſures, make uſe of the Point D, or F, on either ſide, as occaſion requires; as *AD*, for the Door at P; the full meaſure being applied from A to *b*, gives the ſame Point p, as the half meaſure, at *D*.

A*Q* is equal to AQ (No. 1.) which, by drawing a Line to D, cuts AC at *Q*, the place of that Corner, in the perſpective Plan; making A*Q* repreſent twice AQ (No. 1.) ſeen at the Diſtance CE; or equal AQ, at the Diſtance CD.

AC, or CC, being proportioned at b, c, d, &c. perſpectively, for the ſeveral Piers, &c. Lines parallel to AB, being drawn, determine thoſe on the other ſide, at g, k, &c. and againſt the Wall, at n, o, &c. by which means the Plan is compleated, in Perſpective, from which the Picture is delineated, as follows.

The meaſure, on AB, being tranſfered to ab (that is, reſpecting the ſpace between the Wall and the Piers, the thickneſs of the Piers, and diſtance between them, in front) draw a*C*, c*C*, &c. and, at *A*, draw *AB* perpendicular, the vertical Section of the plane of the Piers, on the right hand, with the Picture; which, being fartheſt from the Vertical Line, is beſt, for proportioning the heights of the Piers, Arches, &c. which are ſet off, from *A*, at a, b, c, &c. according to known, or determined, meaſures of their heights, in proportion to the Plan.

Draw *A*C, a*C*, b*C*, &c. and, from the Plan below, draw perpendiculars from g, i, &c. cuting them, at *g*, *i*, &c. Perpendiculars being drawn from all the other correſponding parts in the Plan, as l, m, n, o, &c. parallel Lines drawn through *g*, *f*, &c. determine the true places of all the Plinths, and Borders, at the tops of the Piers, from which the Arches ſpring.

The Arches, on each ſide the Avenue, in front, are Semicircles; wherefore, their Centers being obtained, at S, by biſecting the Diameter R T, they are readily deſcribed; and, by drawing Lines, from S, on each ſide, to C, the Centers of all the receding Arches are determined, in thoſe Lines.

Plate XXX.

In the returning Fronts, the Arches are ſemi-Ellipſes; their heights are the ſame as the other, determined on *AB*, at d; Cd being produced to the crown of the Arch, at *D*, obtained from below, at N, projected. For, ſeeing that the Picture, on A B (No. 1.) is nearer to the Piers than the Wall, conſequently, it does not cut the Arches in the middle; wherefore, all the Wall, with the eliptical Window, on this ſide of *AB*, projects through the Picture and is projected again to it. *C* is the Center of that Arch; the Curve, *Def*, is determined after the ſame manner as if they were circular, it being obvious, that the ſame lengths, from *i* to *k*, or from *f* to *l*, between the Piers, may repreſent any length of the Indefinite Repreſentation, *AC*, according to the diſtance of the Eye (Cor. 5. Th. 12.) *AC* remaining the ſame. Hence it is undeterminable whether thoſe Arches repreſent Circles or Ellipſes, of any proportion, the Diſtance of the Picture being unknown; for the two middle Arches are more excentric than the other two, ſeeing they have the ſame height, and the Piers are wider aſunder*.

The eliptical Windows, over the Arches, are deſcribed after the ſame manner; as the firſt on the right hand, half a Window, inſcribed in the Rectangle *abgh*, indicates ſufficiently.

The groined Arches, over the ſide Avenues, have an appearance of difficulty without the reality. If a Line be drawn through the crown of the Arch, at c, to C, and, diagonal Lines being drawn between the corners of oppoſite Piers, in the perſpective Plan, as r t and m s, interſecting at u, from which, a perpendicular, being drawn, cuts cC at *c*, the center of the Groin. Or thoſe Diagonals may be drawn, at the tops of the Piers, and having drawn perpendicular Ordinates to the circular Arch, at *A*, *B*, they may be transfered, to the Diagonals; where, perpendiculars being drawn, and, to C, Lines drawn through the points a, b, c, &c. in the front Arch, cuting the diagonal Ordinates at *a*, *b*, *c*, &c. through which the Curve may be accurately deſcribed, by encreaſing the number of the Ordinates at diſcretion.

The Arches in the middle Avenue are alſo ſemi-Ellipſes, geometrically deſcribed, being parallel to the Picture; the tranſverſe Diameter of the firſt is *FG*, and *CD* half the Conjugate. Having determined ſeveral Points, 1, 2, 3, at equal diſtances, or otherwiſe draw the Ordinates *o* 1, *A* 2, *B* 3 parallel to *CD*; from which Points, *A*, *B*, *C*, &c. draw Lines to the Center, giving the Diameters of the other Arches, *HI* and *KL*, cut in the ſeveral Points a, b, c; *a*, *b*, *c*; at which Points, Ordinates being drawn, and, from 1, 2, 3, and *D*, Lines drawn to C cuting them at 1, 2, 3, through which thoſe Curves may alſo be deſcribed.

The Apertures through theſe Arches, being Circles parallel to the Picture; their Centers being determined, and their Diameters, are only neceſſary.

The large Beam, at O, is in the middle of the opening, at M N, and tends to the Center, on which the Joiſts reſt, which decline to the Wall on each ſide, geometrically; their meaſures and diſtance from each other being known, are readily determined, by Prob. 10, drawing a Line through *B*, parallel to MN.

The Stairs, at the far end, are managed by Example 8, in front; the returning flight being parallel to the Picture are inclined, geometrically, to the Horizon. The front Steps may be proportioned to the opening between the Piers; as at Y and Z, in the geometrical Plan; the Hand-rails, of which, tend to a Vaniſhing Point, in the Vertical Line, at H; as the Kirbs in Example 8th. The Door is geometrically proportioned, on the Steps.

The middle Avenue of this Building, being loftier than the Sides, the Repreſentation is limited by a Section of the Walls, and Timbers of the Roof, over the Sides, geo-

* At No. 2. is half a circular Arch, that is, a Quadrant, or fourth part of the circumference of a Circle, in a Square a b c d; c is the Center. *abcd* is a Rectangle circumſcribing a fourth part of an Ellipſis, of the ſame proportion as the middle Arches to the circular ones at each ſide; in which, it is obvious that, the Diagonals, ac and *ac*, cut the Curves at the ſame height, at e and *e*; and if the circular Curve be divided into more parts, at 1 and 2, thoſe being transfered by parallel and perpendicular Ordinates, cut the Diagonals of both in the ſame proportion; and conſequently they give correſponding Points in the Curve.

metrically

metrically drawn; the lower part is bounded by the limits of the Picture in length which, on account of the Station not being in a central Line, has its Center towards the left hand, out of the middle; by which means, it becomes distorted, on the right, being extended too far from the Center; by which the optic Angle is enlarged, and the Visual Rays cut the Picture too oblique, as SB in the Plan (No. 1.) in which case, no more should, properly, be taken into the Picture than the Piers, the optic Angle ASf, being about 50 Degrees.

EXAMPLE XLI.

Is the Inside of a Church, in the same position, as the foregoing Example.

At Figure 2, No. 1, is a half Plan of the Building, as far as the Section by the Picture, on AB. The Piers support the Roof as in the last Example, having Columns on them, of the Ionic Order, with an Entablature all around the middle part of the Building, and a coved Cieling. At the far End is a semi-octagonal recess, in which is the Altar and communion Table.

It would be wholly superfluous to form a perspective Plan, as in the preceding Example, or to fix the Station, with all the other Preliminaries to the Ground Plan; suffice it to say, that the situation of the Eye is nearly the same in respect of the Building and Picture. C is the Center, and CE half the Distance by which the whole is proportioned; the full Distance cannot be contained within the Picture, on either side.

The places of the Piers, the Windows, &c. it is obvious, are obtained here, as in the foregoing, by applying the half measures (that is, the full measures of the Plan) to the Ground Line, AB, sufficiently correct, insomuch that, an extra Plan would be unnecessary. The Piers are square, having a Sub-plinth with a Base Moulding, and Impost at the top; on which is a regular Pedestal, which forms the front of the Gallery, receding somewhat from the Pedestal; all which need no other description. The Picture is supposed to cut the Galleries, by which, the Seats, &c. are laid open to view; the Section is drawn geometrically, from which, all the Seats and Backs tend to the Center of the Picture.

The Columns, are supposed to be planed at the bottom of each Pier; but, if a central Line be drawn (as KL) from the middle point of each, and the height of the Column be there ascertained, all the rest are determined by Lines drawn to the Center, giving each its proper Diameter, in proportion to its height.

The Bases of the Columns, being above the Eye, are curved contrary to the usual order when below it; the Curves are still concave towards the Vanishing Line. The Capitals are according to the modern Ionic, in Example 29.

The recesses of the lower Windows, on the Floor, being less than the window Frames, cannot be a difficulty; the sloping part makes up the difference. The Windows above are perpendicular over the other; the heights of which are determined, on the vertical Intersections of the Walls, AD and BG; also the Gallery, and other parts of the Building.

The Nich at the far End, with the Doors and circular Windows over them, are all parallel to the Picture, consequently they are all geometrically proportioned, in front. The receding of the Nich, that is, the perspective widths of the Faces within, are best determined from a perspective Plan, formed below, the figure being determined; or, if the whole be drawn geometrically, and the Station fixed, as in the foregoing, Visual Rays may be drawn, cuting the Picture and giving their proportions thereon.

The Mouldings in the Impost, runing around it, have their Vanishing Points, for two Faces, in the Points of Distance. The middle Face is parallel, and in one, seen on the right hand the Lines tend to the Center. The Compartments in the Sides and in the Head are best described by inspection; the Lines, forming the Head, are drawn after the same manner as in an octagonal Dome, seen externally, as described in the 35th Example.

The

Plate XXX.

The ſemicircular Heads of the upper Windows have been frequently deſcribed; thoſe below have a flat Arch, which is but little ſeen, by reaſon of the ſpherical receſſes in the Cieling of the Gallery, which are repreſented by ſemi-Ellipſes, in the Plane of the Cieling.

The Entablature, ſupported by the Columns, is formed by means of geometrical Sections, at X and Z; the true place being acquired from a Plan below; or from the Ground Line and vertical Interſections, at H and I.

The Profile of the Mouldings, in the Entablature, on one ſide, with the large Cove, and a ſmaller on the other ſide, being drawn, it would be impertinent to deſcribe the delineation of every Line from each Angle to the Center of the Picture, and how to determine the mitre Angles at the End, where they return, over the Arch; for which, ſee Example 16, or 19.

For the large Cove, draw Lines from M and N, the internal Angles of the Building to C, cuting Perpendiculars from the Angles of the Frize, or the Pilaſters, below the Capital, at O and P; from which, draw Lines to the Eye, at the full Diſtance on each ſide, and produce them; then, Lines drawn from Q and R, to C, will cut thoſe Diagonals at S and T, and determine the returning Line of Moulding, which bounds the plane Cieling.

The mitre Angles of the Cove may be truly projected by means of Ordinates from various Points (as *a* and *b*) in the geometrical Curve; but, as it comes ſo ſmall, at the place, it may be done correctly enough, without that proceſs; taking care that it does not curve too much, but leads truly into the Perpendicular and Diagonal, both which are Tangents to the Curve, as RN and NZ, in the geometrical Section, and muſt never cut it.

The Compartments, in the Cieling, are but ſimple Figures, as Rectangles, with Ellipſes inſcribed, (ſee No. 1.) which are managed the ſame as Circles, as it has been ſhewn in the laſt Example, in reſpect of the Arches.

The receſſes in the Cieling, over the Galleries, are no more than a repetition of the ſame thing by a leſs Scale, having only an Architrave below the Cove, which returns at every Beam from the Columns to the Wall; whoſe mitre Angles are determined by the ſame means, from a Profile of the Cove, at G.

Thus, without the real proceſs, I have deſcribed every neceſſary ſtep to be taken in the delineation of this Object; for, as the laſt is, in reſpect of its Plan, the ſame kind of Figure, the ſame kind of Proceſs will anſwer for both, in the general parts; and, for particular ones, the Examples refered to give particular Inſtructions.

EXAMPLE XLII.

Is the repreſentation of an elegant Room, having a large, circular, Bow Window, in the Side, and a Cove-Cieling, inclined to the Picture.

In the two former Examples, the center of View, not being in the middle of the Picture, it may be imagined is wholly owing to the Station being towards one Side more than the other; but I ſhall ſhew, here, that it is not; for, let the Station be taken where you pleaſe, the Point of View, or rather the Center of the Picture may ſtill be in the middle, and yet, take into the Picture as much of the Object; a circumſtance extremely ſimple in itſelf, though to ſome it may appear a myſtery.

In Plate 29, Fig. 117, is a Plan of the Room intended to be delineated. Let E be the Station determined on; from which, if Right Lines are drawn to F and B, the extreme on each Side of the Room, intended to be delineated, FEB is the Optic Angle under which it is ſeen; and, if AB be the Interſection of the Picture, EC, perpendicular to AB, determines its Center, at C, and EC is its Diſtance; in which caſe, the Center of View is not in the middle of the Picture. But, if the Optic Angle be biſected by the Right Line ER, then, FG, perpendicular to ER, is the true Poſition of the Picture, S is its Center, and ES its Diſtance; and the farther End (HI) is conſequently inclined to the Picture; as it is repreſented in the 31ſt Plate. See Figure the firſt.

Let AB be the Ground Line, which is not neceſſarily the bottom of the Picture; but, whatever falls on this ſide is ſuppoſed projected to the Picture.

Draw AD and BK, perpendicular (the vertical Interſections of the Sides) on which, ſet up all the meaſures of the heights (by the Scale of Proportion) for the Dado and Mouldings, the Windows, Chimney Piece, the Entablature, Cove, &c.

On AD, deſcribe, geometrically, the true Profile-Section of all the Mouldings; as DEF for the Cornice, &c. the Lines of which (in this Caſe) not being perpendicular to the Picture, the Section of it, with the Picture, is not the true Profile; the deviation, in this, is inconſiderable. But, when the Lines are more inclined to the Picture, an Expedient for truly proportioning the Mouldings may be neceſſary.

Let ABC (Fig. 118, Pl. 29) be the Profile of the Cornice to be repreſented in the Picture.

The ſection with the Picture, being vertical, makes no variation in the heights of the Mouldings; but, according to the inclination of the Picture, to the Building, their projectures are varied conſiderably.

From each projecture a, b, c, &c. draw Lines perpendicular to BC, conſequently parallel amongſt themſelves; make the Angle BCD equal to the complement of the inclination of the Cornice to the Picture; CD will be the extreme projecture of the ſection of the Cornice, whoſe Inclination to the Picture is ECD; and, FDC is the true Section, in that poſition of the Picture; the projecture of each Moulding being taken from CD, where the parallel Lines cut it, at *a*, *b*, *c*, &c.

On the other ſide, the heights of the Windows, &c. are ſet up from B to I.

Having drawn the Horizontal Line (VL) and fixed the Center of the Picture, the Vaniſhing Point V being determined as uſual; which, on account of the inclination of the Sides of the Room, is not in the Center, as is cuſtomary in Inſide Views; the Room, or Building of any kind, being right angled.

In this Caſe, theſe Preliminaries are beſt determined by the geometrical Plan.

If the Ground Plan of any Building which we intend to delineate be drawn, truly geometrical, the Station may be determined, ſo, as to ſee ſuch parts of the Objects as we require, whether internal or external.

If it be required to ſee part of the ſecond Window in the Bow; draw a Right Line, from J, through the Angle at L, allowing ſo much of the receſs of the Window, &c. as you wiſh to repreſent. It is manifeſt, that the Station muſt be ſomewhere in that Right Line, produced; but if the View was to be central, it would be too near, as at O, for, the Optic Angle, AOB, is too large for the Eye to take in, at one View. Fig. 117.

At a further Diſtance, in the ſame central Line, at P, if PL be drawn, it is plain; that the Window cannot be ſeen at all, as it cuts the Pier at M; whereas, any where in the direction of JL, as at E, if the Station be fixed ſo, that, drawing EF and EB, the Optic Angle does not exceed 50 Degrees, it may be repreſented from that Station, without much diſtortion. Then, making EG equal EF, FG is the poſition of the Picture; and ES, biſecting the Angle FEG, is its Diſtance; wherefore S is its Center (Def. E) for ES is perpendicular to FG†, and SG is equal to SF, conſequently it is in the middle of the Picture. † 9. 1. El.

Now, E, is the Station, or Point of View, and FG is the poſition of the Picture.

Draw ED parallel to AH and BI; conſequently, D, and not S, is the Vaniſhing Point of thoſe Sides. (Def. L.)

The End (HI) vaniſhes in a Point where EK, parallel to HI, would cut the Picture, FG, produced; its Diſtance, from the Center, S, will be to SE, as SE is to SD‡, (Prob. 12.) And, if the Angle DEK be biſected, by the Line EN, N is the Vaniſhing Point of a Diagonal, or mitre Angle. ‡ 7. 6. El.

Then, if AB (Fig. 1, Pl. 31) be equal to FG, make VC equal to DS; or, in whatever ratio AB (Fig. 1) is to FG, ſo make VC to DS, and V will be the Vaniſhing Point of the Sides of the Room; that is, of all the horizontal Lines in thoſe Sides, and of all other Lines parallel to them, whether in the Cieling, or on the Floor, or elſewhere. (Cor. 1, Theorem 3.)

Being thus prepared, we now proceed to the delineation.

Plate XXXI. Fig. 1.

Draw the indefinite Lines AV and BV, and proportion them, by ſeting off the meaſures of the ſeveral parts from A or B, and drawing Lines to the Diſtance Point of the Eye, placed on either Side (at E, or E^2) EV being half, and VE^2 one third part of the Diſtance of the Vaniſhing Point V, there not being room, on the Picture, for the full meaſure.

Make Aa, ab, &c. each one third of the meaſures of the Originals, and draw aE^2, &c. cuting AV at a, &c. which give the Angle of the Chimney, &c.

For its projecture, make Aa equal to, or ſomewhat more than the real meaſure (becauſe, AB being inclined, it cannot be equal, but is the Diagonal of the Inclination; i. e. as CD to CB, Fig. 118) and draw aV, cuting a Line drawn from a to the other Vaniſhing Point, at b. Make ad in the ratio of one third part of the front of the Chimney (as CD to CB, 118) and draw dV, cuting a parallel Line from b at c. Divide bc in the proportion of the Truſſes, &c. and draw Lines to E^2, cuting bV in their perſpective Proportions.

Draw perpendiculars from a, b, and e; and, from all the Angles of the Cornice, &c. draw Lines to V; and return the Mouldings, at the ſeveral Angles of the Chimney, internally, at gh, and externally, at ik and lm, as deſcribed in Example 19; alſo, at no, the length being obtained by a Perpendicular from f.

To deſcribe every particular would be ſuperfluous and unneceſſary, previous Leſſons having been frequently and particularly explained.

Having obtained the mitre Angle at no, return the Mouldings of the Entablature, at the End, by means of the Vaniſhing Point of the End; or by Prob. 13.

The Angle of the Cove may be obtained, with accuracy, having made a geometrical Section, as in the laſt Example, ſeting up its height from D, and making its projecture as CD to CB (Fig. 118) in reſpect of the true projecture; then, draw a Line from the internal Angle to V, cuting the Perpendicular fn at p, and draw the Diagonal (pq) of the Angle, indefinite; and, from the extreme projecture of the Cove, draw a Line to V, cuting it at q.

But, becauſe the Vaniſhing Point of that Diagonal is not determined, and is out of the Picture; from p, draw a Line tending to the Vaniſhing Point of the End; and, where it is cut, at r, make rs equal pr, and draw a Line from the Vaniſhing Point of the other Diagonal; which will give the Point q nearly *.

The other Angle, x, is determined by the Vaniſhing Point of the Diagonal xy †.

After the ſame manner as in the foregoing, *viz.* by drawing ſeveral Ordinates to the Curve, at t, u, &c. the perſpective Curve at the Angles may be deſcribed, and tranſfered from one Angle to the other.

On the other Side, VE being half the diſtance of the Eye, the meaſures on the Ground Line, AB, are applied half the real meaſures, for proportioning the Piers, and the opening of the Bow Window, &c. from B, to e, f, &c. BI is the height of the Windows. Draw Ih tending to the Vaniſhing Point of the End of the Room (as for the projecture of the Chimney) and Bd being made equal to the receſs of the Window, draw dh; regarding the projectures of the Mouldings.

Having obtained the opening of the Bow Window at ik, which is a Segment of a Circle (by drawing eE and fE, cuting BV) find S the repreſentation of its Center (biſecting ik perſpectively, at 2, and drawing $S2n$ to the Vaniſhing Point of the End, and 3V, cuting it at S; B3 being ſomewhat more than its Diſtance from the Wall, *viz.* as CD to CB, Fig. 118) and draw Si, and Sk, which repreſent Radii; by means of which, other Radii may be obtained, as Sl, Sm, &c. making certain Angles with Sk perſpectively (Prob. 10, Caſe 3rd.) and thus, not only the Curve (klmnn) but alſo, the true place of each Window, &c. is acquired.

The Curve at the Top may be deſcribed by the ſame means.

If the Lines, on the Floor, in the receding of the Windows, tend to the Center S, draw SR perpendicular, and divide it, geometrically, into the ſeveral di-

* This is not ſtrictly true, prs not being parallel to the Vaniſhing Line; but, being ſo little inclined, the deviation is inconſiderable. rs is leſs than pr; they may be truly proportioned by Prob. 8, *Cor.* 1.

† This Vaniſhing Point is alſo out of the Picture, being diſtant from the Center as SN to SE (Fig. 117.) the Diſtance of the Picture.

viſions for the Mouldings, &c. and producing 81, &c. to the Vaniſhing Line; then, Lines drawn from each diviſion, on RS, to the reſpective Vaniſhing Point of each Jamb, will divide each Window into its ſeveral diviſions of Mouldings, Saſh Squares, &c. and, by the ſame means, all the Curves, in the Mouldings, &c. at the top, may be deſcribed with the greateſt accuracy.

If the Jambs of each Window be parallel between themſelves, then, a Right Line, So, biſecting the Angle 1Sm, &c. perſpectively, will produce the Vaniſhing Point of each reſpectively, and each Jamb muſt, in that Caſe, be divided geometrically, on the Angle, into the ſeveral diviſions required.

The Entablature may alſo be proportioned on BK, the ſame as on AD, on the other Side; which will be more accurate, than to depend entirely on the proportions being carried around, from the other Side; which, as they diminiſh ſo much, at the far End, are liable to error.

The places and proportion of the Columns, may be obtained by Prob. 8th, thus. The Angle *f* at the foot of the Pilaſter being obtained, draw *f*j parallel to the Ground Line, which divide, geometrically, in the Proportion required; that is, *f*1, equal j4, is the diſtance of the Column, at the Plinth, from the Wall; and 12, equal 34, is the width of the Plinth of the Column; from all which, draw Lines to V, cuting the inclined Line, in which the Columns ſtand, in the perſpective proportions of the Columns and Spaces; which may be compleated from the known proportions of the Order, as in former Examples.

If rs be taken, geometrically, the width of the Door, &c. in proportion to the Columns and ſpaces, Lines drawn to V will give the place of the Door, at the farther End, which may be compleated, by Example 20.

The Cieling has nothing of difficulty in it, ſave the Ornament, being compoſed of Right Lines, regularly diſpoſed; which, from the geometrical Figure, may eaſily be determined. GH, may be conſidered and uſed as the Interſection of the Picture with the Cieling, on which, the diviſions are geometrically diſpoſed; from which, draw Lines to V; and the ſeveral diviſions, in the length, being found perſpectively, on GV or HV, Lines drawn to the other Vaniſhing Point will cut each Line, drawn to V, in the ratio required; by which, the perſpective Figure is formed on the Picture.

EXAMPLE XLIII.

Is the Repreſentation of the inſide of the Piazza, Covent Garden, from the farther corner of the entrance into the Playhouſe.

The Poſition of the Picture being determined, and conſequently, the Inclination of the Lines, in which the Piers ſtand, is known.

Let C be the Center of the Picture, EV is the Horizontal Line, and V is the Vaniſhing Point of one Face of the Piers, found, or determined at pleaſure; the other is out of the Picture, on the Left, found as uſual; the Diſtance of the Picture is ſix Inches and a half, nearly*. Fig. 2.

Let AB be the Interſection of the Picture, i.e. the Ground Line, and, let S be the determined Seat (on the Picture) of the corner of the Pier, on the Ground, its Diſtance from the Picture being known.

Becauſe there is not room, on the Picture, to ſet off its whole Diſtance, take CE half the Diſtance; and make SD half the diſtance of the corner of the Pier; draw DE cuting SC at a, the true place of that Corner, on the Picture.

Draw aV, the indefinite Repreſentation of one Side of the Piers, and aY, to the other Vaniſhing Point, whoſe Diſtance, from C, is nine inches and three fourths.

* Let it be obſerved, that the Diſtance, here uſed, is too little; but, being a true Portrait of ſo public a Place, I thought proper to diſpenſe with it; by repreſenting it as it appears, from the Station determined above. See Preliminary Obſervations, Page 111.

Draw

Plate XXXI.

Draw a*d* parallel to AB; and having transfered the full meaſure of the Piers and Arches from AB, the Ground Line, to a*d* (by means of the Point C, or any other in the Vaniſhing Line) at *b*, *c*, and *d*, draw Lines to *E*, the Diſtance Point, of V, giving their places, at ab, and cd, &c. the reſt may be obtained to any length, required, by Example 4th. Note, a*d* may be the Ground Line, by a leſs Scale.

On the returnining Angle, they are obtained by the ſame meaſures, applied on the other Side, by the Point F the diſtance Point of Y.

Produce V a to the Ground Line, cuting it at A, and draw AH perpendicular, on which, ſet up, from A, the meaſures of the heights of the Piers, &c. at G and I; from which, draw Lines to V. Perpendiculars, from a, b, c, &c. cuting them at e, f, g, &c. give the Piers, on that ſide.

From, ae, the common Interſection, they are returned on the other Side, to the Left.

The circular Arches are all conſtructed by Exam. 21, as the one on the Left; and the Curves over them, by means of Ordinates *a*b, *c*d, &c. their heights being ſet off from e to i and k, &c. the true Curve being determined (ſee Fig. 119) and as many Ordinates drawn as are requiſite, they are returned on both Sides.

The eliptic Arches and Borders, from the Piers to the Wall may be thus deſcribed.

Produce Y a and Y e, and having found the Point J (Prob. 8.) in the middle of the whole width; from J draw a Perpendicular, cuting Y e and Y k, produced, at l and m; m is the middle Point in the Crown of the Arch.

Draw l V and m V; and, from the Vaniſhing Point Y, produce Lines through g, j, &c. from each Pier at the foot of the Arches, cuting l V at n and p, &c. from which, draw Perpendiculars, cuting m V at o and q, &c. the height of each Arch, reſpectively; as l m, n o, and p q.

Then, by means of ſeveral Ordinates, as *ab*, *cd*, &c. perſpectiyely found, on the Baſe Line of the Arch, at *a*, *c*, *e*, &c. (ſee Fig. 119, Pl. 29, for the geometrical conſtruction) their ſeveral heights are ſet off on e k, and projected, by the Vaniſhing Point Y, cuting Perpendiculars from *a*, *c*, *e*; by which means, half the Curve of the firſt Arch is deſcribed, as e*bdf*m. The other half is determined by the ſame means, and all the other Arches after the ſame manner, by drawing Lines from the Seats, *a*, *c*, *e*, &c. of the Ordinates, to V, cuting the Baſe Line of each Arch, as g n, at 1, 2, 3, &c.; from which, Ordinates being drawn, and Right Lines from *b*, *d*, *f*, &c. to V, cuting them at 4, 5, 6, &c. then, through their Interſections, the Curve g 5 o v u is deſcribed.

Having obtained all the Arches, or Borders, the Groins are next to be determined. The middle Points, r, ſ, t, &c. are in the Line m V (allowing ſomewhat for the thickneſs of the Border) and by means of horizontal Lines, whoſe Vaniſhing Point is V, drawn from *b*, *d*, *f*, &c. their lengths, from thoſe Points, as *b*g, *d*h, *f*i, &c. may be determined, by the Point *E*, as in all other ſimilar Caſes whatever. Or, perhaps more readily, by perpendicular Lines from the Diagonals, f u and g x, (at the foot of the Arch) where they are cut by Lines drawn from *a*, *c*, and *e*, to V, cuting the former at *g*, *h*, and *i*, reſpectively.

The Plan of the Plinth of the Column, being determined, from its known meaſure and place (by its Seat on the Ground Line, or otherwiſe) there is nothing ſingular in its conſtruction; ſave the Blocks or Ruſtics (W) which are each equal to a Diameter, in height; the ſpace between is the ſame; they project equal to the Plinth of the Baſe. Y and V are the Vaniſhing Points of the horizontal Lines, in the Cornice, &c. The Picture is ſuppoſed to cut the Column, conſequently its full meaſures are applied, on BL or ſomewhat more, ſeeing it is projected.

The Piazza, on the right-hand, is ſeen to the End; and, through the Arch, at K, is ſeen the front of the next, on the other ſide of James Street.

The diſtant view of the Church and adjacent Buildings, being ſo little ſeen, are beſt ſketched from the place; as it would be attended with unneceſſary trouble to find their places, &c. perſpectively, from their true geometrical proportions and poſitions. The Ruſtics, in the Piers, are a Scale for proportioning them.

EXAMPLE XLIV.

Plate XXXII.

Is the Repreſentation of a Staircaſe, internal; ſhewing the deſcending Stairs, direct.

This Leſſon, is intended, not merely as an Inſide View, but in order to ſhew that a deſcent may be repreſented by the Rules of Perſpective, on the ſame Principles as any other Subject, whatever; in which I have reduced to practice, what was heretofore treated on (Art. 8. Sect. 6. B. 1. P. 97) reſpecting a down-hill Repreſentation.

In this Example the Window Side is parallel to the Picture, and conſequently, the Sides of the Staircaſe are perpendicular to it; therefore, the Center of the Picture is the Vaniſhing Point of horizontal Lines, in the Sides.

C is the Center, and HL the Horizontal Vaniſhing Line; and VL, the Vertical Line, is the Vaniſhing Line of the Sides of the Staircaſe.

The Vaniſhing Lines of the aſcending and deſcending Planes, i. e. of the Stairs, are determined by Prob. 2, making CE equal to the Diſtance of the Picture, and the Angle CEF equal to the inclination of the Stairs to the Horizon. Make CG equal CF, and through F and G, draw Lines parallel to the Horizon; one is the Vaniſhing Line of the aſcent, the other of the deſcent; and becauſe they are, in this Caſe, parallel, the Vertical Line, VL, is common to them all †, and conſequently, F and G are the Centers of thoſe Vaniſhing Lines, reſpectively, and their Diſtance is EF, equal EG. (Theo. 7, and Def. 20.)

† Theo. 7. Cor. 2.

As this is a circumſtance which has occaſioned ſome diſputes amongſt Artiſts, I ſhall diſplay it in the beſt light I can, and doubt not I ſhall do it ſatisfactorily.

In order to which, let Fig. 120 repreſent a ſection of the Staircaſe, geometrical, by half the Scale of the Picture; AB is the deſcent, from the Landing, at AD, on which the Spectator is ſuppoſed to ſtand, at ED; E is the Eye or Point of View, and EC is the Diſtance of the Picture *, of which AH is a ſection. AK is the flight of Stairs, immediately aſcending, as AB is deſcending, and KI is the under ſide of the next flight, over them, parallel to AB.

Fig. 120.

Now, becauſe the Vaniſhing Point of every original Right Line is where a parallel from the Eye cuts the Picture ‡, and EC the Direct Radial, produces the Center, C is the Vaniſhing Point of the ſides of the Half pace BL, and of the extremes of the Steps, as may be ſeen in the Picture; alſo, EF, and EG, being parallel reſpectively to the Aſcent, AK, and Deſcent, AB, conſequently, F and G are the Vaniſhing Points of all Lines, on the Picture, parallel to them (as the Hand-Rails, the Wainſcoting at the Sides, &c.) and conſequently, Planes paſſing through the Eye and thoſe Lines, reſpectively, parallel to the Stairs, muſt cut the Picture in the Lines IK and MN paſſing through F and G, reſpectively, and therefore they are their Vaniſhing Line.. (Def. G.)

‡ Def. 22.

This I preſume is intelligible and clear; and, if Viſual Rays are drawn to the ſeveral parts of the Staircaſe, they will determine what can be ſeen and what cannot; as EB determines how much the Deſcent riſes on the Picture, which it cuts at b; Ab is, therefore, its whole apparent width. The aſcending Flight, parallel to AK, it is evident, is ſeen on the under ſide, as the Viſual Ray EM evinces; and conſequently, it deſcends on the Picture.

Let AB be the Interſection of the Picture with the landing of the Stairs, which, in this Caſe, is the Ground Line, the Picture being ſuppoſed cloſe to the Stairs; all that lies on this ſide is projected to the Picture; as the Door on the Landing, and Cieling, as well as the Floor; and it is obvious, that, whatever appears to deſcend, below the Landing, has its place on the Picture above the Ground Line; conſequently, it riſes on the Picture.

Make BD equal to the known width of the Stairs, the width of the whole, being repreſented by AB; the Station is determined by the Vertical Line, VL.

* The Diſtance of the Picture, it muſt be obſerved, is too little, for taking in ſo much as is here repreſented; but it is manifeſt, that, if the Diſtance was half as much more, or one third lower, we ſhould not ſee the deſcending Stairs at all, as the Eye would be either in the Plane of the Stairs, or on the other Side.

Plate XXXII.

Draw BG and DG, to the center of the Vaniſhing Line MN; the Originals of thoſe Lines being perpendicular to the Interſection, AB; in which Lines are all the upper edges, or noſeings of the Steps; whoſe places are obtained by Prob. 8th.

Make G*E* equal EG,* the Diſtance of the Vaniſhing Line MN, or IK (Def. 20) Make Ba, ab, &c. double Aa, ab, &c. (Fig. 120) and draw a*E*, b*E*, &c. cuting BG at *a*, *b*, *c*, &c. giving their places on the Picture; and parallel Lines, *ad*, &c. being drawn, give the edges of the Steps; the Ends, being at right angles with the Pictures, vaniſh in its Center; therefore, draw C*a*, C*b*, &c. and produce them to the next Step; and thus the deſcending Stairs are compleated.

The Half paces, below, at X, and above, at Y, being horizontal; their Sides, conſequently, vaniſh in the Center; alſo, the ends of all the Steps, every where. The width of the Half pace being equal to the Steps, draw *e*C and *f*E, cuting *e*C at *g*, which gives the Angle *gh*, at the farther end of the Staircaſe.

The height may be determined on a Perpendicular from B, as BJ, and drawing JC.

The aſcending Flight is managed by Example 9th in every reſpect. The Vaniſhing Point of the Hand Rails, Wainſcoting, &c. of that Flight is at F, as well as of the Lines de and fg, of the Plane defg, being parallel to thoſe below; the other, with the Lines in the Plane Z, is at G.

The height of the Hand Rails, &c. are ſet up on the Interſection BJ, as at h; or from D, as Di, allowing for the ramping Curve, equal ik.

The panneling of the Wainſcot is divided in the ratio required, as in any other Caſe, by Prob. 8th, making uſe of the Diſtance EG (not EC) for the inclined part.

The Windows may be proportioned on *gh*, at the farther end, geometrical.

The Cornices, and other Mouldings, are managed by the Leſſons in Sect. 7.

The Architrave around the Door, being projected, is thus done.

Make Bl, Im, equal, reſpectively, to its diſtance from the Steps, and width of the Architrave, and draw El, Em, projecting thoſe meaſures to CB, produced, cuting it at *i* and *k*, and giving *ik* for the Architrave.

The Cieling is projected in the ſame manner; and all beyond BP.

AOPB is the Section of the whole by the Picture; OP is the Interſection with the Cieling, and AO and BP with the ſides of the Staircaſe; on which Interſections, all the meaſures are applied, for each Plane reſpectively.

OF HORIZONTAL PICTURES.

Horizontal Pieces are conſidered, by many, as performed by Rules different from vertical Pictures; whereas, they are the ſame in every reſpect, being founded on an univerſal Theory, which regards no particular poſition of the Picture; but only, the poſitions of the Planes and Lines in Objects, to the Picture, and to the Eye. For, however the Picture be ſituated, the Eye being conſidered as a Point, it muſt either be in the Plane or out of it, and cannot vary in its poſition, otherwiſe; conſequently, its Center and Diſtance, being determined by a perpendicular from the Eye to the Plane, are the ſame in all poſitions of the Plane, reſpecting the Horizon, to which we are naturally partial; but, being properly conſidered, we ſhall find it of no conſequence in the Theory of Perſpective.

Let the Picture be ſituated as it may, all Original Lines perpendicular to the Picture vaniſh in its Center (Cor. to Th. 4.) Planes, or Lines, being parallel to the Picture, have no Vaniſhing Line, or Vaniſhing Point; but, in ſuch Caſe, the repreſentations of all Figures, in Objects ſo ſituated, are ſimilar to the Originals (Theo. 9, Cor. 5) alſo, all Original Lines, however inclined to the Picture, vaniſh in that Point, where a parallel Line from the Eye would cut the Picture; and conſequently, all Lines, which are parallel, amongſt themſelves, have the ſame Vaniſhing Point (Th. 3, Cor. 1.) with every other circumſtance in the Theory given, which is quite general, and cannot poſſibly regard any particular poſition of the Picture, to the Horizon.

* Suppoſe GN produced and made equal to GE.

The Repreſentations of moſt Objects (in common Caſes) on a vertical Picture, are managed by means of the Horizontal Vaniſhing Line, only, but, in many Caſes, other Vaniſhing Lines are requiſite.

A horizontal Plane, paſſing through the Eye, which, in other Caſes, produces the horizontal Vaniſhing Line, is, in this Caſe, parallel to the Picture, and conſequently cannot cut it; for it is the Directing Plane, of Pictures in ſuch poſition. (Def. 4.) The theoretic conſtruction of the elementary Planes is the ſame, however the Picture be ſituated; all the difficulty ſeems to be in fixing the Vaniſhing Lines, in this Caſe, having no particular bias in reſpect of the Horizon.

As, in all common Caſes, the Horizontal Line and Vertical Line cut each other, at right angles, in the Center; ſo, in this, two Lines cuting each other, perpendicularly, in the Center, anſwer the ſame purpoſe, either of which may be conſidered as the Horizontal Vaniſhing Line; for, if the Picture be changed to a vertical poſition, and, the Objects being ſuppoſed changed with it, keeping their poſition to the Picture, it is then a common Caſe, in every circumſtance; the placing of thoſe Lines, moſt conveniently, is therefore the principal buſineſs.

Now, if the Piece be viewed centrally, it is immaterial how they are drawn, for repreſenting a Dome only; but, if we would repreſent the Inſide of any priſmatic Object, whatever, it is moſt eligible to draw the Vaniſhing Lines parallel to its Sides, as AB and DE; becauſe they are, then, the Vaniſhing Lines of thoſe Planes, reſpectively. But, if the View be not central, then, one of the Lines (DE) generally paſſes through the center (C) of its Baſe and divides the Dome equally; which may be conſidered as the Vertical Line of the Picture; the other (FG) as the Horizontal Line; S being the Center of the Picture, whoſe Diſtance is known. Fig. 121.

The chief difficulty, in Cieling pieces, is the repreſenting Objects in ſuch uncommon Poſitions, they being ſeen on the under ſides, inſtead of looking direct at them, as ſtanding before us, upright, in common Caſes; and are repreſented as if lying horizontal, in reſpect of a vertical or upright Picture; in which, the difference conſiſts, wholly. The Cielings, Soffits of Doors and Windows, Planceers of Cornices, &c. are, in this Caſe, parallel to the Picture, as, in other Caſes, they are perpendicular to it; being thus reverſed, and ſeldom practiſed, makes the difficulty greater.

There is one particular circumſtance attending Cieling pieces; the Center of the Picture is the only Vaniſhing Point, generally; not owing to the poſition of the Picture, but of the Objects to be repreſented. For the Baſe of the Dome, or whatever elſe is repreſented, is always parallel to the Picture; and conſequently, all Lines perpendicular to the Baſe (being alſo perpendicular to the Picture) vaniſh in its Center; and, ſince the Objects to be repreſented are always upright, there is no occaſion for any other Vaniſhing Point; as, all Right Lines, in the Objects repreſented, are either parallel or perpendicular to the Picture. Nor can it be otherwiſe, unleſs repreſented on an inclined Cieling; in which Caſe, there may be as much variety of Vaniſhing Lines and Points, as in any other.

Theſe Preliminaries being well conſidered and digeſted, the difficulty will vaniſh, and leave the whole, ſubject to the ſame common Rules already laid down; for there are no other uſed, or neceſſary.

EXAMPLE XLV.

How to repreſent on a Cieling, or horizontal Picture, the repreſentation of a Gallery of Communication; with a Dome and Cupola, as ſeen from below.

ABCD is a geometrical half Plan of the Gallery, &c. to be repreſented, which is a Square, ſuppoſe of 30 feet; and BFHC is a Section of the ſame. The height, BF, to the Cove, is 18 feet, the Cove 3 feet, and the Baſe of the Dome (which is a Hemiſphere) is 24 feet. Through the Arches, in the middle of three Sides, Galleries are ſuppoſed to communicate to various Apartments, above. Fig. 122.

The

Plate XXXII. Fig. 123.

The Baſe of the Gallery being a Square, and the Picture horizontal, conſequently parallel to its Baſe, its repreſentation is therefore a Square.

Deſcribe a Square, AHIB, of the dimenſions you intend the Picture, which is here, double that of the Plan.

The Station being determined, and the Diſtance of the Picture, which, in this Caſe, cannot admit of much variation, from the true, to any Eye which views it. For, I ſuppoſe, every Cieling Piece is calculated to be ſeen from ſome particular Station, below; and every Perſon who would ſee the Piece, truly, muſt ſtand in the ſame place; conſequently, the difference, in Diſtance, is equal to the different heights of the Spectators.

Suppoſe the Hall, or Saloon, to be a Cube of 30 feet; then, allowing 5 feet for the height of the Eye, its Diſtance from the Cieling is 25 feet, which is the Diſtance of the Picture; upon which, is to be repreſented, a continuation of the Walls, &c. with the Entablature, from B to F, near 20 feet, with the Cove, Dome, and Cupola, covering the whole.

Now, the Diſtance of the Picture is 25 feet, which is not equal to its width, but cannot, here, be more, unleſs the Spectator be ſuppoſed to ſit or lie down. Then, as, in common Caſes, the height of the Eye is next to be determined, ſo, in this, the place of the Eye in reſpect of the Picture; which, I would never adviſe to be in the middle of the Piece, as it cannot produce an agreeable Repreſentation; ſeeing that, the longitudinal Lines in the Dome (which in that Caſe are repreſented by Right Lines) will be but ſo many Radii of a Circle tending to its Center, as aC, bC, &c. (Fig. 124) and, the latitudinal Circles will be ſo many concentric Circles, as abc, *abc*, &c. alſo, every Side of the Room will advance equally towards the Center.

Therefore, let the Station be determined ſomewhat towards one Side, where the Cieling may be ſeen moſt advantageouſly; nor is it neceſſary to be ſituated centrally, in that Side; although it is a deviation from my general Maxim, of fixing the Center of View in the middle of the Picture; but, ſince the poſition of the Picture cannot, in this Caſe, be varied, I had rather diſpenſe with it than have duplicate Repreſentations.

Let C, then, be the Center of View; ſomewhat to the right hand. Let AB be conſidered as the Ground Line; and if, through C, a Right Line, EF, be drawn, parallel to AB, it may be conſidered as the Horizontal Line of the Picture; and DG, perpendicular to AB, alſo paſſing through C, is the Vertical Line, i. e. they are the Vaniſhing Lines of the Sides of the ſquare Room reſpectively.

The Galleries, on three Sides, which are ſupported by Truſſes from the Walls, are firſt to be repreſented; which I have made to project but three feet, as they would hide the Walls too much if they projected more.

Now, ſince the Picture is parallel to the Truſſes, and they are ſpaced equally, divide the three Sides AH, HI, and BI, geometrically, into as many ſpaces and Truſſes as are to be repreſented, at X, X, and deſcribe their Seats on the Picture, truly geometrical. Then, the Lines which meaſure their thickneſs or height, being perpendicular to their Baſes, and to the Picture, vaniſh in its Center, as in all common Caſes whatever.

Make CE equal to the Diſtance of the Picture, in the Vaniſhing Line EF; draw the indefinite Repreſentations, aC, bC, &c. and proportion them in height, making a*a* equal to the height, and draw *a*E, cuting aC at *b*; which, by drawing parallel Lines, to AH, HI, &c. will determine them all around.

The Cieling of the Gallery, between each Truſs, is conſequently determined. The under ſide of the Truſs is like an Ionic Modilion, having, alſo, a Cimma reverſa, around the top; which, the Figure ſufficiently deſcribes.

The Cornice of the Gallery is deſcribed as any other; ſave only, that the Facias, or upright Fillets, are contracted, in this Caſe, as the under Sides, or Planceers, in common; the difference is owing to the poſition of the Picture only, the appearance is the ſame; as it is illuſtrated in Fig. 125.

Let

Let A B be the Profile of a Cornice, ſeen from the Point E, by means of the Viſual Rays AE, BE, &c. If the Rays are cut by a vertical Plane (A b) the repreſentations of the ſeveral Angles, at C, D, &c. are where the Rays are cut by the Plane, at c, d, &c. and a horizontal Plane, B a, cuts the ſame Rays at a, b, d, &c. It is manifeſt, that the proportions of the Members, on thoſe Planes or Pictures, differ conſiderably, yet it is obvious, that each appears the ſame, in the Point of View, E; the difference, in the Repreſentations, ariſing from the different Sections of the Rays, as in any other Caſe.

For the iron railing, which is in the plane of the Facia of the Cornice, from each internal Angle, k, draw kC. Make Ad equal to its width, and de equal to its known height, from the Cieling (equal twice *de*, Fig. 122) and draw eE, cuting dC at f; draw f g parallel to AH, and g h parallel to HI; alſo f i, parallel to AB, cuting cC, where it joins the Wall, on the other Side.

The diviſions and proportions of the Railing are determined, geometrically, on dk*, &c. at *c*, *d*, &c. from which, Lines are drawn to C.

The Gallery, with the Railing, being compleated, we next proceed with the decorations of the Walls, ſeen beyond or over it.

Having drawn, geometrically, the Plans, or Seats of the Pilaſters, with all the projectures of the mouldings of the Pedeſtal, &c. at Y, Y; from each exterior Angle, draw Lines to C, as j C, l C, &c. and from l, ſet off, firſt, the height to the Plinth; then, the meaſures of the Plinth, Mouldings, Dado, &c. of the Pedeſtal, at m, n, &c. and draw mE, nE, &c. cuting l C in their ſeveral perſpective Proportions, at *e*, *f*, &c. from each of which draw Lines parallel to their Seat, j l, cuting j C, and return them at right angles. The Diagonals, or mitre Angles are, in this Caſe, parallel to the Picture, i. e. to A I and B H; by which means the Mouldings are determined, every where.

The Baſe Moulding is hid by the projecture of the Plinth, and alſo the Baſes of the Pilaſters, on this Side, being greatly foreſhortened; which, where they can be ſeen, are managed after the ſame manner.

From o, p, &c. draw Lines to C; make pq equal to the height of the Capital, from the Cieling, and draw qE, cuting p C, at n, and compleat the ſquare of the Abacus; which being parallel to the Picture, the curviture of it is geometrical.

The Mouldings of the Capital are managed as the other (except in the Abacus, which are all circular curves, being parallel to the Picture) their meaſures of height, being obtained, from pq, at r, &c. and their projectures from the Plans below, drawing Lines from each, to C.

The Volute may be determined, from its Plan, below, in reſpect of its projectures; but to deſcribe it, particularly, would baffle all deſcription.

The Windows, in this Side, and Niches, &c. in the others, are determined geometrically, on AB, &c. the perpendicular Lines of which, vaniſh in C, and the circular heads are ſemi-Ellipſes, deſcribed, by Prob. 3, Sect. 8.

The other Sides are managed after the ſame manner, from Plans below.

For the Entablature. Let M, N, O, and P, be the Seats of its projecture, on the Picture, in the Diagonals AI and BH. Draw MC, NC, &c. Make MQ equal to its height from the Picture, and draw QE, cuting MC at *n*; draw *no*, *op*, &c. parallel, reſpectively, to the Sides of the Picture, *nopq* is the extreme projecture of the Cornice, around.

Make QR, &c. equal to the meaſures of the Cornice, Frieze, and Architrave, with their ſeveral Mouldings; and draw RE, &c. giving them perſpectively, on MC. Their projectures are determined on the Diagonal, AM, geometrical; or, on Diagonals from *n*, *o*, &c. parallel to AM, &c. m*n*, &c. being divided in the ſame ratio, as AM, (Cor. 2. Th. 9.)

* d k is the Interſection of the Plane d f g k of the Railing, diſtant from AH, equal A d, i. e. equal to its known diſtance from the Walls; conſequently, d is the Interſecting Point of that Angle.

For, becauſe the Picture is parallel to the Baſe of the whole, the ſection of every Plane with it, being perpendicular, is geometrically determined; therefore, the Interſecting Point of every Perpendicular Line in the Objects, as d or k of the Angles of the Railing, is readily found, its diſtance from each Wall being known, or determined; the whole Baſe being a true geometrical Plan.

Plate XXXIII.

The Square of the Cieling above the Cove is thus determined.

From T and U, the Seats of the projecture of the Cove (in the Diagonals AI and BH) draw TC, and UC; and, TV being made equal to its height, draw VE, cuting TC at *r*; draw *r s* parallel to the Cornice, and compleat the Square *r s t u*; in which, inscribe a Circle; and, concentric with it, another, leaving a border around the Dome; and also, compleat the same within the Square *r s t u*.

For, because the Cieling is parallel to the Picture, its perspective form is truly geometrical; consequently, one Side being perspectively found, the rest are also determined, in the ratio of *r s* to the Original.

The Angles of the Cove are determined by Ordinates, on the Diagonals (if it were necessary) geometrically proportioned, alike on each; and others, on AC, &c. perspectively; (See the geometrical construction, Fig. 126, which determines three Points, *a*, *b*, and *c*, in the Curve, equally spaced.) but, as they approach so nearly to right Lines, here, it is an unnecessary process.

To represent the circular Cornice, at the foot of the Dome.

The Frieze or Facia, below the Cornice, is perfectly cylindrical, its height is determined on AC, which represents a Perpendicular from that Angle; on which, every measure of height may be transfered from AB, geometrical, as s, t, u, &c. of the several divisions in the Dome.

From S, the Center of the Base, draw SC, in which Line are the Centers of every Circle in the Dome, &c. for, the representations of all Circles, in Planes which are parallel to the Picture, are Circles; and, because SC represents a Line perpendicular to the Base, in its Center, SC is the Axis of the Dome, and passes through the Center of every Circle in it; which being determined, on SC, or transfered from AC, by lines parallel to the Diagonal AI, as s 1, t 2, &c. draw lines through 1, 2, &c. parallel to AB.

Now v x is the Diameter of the Base of the Dome, cuting the Axis at o; and 1, 2, &c. are the representations of the several Centers, of Circles in the Cornice.

From v, set off the projectures of the Cornice, in proportion to the Radius vo, at *a*, &c. from which draw Lines to C, cuting parallel Lines from 1, 2, &c. which give the Radius of each Circle in the Cornice, as 1 *k*, 2 *e*, &c. by which the Cornice may be compleated; being composed, wholly, of Circles.

The parallel Circles in the Dome are determined after the same manner; which are necessary, for describing the longitudinal Borders.

No. 2. In Fig. 122, draw as many Ordinates parallel to its Base, GI, as are necessary, from the several points a, b, c, equally spaced in the Curve, to the Axis, or central line CH, cuting it at d, e, f. Then, make o*a*, o*b*, and o*c* in the same ratio to ov, as ad, &c. to the Radius GI; from which draw lines to C, as *a*C, *b*C, *c*C; and, from the perspective Centers, 2, 3, 4, and 5, obtained as above, draw parallel lines cuting them at *e*, *f*, *g*, and *b*, which give the Radius of each Circle in its true place, *viz.* *e* 2, *f* 3, *g* 4, and *b* 5, which, last, is the Base of the Cupola.

Divide the Base of the Dome into eight equal parts, at a, b, c, &c. so that, ah and de are parallel to AB, &c. and set off the widths of the Borders regularly from those Points (half on each Side) draw o a, o b, &c. and, from the Centers 2, 3, 4, draw lines parallel to o a, o b, &c. cuting their respective Circumferences at i, k, l, m, through which, Curves being traced, they are the central Lines of each Border *. Their widths may also be set off at each Circle, in proportion to its radius, as at the Base, by which they are compleated.

The Compartments are done after the same manner, from those central Lines, respecting the Margins and Mouldings which are longitudinal; the parallel Mouldings are in Circles parallel to the Base.

* The Eye being in the Plane of those passing through d and h, they are, therefore, represented by Right Lines; those passing through b and f, being fartheſt removed from the Center, are the most curved; for, being viewed centrally, they all appear Right Lines (as in Fig. 124) seeing the Eye is in the Plane of each Curve.

The ſemicircular Windows, at the Baſe of the Dome, are but little ſeen, and may be truly determined by Ordinates, tending to the Center, geometrically divided at the Baſe, and parallel Circles cuting them, as at *i*, *r*, *k*, *l*, *m*. The Margins about them may be done by the ſame means.

The Baſe of the Cupola being obtained, as above, the Windows in it, are geometrically divided at the Baſe, and Lines drawn to the Center.

Their apparent height may be obtained on SC or AC, as the other parts.

Thus have I deſcribed the whole proceſs of this Cieling Piece, from the beginning to the End: to ſay more concerning it would be ſuperfluous, as it muſt be obvious, that the Rules made uſe of are the ſame as have been applied in all other Caſes, and Subjects whatever.

SECTION XI.

Is applied to Houſhold Furniture, Wheel Carriages, Machines, &c.

THE foregoing Sections contain the Elements and Rules for the practice of Perſpective, in all common Caſes; I have alſo ſhewn how to apply them, in familiar Leſſons and Subjects, adapted to any capacity, in the 6th, 7th, 8th, and 9th Sections; and, in the laſt, to particular Subjects, and in a particular ſituation of the Picture; ſo that, there remains nothing more to be done in reſpect of the application of it, to all uſeful Subjects, almoſt whatever.

In this Section, which relates to Objects in particular Profeſſions, yet uſeful to Artiſts, I ſhall only make ſome curſory remarks, in reſpect of the repreſentations of ſuch Objects as are here treated on. The former Leſſons, being well underſtood, will be found applicable to whatever Subject the Artiſt chooſes, or has occaſion for. To multiply Caſes and Subjects, in which the application of the Rules of Perſpective may have the appearance of ſomething particular or ſingular, would be endleſs, as, in almoſt every different Object, there is occaſion for ſome variety in the application of its Rules; which, nevertheleſs, it is obvious, are all founded on the ſame univerſal Principles, though the Rules deduced from them are variouſly applied.

It is a neceſſary requiſite, in many Profeſſions, to be able to give a Deſign, to a Gentleman, of whatever may be wanted, out of the common run of things; and although a perfect Drawing may not be required, yet to give a ſlight ſketch, with propriety, is certainly an Accompliſhment which cannot well be diſpenſed with. In an Architect it is abſolutely neceſſary, to give a correct Deſign, but they are generally contented with geometrical Repreſentations, which does not give the true appearance and effect; that can only be done perſpectively. Geometrical Drawings to a Workman are neceſſary; but, to give a true Idea how the Object will appear, when executed, from any particular Station, requires ſomewhat more; to give ſuch an Idea, without knowing ſomething of Perſpective, requires a much greater ſhare of genius than falls to the lot of many.

Next to the Architect, the Cabinet-maker and Upholder will find his account in a knowledge of Perſpective; nay, I am of opinion, that it is full as neceſſary to the Furniſher as to the Builder; having almoſt as large a field, and as various. There is nothing influences a Gentleman more in the favour of his Workman, when he is pleaſed to want ſomething whimſical and out of the way, than to take his Pencil and ſketch out the Idea the Gentleman had conceived, and was big with, yet could not bring it forth without aſſiſtance. He who can do that, and,

at

Plate XXXIV.

at the ſame time, diſplay a little more taſte, in Ornament, being known, is certain of ſucceſs, or of employ, at leaſt. I have, therefore, given ſeveral pieces of Furniture, of various kinds, for his practice, which will alſo, frequently, be found uſeful to the Artiſt, to furniſh his Family, or Converſation Piece.

The Organ Builder, and Coach-maker, may ſometimes find it neceſſary to give their Deſigns in Perſpective; a ſpecimen is given for each Profeſſion. And, laſtly, the Mathematical Inſtrument-maker, Mill-wright, or Engine-builder of any kind, by Perſpective, may be able to ſhew to much greater advantage, the ſeveral movements, and mechaniſm of his Inventions.

There are but very few Perſons, who have not had much practice, in Perſpective, know any thing more of it, than the application to Objects, whoſe Planes are parallel and perpendicular to the Picture, and conſequently, have ſeldom occaſion for any other Vaniſhing Point than the Center of the Picture; they always, therefore, adapt every thing to that poſition, without thinking about the propriety of it. Some cannot think a Drawing is Perſpective, unleſs an End of the Object is repreſented; for which reaſon, we ſeldom ſee any Object delineated in Perſpective without having an End ſeen, which, in a Building, or other Object of a tolerable length, is abſurd to repreſent, the Front being parallel.

Notwithſtanding I have endeavoured to explode the abſurdity of the Picture, being neceſſarily parallel to the front, or ſome other face of the Object, as not being pictureſque; yet, in the following Caſes, I would rather prefer it; ſuch as giving a Deſign of a piece of Furniture, &c. to a Gentleman; becauſe, the Gentleman, not having judgment in Perſpective, might be greatly deceived in his Ideas of the proportion of the ſeveral parts to each other; whereas, the front of the Object being parallel to the Picture, the true geometrical proportion is preſerved, which is moſt eſſentially requiſite, in order to be clearly underſtood.

Fig. 129. Figure 129 exhibits a repreſentation of a Pedeſtal and Vaſe direct in front, the Eye is directly oppoſite to the Ball at the top, being about the height of a Perſon ſiting; for, to ſtand, when we view ſuch Objects as are wholly below the Eye, although we ſee no imperfection or diſtortion in the Object, itſelf; yet, unleſs the Diſtance be greater than we uſually ſtand at, to ſee ſuch Objects, it will be greatly diſtorted in the Repreſentation, particularly in the lower parts, ſeeing that the Diſtance, in ſuch caſes, is frequently leſs than the height of the Eye.

C is the Center of view, for this Object, the Diſtance is CV, which is leſs than the height of the Eye; but, as the far feet, are not ſeen, and as there are no receding parts ſeen, below, no diſtortion is obvious; the Object ſtanding in little compaſs, the inſide of the feet are ſcarce ſeen at all, owing likewiſe to their tapering downwards; otherwiſe they would be much diſtorted, on the returning ſides.

AE is a Scale of Proportion, on which the heights of the ſeveral parts are ſet up, from A, to B, D, and E, for the Pedeſtal. EH is the geometrical width, by the ſame Scale. From E and H, if Lines be drawn to the Center, and FH be made equal to its depth, in proportion to the front, FV being drawn, or a Ruler applied to F and V, cuts HC at I; from which a parallel Line is drawn, cuting EC, and determines the width of the top.

The Mouldings in the front of this Object are the chief difficulty; but the geometrical figure being known, the place of the parallel ſtreight part is eaſily determined, on EH; and the curved parts, whether convex or concave, are flatter or quicker, as they are nearer to or farther from the Vaniſhing Line, and may be done ſufficiently correct by hand; indeed all attempts at accuracy, by rule, in ſuch minutias, being ſo ſmall, would be fruitleſs and loſt labour; experience and judgment are the beſt and only guides.

The place of the foot of the Vaſe being got, on the Pedeſtal, at FG, let the height of the greateſt ſwell be ſet up, from E to J; where, draw a Line parallel to the Horizon; which, if the ſwell be equal to the ſtreight part of the Pedeſtal,

may be uſed for the ſide of a Square, encloſing the Circle, making KL equal to its diameter. If the Vaſe ſwell more, it will project on this ſide KL, if leſs it will fall within it; the real meaſure, in either caſe, is ſtill the ſame, on KL, whether it be projected larger, on this ſide, or be diminiſhed on the other.

This curve being determined, all the others will be more or leſs curved, as they are higher or lower; each of which muſt be done by the ſame means, if accuracy be required; finding the true place and proportion of each, by drawing a parallel Line, as NO, according to its height, above or below the other. The height of the top is beſt determined by a central Line, and adjuſting the ſcale of proportion to its diſtance by means of the Vaniſhing Point (V) of Diagonals, and a perpendicular Line drawn through K; but, as it is, here, on a level with the Eye, it is determined by the Horizontal Line; and thus, this Object may be compleated.

Figure 130 exhibits the repreſentation of a round Pier Table according to the preſent taſte, with tapering, term Feet. Fig. 130.

No. 1, is half the Plan of the bottom, for the place of the Feet, and No. 2 is the figure of the Top, geometrical, in inlaid work; both together form a Semicircle; but, if it has not a folding Top, which, when open, form, together, a complete Circle, I would rather it ſhould be ſomewhat broader, than half the length; equal to the narrow margin, around; becauſe, the figure within it ſhould be an entire Semicircle, which is not ſo here; conſequently the Figure is not ſo regular.

Let the Center of view be determined, at C, and take FG equal to the diameter of the Top; as much below C as the Eye is ſuppoſed above the Table. The Eye is, here, ſituated ſomewhat to the left hand, ſo that this Object is not viewed direct in front, that is, not centrally, as the foregoing.

Biſect FG, at *E*, and draw *EC*, FC, and GC, and from *E* or F draw a Line to the diſtance Point (which is not here fixed) cuting GC or *EC*, at K or *D*; through which, draw IK parallel to the front Line. In ſhort, complete a Semi-ellipſis (I*E*K) repreſenting a Semicircle, within the Rectangle FIKG; and, within it others, repreſenting the ſeveral Borders, &c. in the Top of the figure; which may be obtained by drawing perpendiculars from a, b, and c to FG, and, from *A*, *B*, *C*, draw Lines to C, cuting the inner curve of the border at *a*, *b*, *c*, from which draw Lines to *D*. On account of the inner Curve, *abc*, not being a complete Semicircle, *bD* does not fall in the Diagonal G*D*.

The Top being completed, take *FG* parallel and equal to FG, at the diſtance of the breadth of the Frame, and within it, take a leſs Diameter, allowing for the projecture of the Top; on which, deſcribe the curve at the bottom of the Frame, and the ſmall Aſtragal; from the extremes of the inner curve, draw perpendicular, to the lower curve Line expreſſing the thickneſs of the Top.

For the place of the Feet, take fg below, at the diſtance Ff, equal to the height of the Table, in proportion to its width (FG) and having deſcribed the repreſentation of a Semicircle, of the diameter of the Frame, the place of the Feet, at i and k, are in the Angles (but let it be noticed, that, the Top not being more than a Semicircle, the curve i*xx*k is leſs, becauſe the Top hangs over, behind the Table; ſo that, d repreſents the Center)

From x (No. 1.) draw perpendiculars to HE, cuting it at Z; which meaſures transfer to fg, and draw Lines to C, cuting the curve, at *x*, which give the place of the front Feet. The Plans, at *x*, *x*, being completed by means of their proper Vaniſhing Points, in the Horizontal Line, draw perpendiculars from each corner, to the Frame; and, within the ſquare of the Foot, deſcribe another, anſwering to the thickneſs at the bottom, from all the corners of which, draw Lines to the Frame, where their full dimenſions are obtained. The Plinth, at a little height from the bottom, is equal to the thickneſs of the Foot, at the Frame, the Lines of which tend to proper Vaniſhing Points, as the Plans, below.

The Compartments in the Frame, and the margins of the Feet, &c. it would be trifling to deſcribe, being divided in the middle, and at each Foot; or otherwiſe, at pleaſure, from the Plan, below; the Figure muſt ſupply the reſt.

Plate XXXIV. Fig. 131.

Figure 131 repreſents a Chair, direct in front; C is the Center of view, the Diſtance is CE. GH is the Ground Line, in which, take G and H for the places of the fore Feet; where, make their Plans, and deſcribe the front, GABH, truely geometrical.

Find V and X, the Vaniſhing Points of the Sides, AF and BD, reſpectively (Prob. 2.) and having ſet off, from V, the diſtance of that Vaniſhing Point, on the Horizontal Line, make B*a* equal to the depth of the Seat, and draw a Line from *a* to the Eye, cuting BV at D, and draw FD parallel to AB, cuting AX which gives the Figure of the Seat, in the naked Frame; but being ſtuffed, they muſt be gently curved at the ſides and behind, as on the left ſide.

The place of the back Feet, on the Floor, muſt be obtained by the ſame means, obſerving that they ſplay, backward, from the Seat, and conſequently, are farther diſtant from the front; alſo the line of direction is thereby ſomewhat varied.

To determine the height of the Back, and the Elbow, draw GB perpendicular, and equal to its height. Draw BC or BX, cuting a perpendicular Line from K or k, at a or c; through which, a parallel Line, ab, determines its height. As to the Figure, 'tis at each Perſon's option, and depends wholly on the Eye.

The height of the Elbow is ſet up from A to d, and by drawing a Line to X, its direction, and where it is joined to the Back are aſcertained; alſo, its place on the Rail may be determined, as the whole length, at D. The reſt depends entirely on the Hand and Eye; for, no Rules can poſſibly determine its apparent Figure, perſpectively.

Fig. 132.

Figure 132 is the repreſentation of a Lady's dreſſing Table.

In this Picture, the Center of view is taken ſomewhat to the left hand, at C, the Diſtance is CE². The height of the Eye ſuppoſes the Perſon ſitting.

The meaſure of its length, in front, may be ſet off, on the ſame Ground Line, GH, in proportion to the Chair; or on another, at MN, being ſuppoſed to ſtand farther from the Picture; on which acconnt, its geometrical Figure, in front, is by a leſs Scale, as MABN.

AB, the front Line of the Top, being determined, let it be biſected, and, AD and BF made equal to half the meaſure, in any poſition required; from all which Points, draw to C; make A*b* equal to the breadth of the Top, and draw *b*E², cuting AC at b; draw bf parallel to AD. From b it is transfered, by a doted Line, to the other ſide. The hollowing and thickneſs of the Covers, in front, are determined geometrically, by perpendiculars to AD, or BF, at f and D, or F.

Make A*a* equal to the diſtance of the neſt of Drawers, which riſe, by a Spring, out of the Frame, and draw *a*E², cuting AC, at a; draw ac parallel to AB, cuting BC; obſerving, in this operation, that there is a margin, of above half an Inch allowed on each ſide, leſs than AB. Draw the perpendiculars ae and cd, and proportion them to the length ac; or ſet up the full meaſure on a Perpendicular, the breadth of the margin within, at B; on which, the ſeveral Drawers may be proportioned, at 1, 2, 3, geometrical. The front of the Drawers, being parallel to the Picture, they are geometrically divided into as many Drawers and in what order you pleaſe.

The dreſſing Drawer, being drawn out, it conſequently projects on this ſide the Picture. Having drawn the fronts of the two Drawers, over it, draw CG, and produce it, indefinite; produce HG, and make GI equal to the meaſure you intend the Drawer to project, and draw E²I, till it cuts CG, produced, at K; draw KL parallel to GH, cuting CH produced, at L; KGHL repreſents the upper face of the Drawer, projected towards the Eye. HO (*the ring*) being made equal to its depth, draw CO, till it cuts a perpendicular from L, at J, which determines the front.

The diviſions, for the Glaſs, in the middle, the Boxes, &c. are determined, by their meaſures on GH, or by a larger Scale, on KL, as K*c* being made equal to the firſt diviſion, *e*E² determines its apparent breadth, at d. After the ſame manner the reſt are determined.

The

The receſs, below the Drawers, being an elliptic curve, is thus determined.

The height being determined, draw ag, the tranſverſe Diameter of the Ellipſes, deſcribe the geometrical curve a 3 5 g; ag being biſected at d, d 4 perpendicular to ag, is half the conjugate Diameter. Take as many Points, 1, 2, 3, &c. on each ſide, as are neceſſary, from which, draw Ordinates to the Tranſverſe at a, b, c, &c. obſerving to make g, f, and e, reſpectively, the ſame diſtances from that extreme, as a, b, and c from the other; from all which, draw Lines to C, and determine the length of each perſpectively, and reſpectively, by means of the Point of Diſtance E'; making b*b* repreſent b 2, and, c*c* repreſent c 3, &c. parallel Lines, *bg*, &c. will determine thoſe on the other ſide, giving the Points a, *b*, *c*, &c. through which the perſpective curve may be deſcribed, as in the Figure.

What remains is ſufficiently explicit by inſpection.

Figure 133 repreſents a Lady's Secretary and Library; of which there needs no particular deſcription. It is viewed directly central, ſo that, the Vertical Line cuts it into two ſimilar Figures. Fig. 133.

Being parallel to the Picture the whole front, of Drawers, &c. is geometrically drawn; the middle part, with the Deſk Drawer, may be projected, as the laſt Figure; or the Picture may be ſuppoſed to be wholly on this ſide, ſo that, only the Fall of the Deſk is projected.

The Shelves, with the Books, recede from the front, it would be impertinent to ſhew how it is managed here; the Books are diſpoſed at diſcretion, and, the Door which is open is determined by Example 39; V is the Vaniſhing Point.

For the reſt, the Figure is more explicit than the moſt elaborate deſcription.

Figure 134 exhibits the repreſentation of a Bed with a Canopy-Teaſter, viewed direct in front; one half ſhews the naked Frame, the other half is furniſhed. Fig. 134.

The Front being parallel to the Picture, it is geometrically proportioned. C is the Center, and C V the Diſtance of the Picture, by which the reſt is proportioned.

The Column, on the left hand, being ſeen entire, is repreſented as in former Examples; the returning Lines, in the Baſe and Capital tend to the Center; alſo the inſide of the Foot.

The length of the ſide (GH) being determined, and the height of the Poſt, at A, draw AC, cuting a perpendicular from H, at B, the apparent height of the head Poſt. The geometrical curve of the Laths being determined, the perſpective curve, of the ſide Lath, is determined by means of Ordinates, *a* 1, *b* 2, &c. the ſeveral heights being ſet up from the corner, at A, and Lines drawn to the Center.

The Laths forming the Canopy, diagonal wiſe, are determined as groined Arches, by Ordinates from the Diagonals, DE and EF, E being the Center.

The hither one being determined from its known figure, dividing DE perſpectively, at a, b, c, &c. as the Baſe of the Original is divided by Ordinates, at diſcretion; by which means, the ſeveral heights a*a*, b*b*, &c. are determined, and the Curve deſcribed through the Points *a*, *b*, *c*, &c. Then, Lines drawn to the Center, from the Points a, b, c, &c. cut the other Diagonal (FE) at k, i, h, &c. at which Points, the Ordinates k*k*, i*i*, &c. being drawn, and from *a*, *b*, &c. Lines drawn to the Center, cuting them, at *k*, *i*, *h*, &c. through which, the curve of the other diagonal Lath is drawn.

In reſpect of the Drapery, the Furniture of the other half, I ſhall only obſerve, that it is proportioned as the other; but as for the outline figure and ornament, I will not attempt to give any Rules; ſuch Subjects are always beſt drawn from a real Object, or they can only be done by a Perſon of experience and judgment. The figures of the Vallens are geometrical, in front, and no other are ſeen. The Canopy is but little ſeen, though it riſes conſiderably; a Right Line, from *b*, ſhews how much, ſave the middle, which appears over it; on which a Vaſe, or other ornament, at diſcretion, may be drawn, crowning the whole.

Figure

Plate XXXV. Fig. 135.

Figure 135 is a Deſign for a large Library Bookcaſe.

In order to give an Idea of the real proportion and figure of ſuch an Object, to a Perſon who, perhaps, knows not what Perſpective means, and who has no other Idea of its proportion and form, than what the Figure really exhibits, it is moſt proper to give it parallel, as it is here repreſented, but by no means to ſhew an End. The Station is central, and conſequently, the Point of View is in the middle. The receding parts, and breaking of the Mouldings around them, ſufficiently indicate that it is perſpectively delineated; and, by reaſon of its regular poſition, I am of opinion that it has a much more natural appearance, than when viewed oblique, from either end, which always occaſions diſtortion; as may be ſeen in Plate 23, Fig. 108, eſpecially when the Object is long.

In reſpect of the delineation, let it be obſerved, that the Plinths of the middle part and the two Ends are in one Plane, which are geometrically proportioned, by the Scale. The parts which recede are determined as uſual.

At either Corner, as A, of the Plinth, draw the perpendicular AG; on which, ſet up all the meaſures of its height, as AB for the Dado part, with its Baſe and Sirbaſe; BD the height of the upper Doors, &c. and DF, of the Cornice, &c.

To the Center, C, draw Lines from each diviſion; make Bc equal to the receding of the upper part; or, becauſe the whole Diſtance cannot be on the Picture, take CE half its Diſtance, and B*a* half Bc, and draw *a*E, cuting BC at b.

Draw b*d* perpendicular, cuting the Lines drawn from B, D, &c. to C, which gives the height of the Doors, &c. perſpectively, according to their diſtance from the Front. For their widths, make B*a*, equal to the projecture of the Mouldings, and *ab* to the width of the Door, and draw *a*C and *b*C cuting a Line drawn from b, parallel to the Baſe.

The other End and middle Doors are obtained by the ſame means. Or, being all in the ſame Plane, if either the height or width of one be obtained in its place, the whole is determined; for, being parallel to the Picture, the Doors are all repreſented ſimilar to their Originals; that is, geometrical, in reſpect of their outward figure, and alſo of the ornamental part, for Glaſs, &c. or as thoſe below.

For the Cornice; if CE, on each Side, be made equal to the Diſtance of the Picture; they are the Vaniſhing Points of the Diagonals, which are drawn through the Angles at the top, obtained from the Perpendicular at F, transfered to f. The general method, in Example 15, for Mouldings parallel to the Picture, is applicable here; X, being the Profile, and *E*, the Point of View.

The figure of the outline of the Pediment is geometrical; which certainly gives the beſt Idea of its true Figure; and ſo is all the Ornament, every where.

Fig. 136.

If it be required to ſee an End of any Object of which a Deſign is to be made, the Front ſhould then be inclined to the Picture, as in Fig. 136; which exhibits a repreſentation of a Library Table, by a larger Scale than the foregoing Figure. For however prepoſſeſed many Artiſts are, in the parallel poſition of the Picture, I am confident, that, were they practiſed a little in inclined poſitions, and ſaw clearly into Perſpective, they would not retain their prejudice long; for we cannot poſſibly ſee two Faces of a right angled Object, and the horizontal Lines, in either, appear parallel. Why, then, are they ſo repreſented? Whenever a ſingle Object is the ſubject of a Picture, being right angled, and ſeen oblique, it is, therefore, moſt palpably abſurd to be placed parallel to either Face; or, in any common Caſe, to have the Center of View out of the middle of the Picture.

The delineating this Object has nothing in it particular. The Inclination of it to the Picture and the Diſtance being determined, the Vaniſhing Points are determined, as uſual, both which are, in this Piece, out of the Picture.

S is the Center, and d the Diſtance Point, by which the Front is proportioned; the meaſures being applied on the Ground Line, AB, as in former Leſſons, at *a* and *b*, for the Front; which give a and b, on A*B*, the line of the Plinth, and on F*B*, of the front Plane, on the Floor, tending to their Vaniſhing Point.

As there is not room on the Ground Line to set off the full measure of the End, take AD a third part, or Ae half; and let f or S, be in proportion, to the Distance of the Vanishing Point of horizontal Lines in the End; then, Df, or eS, cuts AD in the perspective length required.

From each Angle, *A*, *B*, and *D*, Perpendiculars are drawn, and the real measures of the heights of the Drawers, &c. being applied on EF, where the Seat, of the Plane they are in, on the Floor, cuts the Intersection, lines drawn to the Vanishing Point of the front Lines, give their perspective heights.

Perpendiculars from a and b determine the opening; and if, from b, a line be drawn to the Vanishing Point of the End, and another from *D*, to the other, of the Front, cuting the former, at c, the perspective width of the inside End is determined. The Mouldings are done as usual.

EXAMPLE XLVI.

In the representation of a large Chamber-Organ, seen oblique. Fig. 137.

The method of proceeding, in obtaining the general proportions, which is subject to one general Rule, it is unnecessary to repeat; the measures, or their ratio to each other in proportion to the Distance used, being always applied on the Intersection of whatever Plane the Lines are in, or on a Line parallel to it; as on AB, for the Piers, &c. I shall, therefore, pass over that description, and only explain the Parts, where there is any apparent difficulty.

The Finger-Board (U) is usually made to draw out, the length of the Keys, about 6 Inches. Its place being acquired, at a, and V being the Vanishing Point of the End, draw Va, and produce it; draw ae parallel to the Ground Line, make ae represent 6 Inches, in proportion to the length, and draw Ce, (from the Distance Point of V) till it cuts Va produced, at b; and, from b, draw bc to the other Vanishing Point, of the Front, by the Expedients, Problem 13.

Draw bd parallel, on which, set off the number of Keys, as at U; then (having made hc represent the length required) join dc and produce it to the Vanishing Line cuting it at D; to which Point, draw Lines from all the divisions 1, 2, &c. which give the Keys.

The process may be done, more commodiously, below, on the Ground Plane, by dividing *be*, below, as bc above; from which, by Perpendiculars, it may be determined, above.

The Mouldings, breaking about the Piers, &c. have nothing singular in them; their heights, on the Pedestal, are set off on AD, the Intersection of the front Plane, with the Picture; and on *AF* (for the upper part) the Intersection of the Plane of the Pipes, &c. above, at *B*, *C*, &c.

At Z (No. 2) is a geometrical Plan of the Breaks.

The Column of Pipes, at each End, are Segments of Circles containing five Pipes each, which are unequal; let their perspective Plans be formed above, at X and Y; from their extreme edges, *a*, *b*, &c. draw perpendiculars, as shewn by the dotted Lines, giving the places, below, of the cylindrical Part.

The Mouth of each Pipe, at f, &c. is at about one third part of their height, from which it is conical, inverted. The Ornaments, at the tops and bottoms, are regular, from the middle Pipe, which it would be impossible to describe, being circular; their Figure being known, such minutias are drawn by the Eye.

The Pediment, and middle compartments of Pipes, being inclined, it may be necessary to shew how they are described perspectively.

Suppose the geometrical Figure of the Pediment the same as in the Book Case (Fig. 135) which, inscribe in a Rectangle, and draw Ordinates parallel to both Sides, from various points in the Curve, as in that Figure.

The Cornice, at the End, being described, as usual, and the Angle F determined, draw FI to the Vanishing Point of the Front; and make FI represent the distance of the two extreme Angles; at each of which, draw perpendiculars.

Plate XXXV.

fg is the Interſection of the Plane of the front of the Pediment; from *AF* tranſfer the height of the top of the Cornice, to f, and make fg equal to the known height, which, divide in the ſame ratio as HI (Fig. 135) from all which diviſions, draw lines to the Vaniſhing Point of the Front; and having divided GH perſpectively, as IK (Fig. 135) perpendiculars drawn from each diviſion, cuting the other, give the Points *a*, *b*, *c*, &c. through which the perſpective curve is deſcribed.

The middle Column of three Pipes is projected as the other.

If the geometrical figure of the two Sides be alſo determined; then, on *ag*, take the diviſions *c*, *d*, *f*, &c. geometrical, and draw lines to the Vaniſhing Point; let the horizontal Line *gk* be divided perſpectively, at *h*, *i*, &c. from which, perpendiculars being drawn, cuting the other at *m*, *n*, &c. give ſeveral Points in the Figure. The reſt may be drawn, accurately enough, by hand.

The proportion of the Pipes may be determined on any horizontal Line croſſing them, as *gk*, either above or below, as is moſt convenient; dividing it perſpectively, in the ratio of the Pipes (by Prob. 8) as they are all in the ſame Plane.

The mitre Angle, of the ſtreight Mouldings with the circular, muſt be determined by a geometrical Plan, at W, which varies according to what portion of the Circle is taken; alſo, where it falls againſt the Plane, the projectures may vary conſiderably; in a Semicircle they have their true projecture.

It is unneceſſary to give more examples of Furniture, ſeeing that, if the Rules of Perſpective, here uſed, be clearly underſtood, the Examples, given, are ſufficient for any Perſon, who is clear in the Principles on which they are founded; and may, by ſuch, be applied in any Caſe, whatever, in right lined Objects. Nevertheleſs, as Chairs, from the various directions of the Lines in them, are ſomewhat difficult to delineate, I ſhall give another ſpecimen, how far the Rules of Perſpective are applicable to ſuch Objects, any how ſituated to the Picture.

EXAMPLE XLVII.

Fig. 138.

How to repreſent a Chair, perſpectively, caſually ſituated.

The poſition, and inclination of its Front to the Picture being determined, and the repreſentation of the hither Angle, ab, of its Foot, given at diſcretion, to the ſcale of the Deſign you intend.

Let V be the Vaniſhing Point of horizontal Lines, in the front of the Chair. C is the Center of the Picture, and CE its Diſtance.

At No. 1. is a half Plan of the Seat, ſhewing the ſplay of the back Feet, at X, with the inclination of the ſide Rails to the Front.

Draw EV, and make the angle VEY equal to the inclination of the Side to the Front, giving Y for the Vaniſhing Point of the Rails, in that Side; alſo, make the Angle YEX, equal to the inclination of one Side to the other; giving the Vaniſhing Point of the other ſide Rails, &c.

Then, draw aV, and bV, aY, and bY; and cd, through b, parallel to the Vaniſhing Line.

Make bc, equal to the width of the Front, in proportion to the height (ab.)

Make VD equal VE, and draw cD, giving the place of the other front foot, at i; alſo, the breadth of each are ſet off at b*a* and c*b*.

Let bd be equal to the ſide of the Chair, at its Seat, and de to the ſplay of the back foot; make YG equal YE, and draw dG and eG, cuting bY at f and g; g is the place of the foot on the Floor; and if a perpendicular be drawn, at f, it will cut aY in the true width at the Seat.

Make a*c* equal to the width of the ſeat Rails, and b*d* to the height and width of the low Rail; draw *c*V; and *c*Y, cuting the Perpendicular from f; alſo, draw *d*Y, for the low Rail, to the back foot, which ſplays back, from the Seat.

From g and h draw Lines to V, and from i and k to X, cuting them at l and m, which give the place of the other back foot, and compleats the Seat.

Make

Make be, equal to the receſs of the inner Rail from the Front; draw eG cuting bY at f; from which, draw a perpendicular, giving its place on the low ſide Rail, at g; and from g, draw to V, cuting the other Side Rail, the meaſure of which was transfered from d to the other foot, at h.

The height of the low back Rail is obtained on the front Foot; and transfered to the back foot, by means of the Vaniſhing Point Y.

Thus much is practicable, with the greateſt accuracy; but beyond this, little can be done, the whole Back being curved every way.

The height may be truly determined, by means of perpendiculars, from the floor. Having obtained the Line gt, tending to V, over which, a Line, rs, through the corners of the top Rail, is perpendicular, through the Point where it is cut by bY, draw De to the Ground line; make e1 equal to its projecture over the Seat, and 12 to its known meaſure. Draw 1D and 2D, cuting gt at g and t, from which Points, draw perpendiculars. From any Point, as G, in the Horizontal Line, draw Ge, through g; draw eq perpendicular, make eq equal to the height of the Chair, and draw qG cuting gr in the true place of that corner of the top Rail; rV gives the other, at s.

The various proportions of the Baniſter, may, in ſome degree, be obtained, by dividing a line (no) drawn through its middle, into the ſeveral heights, 1, 2, 3, &c. through which, Lines drawn to the Vaniſhing Point, V, determine the raking of the ſeveral parts, as in the Figure. For, whether the Back of the Chair be ſtreight or curved, it deviates ſo little from a Right Line, and from a Perpendicular, that the meaſures may be applied, geometrically, without any ſenſible error.

In reſpect of the modern Chairs, now in uſe, there is no ſuch thing as applying the rules of Perſpective to them, except to aſcertain the place of the Feet, on the Floor; alſo, by inſcribing the Seat in a Square, or other Rectangle, its Figure may be nearly determined in its true place. Fig. 139.

The height of the Back, and of the Elbows may alſo be had, but, as the whole Chair is compoſed of irregular curved Lines, it is abſolutely impoſſible to give its true figure by the Rules of Perſpective; ſuch Objects, muſt therefore depend on the Hand and Eye, for a repreſentation of them; in which, a Perſon who has judgment in Drawing can only ſucceed; but, being well verſed in Perſpective, he will find great aſſiſtance from it.

WHEEL CARRIAGES are the next Subject to be handled; on which, as they are frequently neceſſary to introduce into a Picture, or for the ſake of giving a Deſign to a Gentleman, by thoſe whoſe profeſſion it is to make them, I ſhall make a few Obſervations; but, like many other Objects, not compoſed of Planes and Right Lines, little can be done by the rules of Perſpective, further than to give the proportion of one part to another; alſo the Wheels may be drawn with tolerable accuracy. Nevertheleſs, I am conſcious, that few Perſons who have occaſion to delineate them (except the makers) will take the neceſſary pains to project them by rule; for ſuch as will, I ſhall lie down ſome which are practical.

N. B. It muſt have been obſerved, that to delineate any Object by the Rules of Perſpective, its true geometrical form and proportion of one part to another muſt be known, or determined on; conſequently, the more complicated the Object is, being compoſed of irregular curved Surfaces, the greater is the difficulty in applying the meaſures on the Picture; which, as the ſeveral parts to be delineated are not Planes, or plane Figures, their places can only be obtained, on the Picture, from their Seats on ſome Plane or other, to which they are contiguous. By which means, a ſufficient number of Points, in any curved Line in the Object, being perſpectively found, and joined carefully by hand, the perſpective Figure is determined.

EXAMPLE XLVIII.

How to repreſent a Coach in perſpective.

Let ABCD be a geometrical Plan of the Carriage of a Coach; AB is the Axle of the fore Wheels, and CD of the hind Wheels, whoſe Diameters are ab and de. EFGH is the Plan of the Top of the Coach, all the other Parts are in proportion to them; as in the Figure. Fig. 140.

Plate XXXVI.

S is the Station, from which it is intended to be delineated, and IK the true position of the Picture from that Station. Sv being drawn, parallel to the Axles and front Lines of the Coach, gives their Vaniſhing Point; Sc, perpendicular to IK, gives its Center; and, Viſual Rays from various parts of the Coach, being drawn to S (anſwering the ſame as the Eye, in the Horizontal Plane) give their places on the Picture.

The Vaniſhing Point, of the Line of its direction on the Ground (with which, the Axles, &c. are at right angles) is out of the Picture; its diſtance from c (the Center) is to cS, as cS to cv. (Prob. 12.)

Fig. 142.

Theſe Preliminaries being ſettled, as uſual, let C be the Center of the Picture, and V, the Vaniſhing Point, by a Scale of 2 to 1; AJ is the Ground Line; and A, the nterfecting Point of the line of direction of the Wheels, on the Ground, which are, here, ſuppoſed to be in the ſame Plane.

The Plane of the Wheels not being vertical, but ſomewhat inclined, from the Carriage, let DG be the Interſection of the hither Wheels, with the Picture*.

Draw AD perpendicular, make AB equal to the diameter of the fore Wheel, and AD of the hind Wheel; draw BF and DG parallel to the Horizon, cuting AG at F and G. Draw Ae and Gg, alſo from F, tending to the Vaniſhing Point of the direction of the Coach.

To obtain the true place and repreſentation of the hind Wheel, biſect AG, and draw sH perpendicular. Draw He, to the Vaniſhing Point; make HJ equal to the diſtance of the Seat, on the Ground, of the center of the Wheel, from the interſecting Point H, and draw JE, cuting He at e, (or, if S be its Seat on the Ground Line, draw SC) at which Point, draw a Perpendicular, and from s (in AG) draw a Line to the Vaniſhing Point, cuting it at ſ, the repreſentation of the Center of the rim of the Wheel.

Draw ſG perpendicular to the Interſection AG; from which Point, ſet off the true radius of the Wheel, on each ſide, and draw Lines to the center of the Vaniſhing Line of the Wheels, cuting a Line drawn through ſ, parallel to AD, at e and g; eg is a Diameter of the Wheel, in its true place; from which, the circumference of the Wheel, efgh, may be deſcribed (by Prob. 3, Sect. 8.)

The fore Wheel (abcd) may be obtained by the ſame means.

The end of the Nave is not in the plane of the Wheel, but ſomewhat beyond it, at i and k; alſo, the Spokes diſh inwardly, from the front of the Wheel. Find the circumference of the Naves, *abcd* and *efgh*, in their true places, perſpectively, at the part where the ſpokes are fixed in it, and the place of each; alſo let each Rim be divided, perſpectively, into equal parts, the number of Spokes, at 1, 2, 3, &c. (by inſcribing a Poligon) from which they are drawn to their correſponding parts in the Nave; or to their common Center, cuting the Nave, in their true places; for, as they are not in the plane of the Rim, their Vaniſhing Points are not eaſily determined, nor are they neceſſary.

The hither Wheels being compleated, the Body of the Coach is next to be determined; the Sides of which not being in one Plane each, their Interſections are not eaſily determined. Therefore, from the Seat, on the Ground, of the hither Angle of the Coach, perſpectively determined, at i (from the Plan) draw ik perpendicular, in which, by means of the Vaniſhing Point, V, of horizontal Lines in the Front, determine the Angle k; the real height being IK, in the Interſection of the Front. The other Angles, m and n, may alſo be determined from their Seats; or, by their known diſtances from k, as in other common Caſes.

If the corners of the Coach are perpendicular (as in ſome Coaches) the meaſures may be determined on ik, geometrical; but if they are inclined, the point l may be obtained as k, from its Seat; and, from l, lines are drawn to the Vaniſhing Points, and proportioned as uſual.

The Door is not in the ſame Plane, with the Line kn, but may be determined from the Seat of the hither Line, op, where it opens; and, on any perpendicular

* This Interſection may be truly obtained, by Prob. 6. Sect. 12.

line, cuting the Ground Line, as IK, ſet up from I, its true meaſures, of height, at *a* and *b*; then, from I, draw a Line through its Seat (s) to the Vaniſhing Line, cuting it at L; draw *a*L, and *b*L, cuting the Perpendicular, from s, at o, and p; from which, draw Lines to the Vaniſhing Point of the Side.

The curve of the Bottom being geometrically determined, it may be done perſpectively, by means of Ordinates, as in foregoing Examples. From the Vaniſhing Point V, draw a Tangent to the hither Curve, at r, which alſo touches the other.

The contour of the Top may be determined, by ſeveral vertical Sections deſcribed, perſpectively (as *de*p, and *fgh* at the Door) then, a Curve deſcribed over them all would be the true Contour.

The Axles being parallel to the front of the Coach, their Vaniſhing Point is V; their thickneſs may be proportioned to the diameter of the Wheel, and their Shape by means of Ordinates, at 1, 2, 3, &c.

The geometrical Figure of the Crane Neck being deſcribed (Fig. 141.) then, by means of ſquares (the whole being inſcribed in a Rectangle, ABCD) it may be accurately drawn in Perſpective; with the other appendages of Springs, &c. by the ſame means. Alſo the Circle, at X, on which the fore Wheels turn, is determined, at Y, in its true place perſpectively, as any other Circle, parallel to the Horizon.

The height and true place of the Seat may be eaſily determined from its Plan on the Ground Plane, perpendicular over the fore Axle. The foot-board, at M, will be beſt obtained by the ſame means; or, by its Interſection and Vaniſhing Line, being a plane Figure. The place of the Bar, to which the Traces are fixed, is eaſily determined, at E, which tends to the Vaniſhing Point V.

The other ſmaller Appendages, ſuch as Iron Stays to the Seat and Springs, Rings, Straps and Buckles, &c. it would be trifling to deſcribe.

The true Figure and proportion of the Carriage being obtained, the Ornaments of carving, &c. will depend on the Hand and Eye, wholly.

And, laſt of all, the off Wheels muſt be drawn, after the ſame manner as the hither ones, nearly; obſerving that they incline the other way, from the Picture. But, as they are ſo much hid, by the other parts of the Carriage, their Circumferences and the places of the Spokes therein (with the Point to which they tend) may be obtained from the hither Wheels, by means of the Vaniſhing Points.

Figure 143 repreſents a Phaeton, more foreſhortened, and going from the Picture; by which the hind part is more ſeen, as the fore part in the former Figure. Fig. 143.

A deſcription of the method of delineating it, it is obvious, would be, in a great meaſure, a repetition, conſequently tedious. The Poſition to the Picture being fixed and its Proportions known, the method of proceeding is as in the foregoing Figure, in reſpect of the Wheels and Carriage.

AB is the Ground Line, on which the Picture, ſtanding perpendicular, is ſuppoſed to touch the hither hind Wheel, of which, BD is the Interſection. B*d*, and Bb, tending to their reſpective Vaniſhing Points, are proportioned, at *a*, *b*, *c*, &c. as uſual, for the places and diameters of the Wheels, &c. which are deſcribed as the foregoing; and are transfered to the other ſide, by the Vaniſhing Point of the Axles, the true figures of which, being in vertical Planes, are determined as other Figures, from their known geometrical form.

On ef is ſet off, perpendicularly, the height of the hinder Axle, and the horizontal movement of the fore Wheels, determined at *f* by a perpendicular from *e*; being cut by a Line drawn from f, to the Vaniſhing Point of the Line of direction of the Carriage, on the Ground. *eg*, on the Ground, is a diameter of that Circle, which being deſcribed, it may be determined in its proper place by Perpendiculars; after which, the reſt of the Carriage may be drawn by a Perſon who has knowledge of the ſeveral parts.

Plate XXXVI.

The corners of the Chair, *m*, *n*, *o*, and of the Foot-Board, at *l*, may be aſcertained by perpendiculars, from their Seats on the Ground, perſpectively determined at m, n, &c. and certain other Points which are in a right lined direction, as *p* and *q*. For the reſt, it would be to as much purpoſe to give Rules for determining the figure and poſition of the Man in the Chair, and of the Horſes; ſeeing that, the curve Lines which bound the Figure, ſave thoſe at the top, are ſo variable, that no poſitive determination of their apparent Figures can be deſcribed; but muſt ever depend on the Eye, and judgment in drawing.

EXAMPLE XLIX.

To repreſent a Machine, for driving Piles; which is in the Repoſitory of the Society, for the encouragement of Arts, Manufactures and Commerce, in the Adelphi, Strand.

Fig. 144.

Objects of this kind, I have frequently obſerved, are uſually drawn with one Face parallel to the Picture; not, I am perſuaded, with a view to make it better underſtood, but to ſave trouble; or, as it is moſt probable, for another reaſon, a want of judgment or knowledge in the Art, to do it otherwiſe; for, I am opinion, no Perſon would think that this Object would convey a more adequate Idea of its ſtructure and parts, by being parallel, ſeeing that, all the ſame parts are or may be ſeen, in one as in the other, without any apparent diſtortion; and which, by being parallel, is unavoidable, being ſeen oblique. I therefore think it unneceſſary to give a Specimen, in that poſition to the Picture.

C is the Center of the Picture, whoſe Diſtance is about 8 Inches; the Vaniſhing Points are determined, as uſual, according to the Inclination given.

Let A D be the Interſection of the Picture, with the plane of the Frame at the Bottom; and A, the Interſecting Point of the neareſt Angle of one of the Timbers; alſo, let E and F be the Diſtance Points, for the Vaniſhing Point of each Side, reſpectively.

The Vaniſhing Points being aſcertained, draw A *D* and A *H*, tending to them.

Make A*b* equal to the length of that End beyond the other croſs Timber; and *bd* equal to the width between the Timbers, allowing the thickneſs of one, and draw *b*E and *d*E, giving the Points c and d; through which, the other Timbers are drawn.

Make A*c* equal to the diagonal of the Inclination, and draw *c* V, to the Vaniſhing Point; through c and d, draw Lines to the Vaniſhing Point of the Side, and produce ec, to the Interſection, A B, cuting it at J.

Make A b equal to their thickneſs, and compleat the Ends (v and x); from the two upper Angles of each, draw lines to the Vaniſhing Points, on the left hand.

How that Frame is finiſhed, the Figure deſcribes ſufficiently.

The places of the other Timbers are obtained by ſeting the Diſtances, &c. on the Interſection A B, at *e* and *f*, and thence transfered to A *D*; or, from J, to J *l*.

From J, draw a Perpendicular (J L) the Interſection of the long Timbers; on which, ſet up all the meaſures of the Frame for the Wheels, at *j*, *i*, *k*, and *l*, from each of which, draw lines to the Vaniſhing Point of the Side, cuting Perpendiculars from c and f, giving the height of the Platform, &c. the length of which, (*mn*) may be obtained from the Timber, below; and the breadth (*no*) by the Vaniſhing Point of a Diagonal (*mo*,) at e.

The feet of the Supporters (r and f) being got, and the Bearer, or Head piece (X) drawn to the Vaniſhing Point of the Front (giving what thickneſs is neceſſary, at *l*) draw the Supporters, as in the Figure; alſo, the Bearers (y) and Stays (*z*) of the Platform. Their Vaniſhing Points may eaſily be obtained, knowing their inclination, in the vertical Vaniſhing Line of the End; by Prob. 4.

The top of the croſs Braces being got, by a line from *k*, the Figure ſhews how they are drawn; ſo as to ſhew their thickneſs, properly; firſt drawing the Front.

Produce

Produce the inſide of the long Timber to the Ground Line, and ſomewhat beyond it, at I, draw IK, perpendicular, the Interſection of the vertical toothed Wheel, which begins the motion. Fig. 142.

Make I*p* equal to the height of its Center, and, let *pq* be its Radius, and *qr* its Diameter. Draw from *p* to the Vaniſhing Point of the Side; and, having biſected the head of the Frame, at g, draw, from g, to the Vaniſhing Point of the End, which will paſs through the Center of the Wheel, at *s*, through which, draw *tu* its Diameter, and compleat the Rim hikl (by Prob. 3, Sect. 8.) Its Vaniſhing Line paſſes through the Vaniſhing Point of the ſide Timbers, perpendicular.

The croſs Bars (hk and il) have their Vaniſhing Points in it; making right angles, perſpectively, with each other, at diſcretion.

The Teeth, where they are ſeen, may be done by hand; for although the Rim may be divided into equal parts, yet, to divide it into ſo many would be unneceſſary trouble; obſerve, as the Wheel is contracted ſo are the Teeth.

The other vertical Wheel (Z) of a larger Diameter, on the ſame Axle, is managed by the ſame means; having the ſame Vaniſhing Line, and Vaniſhing Points. Its uſe is to accelerate the motion, by its weight.

In ſuch Objects, great regard ſhould be had to deſcribe the hither parts firſt; taking particular care that they do not croſs each other, improperly.

The middle, upright piece, m n, and the Bearer, n C, being drawn, the place of the long Axle is oppoſite the middle of the vertical Wheel; its place may be found in a Line drawn, on the Ground, through the foot of the upright Supporter, at r.

The ſmall horizontal Wheel (pq) being nearly on a level with the Eye, appears almoſt in Right Lines. Its Diameter is, in proportion to the vertical Wheel, nearly geometrical.

The height of the upper horizontal Wheel may be had from any perpendicular Line, cuting the Ground Line, as J L; drawing a Line through r, the Seat of its Center, on the Ground, to the Horizontal Line, cuting it at E.

Make IL, equal to its height, and draw LE, cuting a perpendicular from r at s, and gives s for the Center of the Wheel.

Draw LM parallel to the Ground Line, make LM equal to the radius of the Wheel, and draw ME, cuting a Line drawn through s parallel to LM, at t; st is the Radius, in its true place; by which, the Circumference, tuvw, may be deſcribed. The thickneſs of the Rim, and height of the Teeth, are ſet up from L; and the croſs Bars tend to the Vaniſhing Points of the Timbers; or at diſcretion.

The place of the Stay at u is alſo determined on IL, at N.

The large upright Timbers may be determined, in height, by the ſame means.

Produce the bottom Line of the hither croſs Timber (at its foot) to the Ground Line, at O; draw OP perpendicular, and equal to the height. Then, having obtained the place of the upright Timbers, in the middle of the Rails, below, at R and S, and the perpendicular Lines being drawn, a line drawn from P to the Vaniſhing Point, on the left-hand, determines the height of the hither one, at T; and, from T to the other Vaniſhing Point, they may be compleated.

The Center and Diameter of the Wheel V may alſo be obtained on OP, at *p* and *q*, and tranſfered to its place, giving *s* for its Center, and *rs* for its Radius; by which it may be compleated. (Prob. 3, Sect. 8.)

Obſerve, it is the circle of the back part of this Wheel that is firſt got.

The place of the Weight, or Ram, W, is at pleaſure; its height is in proportion to the upright Timbers; alſo the Stays between them, above.

The Roller (Y) for winding up the Ram, is a Cylinder, on the Axis of the vertical Wheel; and it is ſo contrived, as to revolve on the Axis freely, while the Ram deſcends, without retrograding the motion of the Wheels. When the Ram is down, it catches, and when the Ram arrives at the Top, the Cylinder is ſet at liberty again; ſo that, the motion of the Wheels is continual, the ſame way.

Behind the Frame is another heavy vertical Wheel (X) on the Axis of the Cylinder, intended to accelerate the motion, whilſt the Ram deſcends.

EXAMPLE

Plate XXXVI.

EXAMPLE I.

Is the repreſentation of a Crane, from the ſame Repoſitory.

Having been minutely particular in the deſcription of the foregoing Machine, leſs will ſuffice in this. It is worked after the ſame manner, by means of Winches; the Model has three, here is but one; which, is ſufficient for the purpoſe of deſcribing the manner of delineating it.

A geometrical Plan, of the place of each upright piece of Timber, &c. in the Frame, being drawn, to any Scale, and the poſition of the Picture determined, let Perpendiculars be drawn, from each, to the Picture, giving their Seats. Or, being ſo prepared, let the Lines, of ſome of the principal, be produced to the Picture; which, where it can be conveniently done, facilitates the proceſs.

Fig. 145. The Center of the Picture is at G, in the ſame Horizontal Line; the Diſtance is about 6 Inches and a half. V is the Vaniſhing Point of horizontal Lines at the Ends, determined at diſcretion; the other runs out, on the right-hand, diſtant from the Center almoſt 14 Inches (a third Proportional to CV and the Diſtance of the Picture, always, when the Lines are at right angles with each other) BK, on the Floor, is the Ground Line of the Picture, on which it ſtands.

From A, the interſecting Point of the end Line, at the foot of the hither Supporter of the Frame, draw AV; and in it, find a, the neareſt Angle (Prob. 7) from which draw a Line to the other Vaniſhing Point, and produce it to the Ground Line, at B; and draw BF, perpendicular; on which, the ſeveral meaſures, of the heights, are ſet up, from B, and Lines drawn to the other Vaniſhing Point cuting Perpendiculars from a and b.

The Axle (RS) of the large Wheel between the Supporters, is determined by a line drawn from G, the Seat of S on the Ground Line, to the Center, giving its place, at S (in the central Line LS) its Diameter is got by the ſame means. Or thus.

The height of the Wheel from the Floor being known, draw its Interſection HI, parallel to the Ground Line; and from G, draw Gg, perpendicular, and gC, cuting the Axle in the Center of the Wheel. Make gH and gI each equal to the Radius of the Wheel, and draw HC and IC, cuting a line drawn through the Center (J) parallel to HI, at h and i; by which Diameter (hi) the Circumference may be compleated. (Prob. 3. Sect. 8.)

Then, if V be made the Vaniſhing Point of one of the Spokes (kl) the Vaniſhing Points of the other are found by making Angles at the Eye, with the Radial of that, equal to the Angles they make with each other; which, in this, are equilateral, having ſix Spokes, forming a regular Hexagon. (Prob. 24.)

The Winch Wheel (U) touches the Picture; its place being obtained, at K, from the Plan, draw its Interſection Kt, and ſet up the height of its Center Ks: make rs, and st, each equal to its Radius; and, on the Diameter rt, deſcribe the Circumference, and finiſh the Wheel from its known Figure.

On the Interſection Kt, of the Face of the Wheel, the height of the Supporters, and alſo the place of the croſs piece (T) may be determined.

The other vertical Wheel, at Y, is got after the ſame manner, on PQ. Its uſe is to ſtop the Crane from going backward, by means of the Catch, at z.

At Z is a Lever, which, by a Cord and Pully, raiſes the Catch.

The upper Gudgeon-piece (X) being drawn; find the foot of the Crane, at L, in the central Line; its dimenſions being obtained, draw the upright Piece, LM, and ſet up the meaſures of the Brace and Headpiece, at *d*, *e*, and *f*.

On the Floor, deſcribe a portion of the circumference of a Circle, perſpectively, on the Center L (Radius, the projecture of the Crane) and, according to the direction of the head of the Crane, its Seat being ſomewhere in that Curve, as at N, draw NO perpendicular, and make NO repreſent its height from the Floor.

The Rollers, which direct the Cord, are Cylinders; their upper Baſes are determined at W, from which, perpendicular Lines are drawn to the head Gudgeon-piece at X, which is level with the Eye, ſo that, its Plane is not ſeen.

The Cord, the Pullies, Teeth of the Wheels, the canting of the Timbers, and all ſuch minutias, I ſhall paſs over, as the Figure deſcribes thoſe parts better than the Pen. For thoſe who would be particular, in large Drawings, various Leſſons may be found in the Work, for truly projecting every part.

SECTION XII.

On INCLINED PICTURES and PLANES, in general.

AS the Plan which I have hitherto purſued, has been ſolely confined to the uſeful ſtudy of Perſpective, ſo, this Section is adapted more for the curious Artiſt than the uſeful; yet, the inclined Picture may, in ſome Caſes, be neceſſary. However the univerſality of the Principles are, here, more fully evinced; it being demonſtrated that the poſition of the Picture to the Horizon, is not of the leaſt conſequence, nor has any meaning in the Theory of Perſpective, as it has already been ſhewn, in reſpect of horizontal Pictures, and is, in this Section, made general.

For, whether the Picture be vertical or horizontal, and the Planes or Lines, in Objects repreſented on it, be inclined to the Horizon or to the Picture, or both; 'tis the ſame thing if the Picture be inclined, and the Planes or Lines be either parallel or perpendicular to the Horizon; ſeeing they are, or may be, nevertheleſs, inclined to the Picture. Wherefore, however either the one or the other be ſituated to the Horizon, it matters not, if the inclination of one to the other be known, and the Interſecting Points of any two Lines in either, with the other, be determined; the ſituation of the Eye being known, in reſpect of either, as ſhall be made manifeſt, to conviction.

PROBLEM I.

The Interſecting Points of any two Lines in a Plane being given, and the Inclination of the Plane, to the Picture, known; to find the Vaniſhing Line of that Plane, its Center and Diſtance; the Center and Diſtance of the Picture being given.

The Interſecting Points of two Lines, in any Plane with another, being given, the common Interſection of thoſe Planes is determined. (Theo. 10.) Therefore, if A and B are the interſecting Points given, or found, AB is the Interſection of the Plane they are in, with the Picture. (Cor. 3.)

Plate XXXVII. Fig. 146.

Let A and B be the Interſecting Points of two Lines, in ſome Plane; and C the Center of the Picture, whoſe Diſtance is known.

Draw AB, and CE parallel to AB, equal to the Diſtance of the Eye; through C, draw DF perpendicular to AB; make the Angle CEF equal to the complement of the inclination of the Plane to the Picture, cuting DC, produced, at F. Through F draw GH, parallel to AB, which is the Vaniſhing Line required; its Center is F, and its Diſtance is EF.

Turn up the Triangle CEF, on CF, till EC be perpendicular to the Picture; then, E, is the true Place of the Eye, and EC is the Direct Radial.

DEM. Now, becauſe DF is perpendicular to AB (the common Interſection) and CEF is a Plane, perpendicular to the Picture, paſſing through DF, it is perpendicular to the other Plane †, for, it is perpendicular to their common Section; therefore, DF is the Vertical Line of that Plane, (Def. 11.) † 9. 7. El.

Then, becauſe CEF is the Complement of the angle of Inclination of the Plane to the Picture, EFD is the real Angle, (Cor. 3. 10. 1. El.) for, ECF is a Right Angle, by Conſtruction.

Conſequently, ſince the Vaniſhing Line of any Plane is parallel to the Interſection of that Plane, (Th. 2) if a Plane be ſuppoſed to paſs through the Eye, at E, parallel to the Original Plane, it is inclined to the Picture in the Angle, EFD; and the Line GH, in which it muſt cut the Picture, is the Vaniſhing Line of that Plane, (Def. 8.) for, the Sections of parallel Planes, by another Plane, are parallel between themſelves. — — — — — 8. 7. El.

But, EF is perpendicular to GH; therefore F is its Center (Def. 19.) and conſequently, EF is its Diſtance (Def. 20.) Alſo, CF is perpendicular to GH, (Th. 7.) for, it is a Line drawn in the Plane CEF, from the Point F, in which a perpendicular (GH) to that Plane, cuts it. 2. 7. El.

Plate XXXVII.

Cor. Hence the Corollary to the 11th Theorem is manifeſt; as the Center of the Picture is the Vaniſhing Point of all Lines which are perpendicular to the Picture; ſo, the Center of every Vaniſhing Line is the Vaniſhing Point, of all Lines which are perpendicular to the Interſection of any Plane, of which it is the Vaniſhing Line, and of all Lines parallel to them.

Becauſe, EF, producing the Center, (Def. 19.) is parallel to all ſuch Lines.

It muſt be obvious that this Problem is univerſal, and cannot poſſibly regard any poſition either of the Picture, or of the Original Plane to the Horizon; for, let them be any how ſituated in that reſpect, every thing remains the ſame. But, nevertheleſs, in many Caſes, the poſition they have to the Horizon is neceſſary, in practice, to be known; ſeeing that, all plane Objects, as Buildings, &c. are determined from the poſition of their Planes to the Horizon, as well as to the Picture; nor is it poſſible to determine any thing, in reſpect of their Projections on the Picture, without the previous determination of ſome Plane or other in reſpect of them and the Picture, and which is beſt done, by means of a Plane perpendicular to the Picture, either horizontal or vertical.

PROBLEM II.

The Center and Diſtance of the Picture being given, and the Vaniſhing Line of ſome Plane which is inclined to the Picture; to determine the Inclination of the Plane, and to find the Vaniſhing Point of Lines perpendicular to that Plane.

Fig. 147. Let AB be the Vaniſhing Line given, and C the Center of the Picture.

Through C, draw DF perpendicular to AB, indefinite; draw CE perpendicular to DF, i. e. parallel to AB; make CE equal to the Diſtance of the Picture, and draw ED; alſo, EF, perpendicular to ED.

EDC is the angle of the Inclination of the Original Plane, to the Picture; and F is the Vaniſhing Point, of Lines perpendicular to the Plane.

Turn up the Triangle DEF perpendicular, alſo turn over the Plane X on AB.

Dem. Becauſe CE is perpendicular to the Picture, and equal to its Diſtance, EC is the direct Radial, and E is the Point of View (Def. 15 and 16) and, becauſe the Plane A*E*B paſſes through the Eye, and the Vaniſhing Line (AB) it is parallel to the Original Plane, (Def. 8.) and its Inclination to the Picture is the ſame, equal EDC.

But, the Plane DEF is alſo perpendicular to A*E*B, and conſequently to the original Plane; therefore, it is the Vertical Plane, (Def. 5.) and DF is the Vertical Line, (Def. 11.) which makes equal Angles with them both; and, becauſe it is the Interſection of a Plane cuting two parallel Planes, perpendicularly; therefore, the Angle EDF is equal to the inclination of the Original Planes.

But, EF is in the Vertical Plane, and it is perpendicular to ED (by conſtruction) conſequently, EF is perpendicular to the Plane A*E*B, and conſequently it is parallel to all Lines perpendicular to the Original Plane; and therefore, F is their Vaniſhing Point. (Def. 22.) Q. E. D.

PROBLEM III.

The Center and Diſtance of the Picture being given, and the Vaniſhing Point of Lines, which are perpendicular to ſome Plane, to find the Vaniſhing Line of that Plane.

Fig. 147. C is the Center of the Picture, and F is the Vaniſhing Point given.

Join FC, and produce it; draw CE perpendicular to DF, and equal to the Diſtance given; draw EF, and ED perpendicular to EF, cuting FC, produced, at D; and through D, draw AB perpendicular to DF.

AB is the Vaniſhing Line required; which is manifeſt, from the foregoing.

PROBLEM IV.

The Vaniſhing Line of a Plane being given, and the Vaniſhing Point of the Interſection, of that Plane, with another Plane, perpendicular to each other; with the Center and Diſtance of the Picture; to find the Vaniſhing Line of the other Plane.

Fig. 147. Every thing remaining as in the laſt Figure; let AB be the Vaniſhing Line given, and A the given Vaniſhing Point.

Find the Vaniſhing Point, F, of Lines perpendicular to any Plane, whoſe Vaniſhing Line is AB (by Prob. 2.) and draw AF the Vaniſhing Line required.

Dem.

Dem. For, ſince the Plane, whoſe Vaniſhing Line is required, is perpendicular to that Plane whoſe Vaniſhing Line is AB, F will be the Vaniſhing Point of Lines perpendicular to the other Plane; and A is the Vaniſhing Point of ſome Line in it, by Suppoſition; conſequently, AF is the Vaniſhing Line ſought. — — — — Th. 10.

N. B. If the Vaniſhing Line given was of a Plane perpendicular to the Picture, ſeeing it would paſs through the Center of the Picture. (Theo. 4.) wherefore, Lines perpendicular to it, would have no Vaniſhing Point; for EG, perpendicular to EC, being parallel to DF (a Line in the Picture) can never cut the Picture and produce a Vaniſhing Point; conſequently, AF, the Vaniſhing Line (in that Caſe) would be perpendicular to AB.

PROBLEM V.

The Vaniſhing Line of a Plane being given, and the Vaniſhing Point of the common Interſection of another Plane with that Plane, whoſe Inclination to the former is known; to determine the Vaniſhing Line of the other Plane; the Center and Diſtance of the Picture being given.

Let AF be the given Vaniſhing Line; and F the given Vaniſhing Point. Fig. 148.

Find AB, the Vaniſhing Line of a Plane, to which, Lines whoſe Vaniſhing Point is F are perpendicular (by the third.)

Make DG equal to DE (the Diſtance of AB) and draw AG; alſo draw GB, making the Angle AGB equal to the inclination of the two Planes, cuting AB at B, and draw BF, the Vaniſhing Line required.

Dem. If the Triangle DEF be turned up, as before, and the Plane AGB be alſo turned over on AB, DG will coincide with DE. Then, if the Plane AHF be turned over, on AF, AH will coincide with AG, and AHF will be parallel to the Original Plane, whoſe Vaniſhing Line is AF (Def. 8.) ſeeing it paſſes through the Eye, E, and the Vaniſhing Line.

But, AGB is perpendicular to AHF (9. 7. El.) for EF is perpendicular to AGB; conſequently the Angle of inclination of any other Plane, with that Plane, may be determined in the Plane AGB†.

But, AGB is the Angle given, of the Inclination of the Planes; wherefore, if BIF be turned over on BF, till BI coincides with BG; then BIF, paſſing through EF, is alſo perpendicular to AGB; conſequently, it inclines to AHF in the Angle AGB.

But, Planes which produce the Vaniſhing Lines of two Original Planes are inclined to each other as the Originals; and, their common Interſection paſſes through the Eye, parallel to the common Interſection of the Original Planes. - - - - - Theo. 6.

Wherefore, EF is parallel to the common Interſection of the Original Planes; conſequently, F is its Vaniſhing Point, (Cor. 2.) for, it is the Interſection of the Vaniſhing Lines; and BF, the Line in which BIF cuts the Picture, is the Vaniſhing Line ſought; (Def. 8.) for, BIF is parallel to the Original Plane, and paſſes through the Eye, at E. Q. E. D.

† See Art. 4. Page 42.

N. B. If AGB was a Right Angle; then, AF, AB, and BF, would be the Vaniſhing Lines of a ſolid Right Angle; each Plane, of which, being inclined to the Picture, reſpectively, as the Planes AGB, AHF, and BIF; all which paſs through the Eye, at E.

The Center of each Vaniſhing Line is where a Perpendicular from C cuts it; as D, K, and L, (Theo. 7.) and DG, KH, and IL are their Diſtances reſpectively; G, H and I, are conſidered as the Eye for each Vaniſhing Line, in the application of them to practice; each Diſtance being the Hypothenuſe of a right angled Triangle on CE; with CD, CK, and CL, reſpectively. (Def. 20.)

Note. When the given Vaniſhing Line paſſes through the Center of the Picture, it is the Vaniſhing Line of a Plane perpendicular to the Picture, which alſo paſſes through F, as DF; in which Caſe, A and D coincide; and, the Angle is made with DG, as DGA or DGB; according to which ſide of that Plane the other Plane inclines.

PROBLEM VI.

The Interſection of any Plane, which is perpendicular to the Picture, being given, and the Interſecting Point of the common Interſection of that Plane with another Plane, whoſe Inclination to it is known, and, the Angle which their common Interſection makes with the Picture; to determine its Interſection and Inclination to the Picture.

Turn over the Plane Y out of the way, it being of no uſe in this Problem.

Let AB be the Interſection given, and A the Interſecting Point. Fig. 149.

Make

Plate XXXVII.

Make BAE equal the Angle which the common Interſection of the two Planes makes with the Picture; and, at any point (E) in AE, make a right Angle, AEB.

Make BED equal to the angle of Inclination of the two Planes, and draw BD perpendicular to BE, cuting ED.

Draw BG perpendicular to AB; make BG equal to BD, and draw AG.

AG is the Interſection of the inclined Plane, with the Picture.

Turn up the Picture (X) on AB, perpendicular, and the Triangle BDE, on BE, alſo perpendicular; then, BD will coincide with BG; alſo, turn over the Triangle AIE; they will form a ſolid Angle, at the Point E.

Dem. BAE is the given Angle, which the common Interſection, AE, makes with the Picture; and, the inclination of the Plane AIE to AEB (which is perpendicular to the Picture, cuting it in AB) is BED; conſequently, AG is its Interſection with the Picture.

For, if the Plane AEI be ſuppoſed continued; beyond the Picture, it will cut the perpendicular Plane in a continuation of EA, making equal Angles on the other Side; wherefore, A is the Interſecting Point of their common Interſection, and the Interſection AG remains the ſame.

Secondly. From any Point (E) in AE, draw EC perpendicular to AB, and CH perpendicular to AG, cuting it in H; make CF equal CH and draw EF. CFE is the Angle of inclination of the inclined Plane to the Picture.

Dem. For, if the Picture be turned up, and the Plane AIE, meeting it in AG; then EH is the Hypothenuſe of a right angled Triangle CHE, congruous with CFE; conſequently, thoſe Angles are equal.

But, CH and HE are both perpendicular to the common Interſection of the Plane AIE with the Picture, therefore, CHE (equal CFE) is the Angle of its Inclination to the Picture.

This Problem, is general; for, the Interſection, AB, given, is not, neceſſarily, either horizontal or vertical; nor is the Picture neceſſarily either, all that's required is, that the Plane be perpendicular to the Picture, whoſe Interſection is given.

But if it was inclined to the Picture, that inclination being known, and which Side it inclined on, the reſt is determinable; whereas, the given Interſection being of a Plane perpendicular to the Picture, it cannot poſſibly be miſunderſtood and applied.

In Prob. 5. Sect 3. is ſhewn how to find the Vaniſhing Line of a Plane which is inclined to the Horizon, and to the Picture, when its Interſection with the Horizon is alſo inclined to the Picture, as in the laſt Problem. The horizontal Plane, being conſidered, ſimply, as a Plane perpendicular to the Picture, the Problem becomes general, and univerſally applicable; however that Plane or the Picture be ſituated in reſpect of the Horizon.

I ſhall here, as it is there propoſed, give a brief Demonſtration of that Problem, which will now, I preſume, be better underſtood, the foregoing being previouſly neceſſary; and the aſſiſtance of moveable Planes will render it far more intelligible.

Let the Picture be turned up perpendicular, and the Plane Y perpendicular to it; alſo, turn over the Plane V, till *FB* coincides with *EB*; and W being turned over, *A*E will coincide with *AE*, &c.

The Planes being thus conſtructed, let the former be alſo placed as was directed.

Then, if the Original Plane, Z, be horizontal, Y, i. e. *AEB* being parallel to it, is the Horizontal Plane; or, Z being conſidered, ſimply, as a Plane perpendicular to the Picture, Y is its Vaniſhing Plane; for *AB* the Vaniſhing Line, produced by it, paſſes through C, the Center of the Picture (Th. 4)

The Plane W paſſes through the Eye, parallel to AGE, and therefore produces its Vaniſhing Line, *AG*; and, *EB* being parallel to EB, alſo, the Angle *BEG* being equal to BEG, *EG* is parallel to EG; and conſequently, *G* is the Vaniſhing Point of EG (Def. 22.) alſo, *EA* being parallel te EA, *A* is the Vaniſhing Point of EA. Wherefore, ſince *A* and *G* are the Vaniſhing Points of two Lines in the Plane, EA, and EG, conſequently, *AG* is the Vaniſhing Line of the Plane thoſe Lines are in. (Theo. 10.)

Alſo, becauſe *E* is the Eye, and *EA*, *EG*, are parallel to EA and EG, reſpectively; therefore, the Plane *AEG* is parallel to AEG (7. 7. El.) and it paſſes through the Eye, at *E*; therefore, *AG* is the Vaniſhing Line of the Plane. (Def. 8.)

Now, if the Plane V, be turned on *BG* into the Plane of the Picture, on either Side, *BF* being equal to *BE*, and the Angle *BFG* equal BEG; conſequently, the Point *G* is the ſame, however the Plane V be ſituated to the Picture, in its revolution on *BG*; therefore, the Vaniſhing Point *G*, is truly aſcertained; and conſequently, the Vaniſhing Line *AG*; being parallel to the Interſection AG of the Original Plane, alſo *BG* to BG. (Theo. 2.)

For, the Original Planes, being ſituated any where on the other ſide of the Picture, being parallel, reſpectively, to their preſent Situation, their Vaniſhing Lines are the ſame. (Theo. 3.)

PROBLEM VII.

The Inclination of a Line to the Picture being given, and the Angle of inclination of any Plane that Line is in, to the Picture; to determine the inclination of the Line, to the Interſection of the Plane it is in, with the Picture.

Let AB be the given Line (in the Plane AEB) inclined to the Picture in a given Angle. Fig. 150.

Make ABD equal to the Angle known, and, from any Point in AB, draw AC perpendicular to BD; make the Angle CAD equal to the Complement of the inclination of the Plane to the Picture, and ADC, is the real Angle.

On AB deſcribe a Semicircle; and, with the Radius AD, deſcribe the Ark DE, cuting the other at E; i. e. make AE equal AD; draw EB, and ABE is the Angle required.

Draw AE and produce it; make EF equal to CD, and draw BF.

Dem. Turn up the Triangle ADC, on AC, and ACB, on AB, till AD coincides with AE; then, turn up the Plane X, till BC coincides with BF.

Now, X being conſidered as the Plane of the Picture, and ACB as a Plane paſſing through the Line AB perpendicular to the Picture, ABC is the angle of its inclination to the Picture, given. But, ADC is the inclination of the Plane it is in, to the Picture, and EB is the Interſection of that Plane with the Picture; conſequently, ABE is the inclination of the given Line, AB, to the Interſection, EB.

From theſe Problems it muſt be obvious, that the poſition of the Picture, to the Horizon, is of no conſequence in the Theory of Perſpective; but is very much ſo in common Practice; becauſe, all perpendicular Lines, in Objects, are parallel to the Picture, being vertical; and horizontal Lines are eaſily determined, whether perpendicular or inclined, their inclination being known. To an inclined Picture, their inclination is the Angle they make with their Seats, on the Picture.

N. B. The Vaniſhing Line or Interſection of ſome Plane in the Object, muſt be given, to determine others, if neceſſary; for which end, the Center and Diſtance of the Picture are abſolutely neceſſary. The Vaniſhing Line of horizontal Planes, being perpendicular to the Picture, is therefore firſt determined, and the Interſection of the Ground Plane; which of all other are fitteſt. Without them, there would be great difficulty in proceeding. For want of that conſideration, the work of Dr. Brook Taylor is almoſt uſeleſs to a Practitioner; he not giving, properly, one ſpecimen, how to find the repreſentation of a Line, from its known length, ſituation, and place, in reſpect of the Picture; but only, by means of the Interſecting Points of other Lines, and drawing Viſual Rays, from the Eye to the Original Points, in their true places, which is not practical in many caſes; or, to determine the Figure from ſome Line given in it, on the Picture; ſo that, the Student knows not how or where to begin the proceſs, from the geometrical proportions of the Object, and its known or determined place and poſition to the Picture, and to the Horizon.

EXAMPLE I.

To find the repreſentation of a right angled Parallelopiped, whoſe Sides are known, ſituated at ſome diſtance from the Picture; whoſe Faces are all inclined to the Picture, the inclination of one being determined, and the inclination of one Side in that Face, to the Picture. The Center and Diſtance of the Picture, with the Seat, and diſtance of the neareſt Angle of the Object, to the Picture, being given.

Let C be the Center of the Picture, and S the given Seat of the hither Angle. Plate XXXVIII. Fig. 151.

Find a, the repreſentation of that Angle. (Prob. 6. Sect. 4.) Through C, draw DF, at pleaſure, and CE, perpendicular to DF; make CE equal to the Diſtance of the Picture, and the angle CED equal to the Complement of the given Angle of inclination of a Face to the Picture.

Through D, draw AB perpendicular to DF (the Vaniſhing Line of that Face, Prob. 1.) and, perpendicular to DE draw EF; F is the Vaniſhing Point of Lines perpendicular to that Plane. (Prob. 2.)

Find the repreſentation of that Face whoſe Vaniſhing Line is AB, D is its Center, and DE its Diſtance. (Prob. 21.) as follows.

Produce FD; make DG equal to DE, and draw GH parallel to AB.

Plate XXXVIII.

Make the Angle HGA equal to the inclination of the given Line to the Interfection of that Face, whofe Vaniſhing Line is AB (Prob. 7.) and, make AGB a right Angle; A and B are the Vaniſhing Points of its Sides; and, F being the Vaniſhing Point of Lines perpendicular to that Face, AF, and BF are the Vaniſhing Lines of the other Faces; as it is manifeſt by Problem 4.

Draw aA, aB, and aF, the indefinite Repreſentations of three Sides, forming the hither Angle; how to proportion them, I ſhall ſhew, as follows.

The meaſures of thoſe Sides are known, and the diſtance of the Angle a, from the Picture; which was found by its given Seat, S, and its Diſtance S*a*.

Now, A, B, and F, are the Vaniſhing Points of the Sides of the Object, any one of which may be in the ſame Plane with SC (Ax. 6.) wherefore, AC*S* is the Vaniſhing Line of ſuch a Plane. (Theo. 10.)

Make ab to repreſent a Line in proportion to that which Sa repreſents, as the Original of ab is to the diſtance of the Angle a (Prob. 10.) thus.

Make C*E* equal to CE, and perpendicular to AC; draw A*E*, and produce it; make *E*M, to *E*N, in the ratio of the Side, to the diſtance of the Angle a; join MN, draw *E*J parallel to MN, and draw SJ, cuting aA at b; ſo ſhall ab repreſent a Side of the Object, whoſe length and inclination to the Picture was given.

Draw bB and bF; and, by means of the Radials AG and GB, make ad, or bc, to repreſent a Line, in proportion to the Original of ab, as one Side of that Face is to the other (Prob. 10.) *viz.* as GI to GK; and draw Ad (through c) and dF.

The Face abcd, whoſe Inclination was given, being compleated; find e or g, ſo, that de, or bg, ſhall repreſent the proportion of the other given Side, by means of ab, or ad, as before; the Center and Diſtance of either Vaniſhing Line being determined, (by Prob. 1.) as *S*, the Center of BF, by a Perpendicular from C (Th. 4.) and *S*O the Diſtance, in A*S* produced; making BO equal BG, and joining OF, BO and OF are the Radials of the Sides in that Face, forming a right Angle BOF, at the Eye.

Make OP to OQ in the ratio of the Originals of ad to de; draw PQ and OR parallel to PQ. Draw aR, which will cut dF at e; and, through e, draw Bf, cuting aF at f; and laſtly, draw fA, cuting bF at g, which compleats the Parallelopiped, abdf, required.

For, becauſe of the Vaniſhing Points A, B, and F, the Sides ab, ad, and af, form a ſolid Right Angle, at a; AGB, ATF, and BOF being Right Angles, which bad, baf, and daf repreſent, reſpectively; and the other Sides vaniſh in thoſe Points, reſpectively; as dc and de, &c. which repreſent Parallels to ab, af, &c. (Cor. to Th. 3.) Therefore bdf repreſents a right angled Parallelopiped, whoſe proportion was known, and poſition to the Picture determined.

ac, ae, and ag, repreſent Diagonals, in each Face, reſpectively, which are in proportion to the Sides, as IK, PQ and TV, reſpectively, to GI and GK, OP and OQ, TX and TY, reſpectively.

The Parallelopiped, bdf, is truly determined, according to its poſition given, in reſpect of the Picture, its place in reſpect of the Eye, and its proportion in reſpect of its Diſtance; no regard being had to its poſition reſpecting the Horizon. Wherefore it is obvious, that, to determine its Poſition, in that reſpect, the horizontal Vaniſhing Line is eſſentially neceſſary, and the Vaniſhing Lines of its Faces are determined in reſpect of their poſition to the Horizon, as well as to the Picture, as ſhall be exemplified in the next.

EXAMPLE II.

To repreſent an Octaedron, perſpectively, ſituated on a Plane inclined to the Horizon, and to the Picture; the angle of Inclination to the Horizon, being given, and the Angle which its Interſection with horizontal Planes, makes with the Picture; together with the Seat of the Object on the inclined Plane; and, its ſituation in reſpect of the Picture.*

Fig. 152. No. 2.

Let ABC (No. 2.) be the Seat of that Face on which the Object reſts, on the Plane Z, which is inclined to the Horizon in the Angle X; DF is its Interſection with a horizontal Plane, and DFG the Angle that Interſection makes with the Picture; FG is the Interſection of the Picture with the horizontal Plane. The Station Point is S, the Diſtance is SG, and SE the Height of the Eye.

* An Octaedron is a regular Solid, one of the five Platonic Bodies, having eight Faces, which are equal, equilateral Triangles; about which Solid, if a Sphere was circumſcribed, every Angle of the Solid would be in the Surface of the Sphere.

No. 1.

Its geometrical Conſtruction is neceſſary to be underſtood, before it be poſſible to deſcribe it perſpectively. Its geometrical Plan, on the Plane on which it reſts, is a regular Hexagon, AEBFCD (No. 1.) ABC is the Face, on which it reſts. DEF is the upper Face, EAD, EBF, and FCD are inclined Faces, above; and AEB, ADC, and BFC, below, out of ſight; IGHC is its geometrical Elevation, or Section through EC, ſhewing the Inclination of its Faces, to each other; viz. the Angle GIK equal IGH.

These preliminaries being determined, let AB be the Interſection of the horizontal Plane, or Ground Line; and, at the height of the Eye (SE, No. 2.) draw the Horizontal Line, DF, parallel to AB; let C be the Center of the Picture.

Draw CE perpendicular, equal to the Diſtance of the Picture; and find the Vaniſhing Point, D, of the inclination given. (Prob. 2. Sect. 3.) Make DEF a right Angle, and through F, draw FG perpendicular to DF. FG is the Vaniſhing Line of a Plane, to which, Lines vaniſhing in D are perpendicular. (Prob. 3.) Fig. 142.

Make F*E* equal EF, and the Angle F*EG* equal X, (No. 2.) and draw DG, the Vaniſhing Line of the inclined Plane, (as by Prob. 5. Sect. 3.) indefinite.

Produce EC, cuting AB at S; make SB equal GF (No. 2.) draw BD, the indefinite Repreſentation of DF, (No. 2.) and find the Points *a* and *b*, repreſenting a and b, in which, BA and CA (No. 2.) cut DF. (Prob. 8.)

Find the Center (*C*) and Diſtance of the Vaniſhing Line DK (Prob. 1.) make CE^2 (in CC produced) equal to its Diſtance, and draw DE^2.

Make the Angle DEH equal BaD (No. 2.) H is the Vaniſhing Point of AB (No. 2.)

Make the Angles HE^2I and IE^2K each of 60 Degrees; H, I, and K are the Vaniſhing Points, of the Sides of the two Faces ABC and DEF (No. 1.) i e. of AB, AC, and BC, (No. 2.) and conſequently of the Sides of the oppoſite Face.

Find the repreſentation of that Face, a b c (Prob. 18) having obtained the Points, *a*, *b*, and *c* (repreſenting a, b, and D, No. 2.) by means of the Interſection AB; making DE^1, equal DE; or, by the Interſection BJ, of the Face, a b c.

Find the Vaniſhing Line HM, of the contiguous Face, whoſe common Interſection, with that found is a b, and its Vaniſhing Point H. (by Prob. 5.) Its Inclination is the acute Angle GIK (No. 1.) determined by a perpendicular from E, and IG being made equal IC; which Angle is 70 Degrees.

Find the Center (O) and Diſtance (OE^4) of the Vaniſhing Line HM, (Prob. 1.)

Draw E^4H, and find the Vaniſhing Points P and Q; as I and K, above; or make equilateral Triangles, X and Y, at the Eye (E^4) and produce their Sides to the Vaniſhing Line HM, cuting it at P and Q.

Through a and b, draw Qa, and Pb, cuting at d; giving a b d for that Face.

The Point d is in the upper Face, and the Sides of oppoſite Faces are parallel, reſpectively, two and two; therefore, draw dI, and dK; and Pc, cuting dK at e; draw eH and join ae, which compleats the Figure.

N.B. This Figure having eight Faces, two and two of which are parallel, there are conſequently (if none are parallel to the Picture) four Vaniſhing Lines; but here are only two required, DL, and HM; a third will paſs through I and P, and the fourth through K and Q, which, being produced, would meet in the Vaniſhing Point of ae.

It may be obſerved, that the three Sides of each Face, vaniſh in its reſpective Vaniſhing Line, (Theo. 10.) and the Vaniſhing Point of the common Interſection, of any two Faces, is the interſection of the Vaniſhing Lines of thoſe Faces, (Cor. 2. Theo. 6.) as H, of a b and e f, the interſection of the Vaniſhing Lines DL and HM; and K, of DK and QK, the Vaniſhing Point of d e, and b c.

The next Figure exhibits two of the ſame Objects, on a level Plane (or any Plane perpendicular to the Picture) ſituate alike to the Picture, but on different ſides of the Station Line; the Sides, in both, have conſequently the ſame Vaniſhing Points, and, their Faces have the ſame Vaniſhing Lines, reſpectively. Fig. 143.

AB is the Vaniſhing Line of the Faces which are perpendicular to the Picture, and C its Center; a g is the Interſection of one of thoſe Faces. The Angle a, of one Object, touches the Picture, the other is at ſome diſtance beyond it.

FG being the Vaniſhing Line of a Plane perpendicular to the common Interſection a b, and F*E* (equal EF) its Diſtance; the Angle F*E*G is made equal to that of a diagonal Plane with the Faces, (equal HIC, No. 1.) *viz.* 55 Degrees. AG is the Vaniſhing Line of the diagonal Plane a b c d, which is a Square.

The Vaniſhing Points, A, H, and B, of the Sides a b, a f, and b f, or e d, being found (as above) and *E*K drawn, perpendicular to *E*G, cuting GF, produced, at K, the whole is determined. BK and GH interſect at I, the Vaniſhing Point of e b and d f; and BG, KH, and IA, being produced, would meet in the Vaniſhing Point of a e and f c.

Plate XXXIX.

AB, AI, BG, and GI, are the Vaniſhing Lines of the Faces; and AG, BI, and KL of the three Diagonal Planes. The reſt the Figure deſcribes.

The Vaniſhing Lines of the Faces produce, by their Interſections, the Vaniſhing Points of every Side of the Figure, *viz.* ſix, A, B, G, H, I, and, if BG and IA were produced, they will meet in the Vaniſhing Point of a e and f c.

These Examples are ſufficient, for finding the Vaniſhing Lines, and applying them to uſe, in projecting Figures in inclined Planes, in ſuch Objects. I do not intend to lead the Reader through all the Mazes, attending the projection of the Dodecaedron and Icoſaedron, becauſe I know no end it can anſwer to any Perſon; ſuch Objects never coming in the courſe of their ſeveral Studies. But, if any Perſon be inclined to amuſe himſelf with them, he may find Rules in this Section and the 5th. for his purpoſe, in every poſition he can deviſe. As they require a geometrical conſtruction, previous to the projecting them perſpectively, I recommend the Reader, who has curioſity, to the Work of Mr. Hamilton; or, to the ingenious Mr. Highmore, they being foreign to my Plan.

EXAMPLE III.

How to repreſent a piece of regular Fortification, in Perſpective.

The foregoing Examples are calculated more for Leſſons than real uſe; in this, the knowledge and application of inclined Planes will be found neceſſary.

In Fortification or military Architecture, as the Walls of the Poligons and other Outworks are all inclined to the Horizon, and moſtly to each other, it ſeems to be the fiteſt Subject of any I know. Indeed, the vertical Planes of other Buildings are, frequently, as much inclined to the Picture, but, their poſition to the Horizon familiarizes them, on a vertical Picture; and yet, in reality, there is no difference, in Theory, and very little in Practice.

Fig. 154. The figure, and poſition of the Poligon (No. 1.) to the Picture being determined, find the perſpective Plan of its Seat on the Ground Plane (Sect. 5.) Then, EC being the Ground Line, and A the interſecting Point of one Line in the Face X, find the Interſection (AD) of that Plane (Prob. 6.) (V is a ſection of the Wall, ſhewing its inclination to the Horizon) alſo, F being the Vaniſhing Point of a b, draw FG parallel to AD; FG is the Vaniſhing Line of that Face (Th. 2.) S is its Center, *E*S its Diſtance, and *E*SC the inclination of the Plane X, to the Picture.

Now, a b is determined, and if the inclination of the Sides a c and b d, to the horizontal Lines, a b and c d, which are parallel, be known, their Vaniſhing Points are determinable (Prob. 4. Sect. 3.)

Produce SC, and make S*E*' equal S*E*. Draw F*E*', and make the Angles F*E*'G and F*E*'I, equal reſpectively, to the known Angles, producing the Vaniſhing Point G, of a c; and, if *E*'I be produced till it meets FG, it will give the Vaniſhing Point of b d; which does not fall within the compaſs of the Picture, yet, by means of *E*'I, b d may be drawn, by Prob. 13.

Then, J being the diſtance of the Vaniſhing Point G, draw J a to the Interſection AD, cuting it at *a*; make *a b* equal to the length of the Side a e, and draw *b* J, which determines the Angle e; and e F gives the Angle d, b d being drawn indefinite. Or, if the Interſecting Points, D and *b* be obtained, by making AK equal to the perpendicular height of a e, (as at V) and drawing KD parallel to AB, the reſt is unneceſſary.

By the ſame means, any other Face (as Y) may be obtained, and continued around. If the inclination of the Faces, X and Y, to each other be known, the Vaniſhing Line GH is determined, by Problem 5; or, H, the Vaniſhing Point of a h, being found, draw G H; by which, the Face Y is deſcribed; as X, by means of the Vaniſhing Line F G.

The method uſed by the old Authors, for delineating ſuch an Object, is, by forming a perſpective Plan of the whole on the Ground, and drawing perpendiculars from the Seats of the interior Angles, as c d, &c.

Of

Of the INCLINED PICTURE.

I preſume, the Reader will, ere now, be fully convinced of the univerſality of the Principles on which the Theory of Perſpective is founded; and, that it is the ſame thing, whether the original Plane or the Picture be inclined, or both, in reſpect of the Horizon; ſeeing, it is the poſition of the original Planes and Lines to the Picture, only, that is conſidered, in projecting them.

In Example 1ſt. if the Parallelopiped, being right angled, had any of its Faces parallel to the Picture; or, if they were parallel to the Horizon, and the Picture vertical; or, which is the ſame thing, if any of its Faces are perpendicular to the Picture; then, the Sides which are perpendicular to them, are parallel to the Picture, and conſequently, have parallel repreſentations (Theo. 9.) But, the Picture being inclined to the Horizon, whilſt ſome Face of the Object is parallel to it, the Sides may then be all inclined to the Picture; and, the perpendicular Lines, when the Picture is vertical, are parallel to it, and have no Vaniſhing Point; but, when the Picture is inclined to the Horizon, they are, conſequently, inclined to the Picture, and vaniſh either upwards or downwards, according as the Picture is inclined, towards the top or bottom of the Object; of which I ſhall give ſome Examples.

EXAMPLE IV.

How to repreſent, an upright Object, on an inclined Picture.

Let AB be an object perpendicular to the Horizon, and let DF be a Section of the Picture, inclined towards the top of the Object; E is the Eye of a Spectator, EC, perpendicular to the Picture, produces its Center and meaſures its Diſtance; ED, parallel to the Horizon, determines the Horizontal Line, and EF, being perpendicular, that is parallel to AB, determines the Vaniſhing Point (F) of Lines perpendicular to the Horizon (Prob. 2.) Fig. 155. No. 1.

Let the Picture be prepared accordingly, drawing the Horizontal Line as uſual, and the Vertical Line, at right Angles, cuting it at D. No. 2.

Take DC and CF in proportion to CE the Diſtance taken (as in No. 1.)

In this Caſe, it is obvious, that the Center of the Picture, is below the Horizontal Line; as, if the Picture inclined towards B, it would be above it, at S (No. 1.) but, ſeeing that the Vertical Plane is perpendicular to the Picture, its Center is always in the Vertical Line (DF.)

At No. 3, is ſhewn the poſition which the Object has to the Picture (ab) laterally; which Inclination being known, find the Vaniſhing Points, H, and I, of horizontal Lines in the Object: (Prob. 2. and 4. Sect. 3.) D*E* (equal DE) is the Diſtance of the Vaniſhing Line; proceed, in every reſpect, as in Example, 1ſt.

F, being the Vaniſhing Point of Lines perpendicular to the Horizon, and, H and I, the Vaniſhing Points, of horizontal Lines in the Object, at right angles with each other; the Object being right angled; conſequently, HF and IF, are the Vaniſhing Lines of the upright Planes in the Object (Prob. 4.)

The Seat (s) of any Angle (the neareſt to the Picture is the moſt convenient) being determined; or, the interſecting Point (A) of *AB* (No. 3.) draw AB, the Interſection of any Plane that Line is in†. Find *a*, the repreſentation of the Angle *A*, its diſtance from the interſecting Point (a*A*) being known. (Pr. 6, or 7. Sect. 4.) or by its diſtance, sd, (equal *AS*) from its Seat, i. e. from the Picture. † Theo. 2.

By means of the Interſection, AB, of the upper Plane of the croſs Arms, is got *a*, *b*, &c. the real meaſures being applied on AB, as uſual; or, on AL, the Interſection of the upright Plane (parallel to IF, Th. 2.) in which is the ſame line, ab; on which Interſection is alſo applied the meaſures of height (Ab, ac, &c.) and projected by means of the Point Q, the Diſtance of the Vaniſhing Point of thoſe Lines; or, the interſecting Points, K, J, and L, may be found, thus.

Plate XXXIX. Fig. 155.

A being determined (by its height above the Eye, the Seat of *A*, (No. 3.) and diſtance from its Seat, with the inclination of the Line it is in, to the Picture (dac) or to its Seat) draw Ab through A, perpendicular to HI; make Aa, Ac, and cb, in the ratio of Ab, bd, and dB; as CE, to CE, No. 1; draw aK, cJ, and bL, parallel to HI, cuting the Interſection AL, at K, J, and L; from which, draw lines to the Vaniſhing Point I, and through *a*, *b*, &c. to F, cuting them at *f*, *d*, *h*, &c. From *f*, and through *b*, *d*, &c. draw to H.

Find the Center and Diſtance of the Vaniſhing Line HF, and the place of the Eye at O (Prob. 1.) make OM, and ON, in the ratio *Ad* to *Ae* (No. 3.) draw OP parallel to NM; and, through *d*, draw P*e*, and compleat the End *ek*, by means of the Vaniſhing Point R (in IF) of a diagonal of a Square.

The reſt is obvious, from inſpection of the Figure.

No. 4. is the ſame Object on a vertical Picture, whoſe Diſtance is ED. (No. 1.)

Here, C is the Center of the Picture, anſwering to D (No. 1.) AB is the Interſection of the upper Plane of the Croſs; and, AD and BE of the upright Faces.

The Interſecting Point of every horizontal Line is geometrically determined; the Interſections of the Planes cuting each other in the Interſecting Points A, B, D, and E, and the reſt are ſet off geometrically from them; or the Lines are determined, and proportioned, otherwiſe, as uſual.

No. 5. exhibits the ſame Object, when the Picture is inclined towards its bottom, (as GH, No. 1.) its Center is at S, above the Object, as in No. 2, it is below.

SE is the Diſtance of the Picture; and, if the Angle SED be made equal to SED (No. 1.) the Horizontal Line D*I* is determined; alſo, EF, perpendicular to DE, cuts the Vertical Line, DF, at F, the Vaniſhing Point of perpendicular Lines, in the Object.

In this Example, the Ground Line (GH) may be uſed, conveniently; whoſe diſtance from the Horizontal, is DB, not EK (No. 1.) i. e. it is in proportion to the Diſtance of the Picture, ES.

AB is the Interſection of the Plane of the croſs Arms, in which the Interſecting Points, A and B, &c. may be determined, as in No. 2. and, BF is the Interſection of a vertical Plane of the Object. The reſt it is needleſs to particularize; the Vaniſhing Points of horizontal Lines being determined as uſual; obſerving, that the Diſtance of the Horizontal Line is not the Diſtance of the Picture, as in a vertical Picture; ſeeing that, horizontal Planes are not perpendicular to the Picture, the Picture being inclined to the Horizon. Wherefore ED, in both Pictures, is the Diſtance of the horizontal Vaniſhing Line; being conſidered, in every reſpect, as an inclined Plane, in reſpect of a vertical Picture.

In theſe Examples may be ſeen the univerſality, and the ſuperiority of Brook Taylor's Principles to all other; by which is ſhewn (in this laſt Section) that any Plane Object, whatever, may be projected, from the known poſition of one Plane to another, in the Object, and the proportion of the Lines in thoſe Planes, together with their poſition to each other; without regarding their poſition to any other Plane, whatever, except the Picture. Whereas, by the Old Authors, it was almoſt impoſſible to project them at all; or, with the utmoſt difficulty, by the Seat of each Angle on the Ground Plane, and its height above it, a troubleſome and laborious proceſs; by which was obtained the ſeveral Angles, and then joined by Right Lines; without Vaniſhing Points, of which they had not the leaſt conception, in any other Lines but horizontal; and thoſe they called accidental Points, for they had no certain method of producing them. Having found the two extremes of an inclined Line, then, drawing the Line, and producing it to the Horizon, they found its Vaniſhing Point (properly called accidental) which is now fixed with abſolute certainty, in all Poſitions of Lines to the Picture, and directs the certain poſition and place of each Line on the Picture, indefinite. And, by means of Viſual Rays, drawn on the Picture, certain portions are cut off (truly mathematical) which repreſent the Originals, as they appear to the Eye, in the true Point of View.

BOOK IV.

Of SHADOWS, in general; of LIGHT and SHADE, REFLECTION, KEEPING, &c.

SECTION I.

An introductory PREFACE, concerning LIGHT and SHADE.

IN the foregoing Work is contained the full knowledge and practice of linear Perſpective; the laſt Book containing Rules, neceſſary for the projection of any regular Object, almoſt, whatever; with Examples, varying the application of the Rules, frequently, as occaſion requires. The linear part being projected, ſtrictly by the Rules there preſcribed, will convey a juſt Idea of the figure of the Object, and the proportion of its parts to each other (the Eye being in the true Point of View.) Nevertheleſs, there requires ſomewhat more to be done, in order to give an appearance of ſolidity, and receding of one part behind another; which indeed is effected, in ſome meaſure, by their perſpective proportions; and perfected, by a juſt gradation of Light and Shade, properly diſtributed to each part of the Object; and which, with the projection of Shadows, &c. is the Subject of this fourth Book.

It muſt not be expected that I ſhould define what Light is, having already given my Opinion on that Subject (Sect. 1. Book 1.) yet, it may be proper to give ſome general Idea of what is underſtood and meant by Light, as it is uſed by Artiſts in general, in the application of it to a Picture.

LIGHT, in that reſpect, means nothing more than the bright parts of Objects; which differs greatly in degree, according as their Surfaces are ſituated in reſpect of ſome luminous Body; which is effected either, directly, from the Luminary, or reflected, from ſome illumined Object.

SHADE is a privation of Light, the dark parts of Objects; occaſioned either by the Object itſelf, on thoſe parts which are not towards the luminous Body, or, by ſome other opake Subſtance, interpoſed between the luminous Body and the Object; depriving it, either wholly or partly, of Light.

To give or preſcribe Rules, abſolutely, for perfecting a Picture, in reſpect of Light and Shade, is as impoſſible as in reſpect of Colour; yet, by adhering to Reaſon, and carefully obſerving Nature, we may arrive at a tolerable degree of perfection. In the firſt place, it is neceſſary to conſider how the Object is ſuppoſed to be ſituated to the luminous Body, from which it is to receive the Effect.

It has been almoſt a general Rule, amongſt Artiſts, to ſuppoſe Light to flow from the left hand to the right; but that is entirely arbitrary, and can have no foundation in the nature of things, but merely an habitual cuſtom; it is, however, proper to imagine it to flow from one hand or the other; for, to ſuppoſe the Luminary, or luminous Body, directly oppoſed, either on this or on the other ſide of the Object, can never produce a pleaſing effect; ſeeing that, in the firſt caſe, it is almoſt wholly illumined, in the other, it is wholly deprived of Light; an agreeable mixture of both, judiciouſly diſpoſed, is what contraſts one Object, or part of an Object from the other, and renders the whole agreeable to the Eye, as a pleaſing imitation of Nature.

Plate XL. There is, moreover, an inconveniency arising from the cuſtom of ſhading Pictures, on the left hand, generally, ſeeing they cannot ſuit both ſides of a Room; for, being placed, in that reſpect, properly, adds greatly to the effect of the Piece.

Let it be carefully obſerved, that, that part of an Object, to which any luminous Body is moſt directly oppoſed, will be the brighteſt; but, when, from the ſituation of the Eye, it is much contracted, or foreſhorted, it may appear as dark, or perhaps darker than other parts, on which, the Light is not ſo direct.

Fig. 1. For Example. Let Z be the Plan or Baſe of ſome priſmatic Object, and AB, &c. Rays of Light, falling thereon. Becauſe thoſe Rays fall more perpendicularly on the Face BC, than on BD, that Face is, conſequently, more illumined than BD, and BD than DF, whilſt FG is wholly deprived of Light; aud GH, being farther removed from the Light, will be darker than FG, from the effect of Light ſimply, or direct. Now, if a Spectator be ſo ſituated, at E, that the Face BC, which is moſt illumined, is much contracted, it may in ſome ſituations of the Object to the Light, appear darker than BD, which is more oppoſite to the Eye; whereas, if the Station be at *E*, where both appear equally contracted, the Face BC will appear the brighteſt; but, if the Light was in the direction of *EB* they would be, and alſo appear equally bright, without diſtinction.

Any direction to the Rays of Light may be given, at diſcretion, as AB; and, by drawing others, parallel to AB, it is eaſy to know which Face ſhould be the brighteſt, and which the next, &c. but to determine in what degree, poſitively, is not poſſible; a careful obſervation of Nature is the beſt criterion by which to judge of that; and, even in that caſe, it is not eaſy to determine, without experience and ſound judgment.

After the ſame manner, the inclination of the Rays of Light to the Horizon, being taken into conſideration, may be determined, whether horizontal Faces of Objects, below the Eye, or inclined Faces, are more illumined than the vertical. As Light flows from above, the whole Hemiſphere being illumined (when the Luminary itſelf is not much elevated) the horizontal faces of Objects, or ſuch as are much inclined, are, generally, the brighteſt; provided they are oppoſite to that quarter from which the principal Light flows. Notwithſtanding it is uſual to ſhade the Roofs of Buildings, Pediments, &c. more than the vertical Planes, it can only be from a ſuppoſition of their being of darker materials, as Lead, Slate, &c. for, if they were compoſed of the ſame, that is, if they are of one uniform Colour (and otherwiſe, no poſitive determination can be made, in reſpect of Light, from obſervation) inclined Planes, will be brighter than either vertical or horizontal, being, generally, more oppoſed to the Light.

In reſpect of curved Surfaces, from the effect of Light, ſimply or directly, being convex, thoſe parts which are moſt towards the luminous Body, are the brighteſt; and conſequently, thoſe which are fartheſt removed from the Light will (without reflection) be the darkeſt. On concave Surfaces, the effect is reverſe.

Fig. 2. On cylindrical Surfaces, as Columns, ſtreight Mouldings, &c. the Light is always parallel to the ſides, or edges of the Mouldings; the greateſt Light being on that part which is moſt perpendicularly oppoſed to the luminous Body (as AB) from which it gradates regularly, on each ſide; conſequently, that ſide which is fartheſt from the Light will be the darkeſt, ſuppoſing the Luminary to be ſituated on this ſide of the Object, on either hand. But, if it be ſuppoſed ſituate on the other ſide, the Edge, AB, towards the luminous Body, will be the brighteſt; and it will be gradated from that Edge, not to the other, as in No. 2. (which has not the appearance of an entire Column or Cylinder, but of a Segment, cut off parallel to its Axe) but, it will be darkeſt ſomewhat from the other Edge (as No. 3.) not owing to reflection from other Objects, but from the luminous Body, being on the other Side; which, as it is more or leſs direct, will occaſion the darkeſt part to be more or leſs removed from the middle.

Convex ſpherical Surfaces are brighteſt on that part to which the luminous Body is perpendicular, from which it gradates every way equally; inſenſibly varying, to the extremes, provided no other Object interfered, to reflect Light. If a Hemiſphere, or leſſer Segment, be properly ſhaded, it will repreſent equally as well a concave as a convex Surface, by ſuppoſing the Light on one ſide or on the other; that is, when the Eye is ſo ſituated as to ſee the whole circumference of its Baſe, or nearly; as Fig. 3. Fig. 3.

Reſpecting ſtreight mouldings, which are compoſed wholly of Planes and cylindrical Surfaces, their different ſituations, towards the flow of Light, occaſion variety of Shades. As for example; the Cima-recta (compoſed of a convex and a concave cylindrical Surface) in its proper poſition, in a Cornice, &c. has two ſtrong Shades and one Light, in the middle; whereas the very ſame Moulding, reverſed, as for a Baſe, has two Lights, and one Shade, in the middle, with a faint one at each Edge. The Cima-reverſa, in its proper poſition, has two Lights and one ſtrong Shade, in the middle; the ſame Moulding reverſed, has two faint Shades and one Light, in the middle.

Figure 4 is the Profile of a Cornice; ſuppoſe the Light to flow in the direction AB, at diſcretion. It is eaſy to determine, what parts of the Cornice will be light, and which ſhaded. For, drawing ſeveral Lines, touching the projectures and prominances, parallel to AB, it is obvious that the edge a, obſtructing the Light, muſt neceſſarily occaſion a ſtrong Shade, below; which gradually dies away into the Light, at b, in the middle, where the Rays fall oblique on it, and where, the prominance or ſwell of the convex part occaſions another Shade, at c; begining faint, and gradually ſtrengthening, reverſely. Fig. 4.

The ſame reaſoning accounts for the Shade in the middle of the Cima-reverſa.

The great projecture of the Corona, (X) throws all below in Shade, which gives great expreſſion and force to the whole, making the upper part appear to ſtand off, from the Canvas or Paper. The whole, below, being immerſed in Shade, would be totally loſt to ſight, was it not, in ſome degree, illumined by reflection; on which I ſhall ſpeak in its place; let it ſuffice, here, to obſerve, that the effects are almoſt wholly reverſed, and but faintly expreſſed.

Let No. 2. repreſent the upper Mouldings reverſed, for a Baſe Moulding. No: 2.

Here, it is obvious, that the Rays, falling on it in the ſame direction, illumines the whole; inſomuch that, no part can be ſaid, with propriety, to be in Shade; nevertheleſs, the parts *a*, *b*, &c. which are moſt directly oppoſed, will be brighter than the other, which are faintly ſhaded, as the Surface falls off.

I ſhall juſt make an obſervation on the prevalent Cuſtom of ſhading Mouldings, in architectural Deſigns, as ſtrongly illumined by the Sun; which entirely deſtroys their proper effect.

Suppoſe AB (No. 3.) to repreſent a Ray from the Sun. It is obvious, that all the part, from B to *b*, will be in Shade; and, being a Shadow, projected by the upper edge (BC) ſtrongly defined by a Right Line (*bc*) has not a very agreeable effect. The Shade, below, is alſo more ſudden and hard. No. 3.

Now, if by means of Light and Shade, it is intended to give an Idea of Mouldings, I would aſk, ſeriouſly, which has the moſt natural appearance and effect? Suppoſe the Profile cut off, or covered; would any Perſon conceive what this Moulding is intended to repreſent? But, where is the neceſſity to ſuppoſe the Sun to ſhine on them? as there are ſtrong Shades when it does not; nor are they intended as Pictures, but Deſigns, which ought to exhibit what is intended, in the moſt expreſſive manner poſſible.

If my opinion might be allowed to have any weight, I ſhould ſuppoſe that a Section ſhewing the inſide of a Building, geometrical, would be more expreſſive, by means of penumbral, inſtead of ſtrong edgy Shades, which are by no means natural, in ſuch Caſes. Nay, ſo fond are ſome Architects of forced Effects, that we frequently ſee a ſtrong right line of Light introduced into a Section of a Dome,

Plate XL. Fig. 5. through an opening at the Top (AB) as ſtrongly defined on the part at C, which is nearly horizontal (where it is abſolutely impoſſible for Light to come) as elſewhere, at D; where, an elliptical figure of the opening would, only, be deſcribed; the concave Cylinder, below, being ſhaded without any edge at all.

Similar to this are the forced and unnatural Effects, which may frequently be ſeen, in Prints and Paintings, of a ſtrong Light being introduced, ſtriking in a Right Line, from the upper edge of the Cornice, acroſs the Side of a Room, as if it was open at the top. Alſo, on exterior Objects, a ſtrong diagonal Shade frequently croſſes a Building, without the leaſt apparent Cauſe for ſuch an Effect.

There is (to me) another great impropriety; which, though it does not come immediately within my cognizance, I ſhall beg leave juſt to mention; which is, the introducing Landſcape, and ſome appearance of Perſpective, in a geometrical Deſign. I am well aſſured that it need but be pointed out, to convince any ſenſible Perſon of its inconſiſtency, as a Picture; exhibiting a geometrical Elevation of a Building, placed upright in a Garden, &c. where every thing elſe appears real.

When I ſee a number of Drawings, geometrical Elevations of Buildings, at an Exhibition, vying with each other, which ſhall make the beſt figure, as a Picture, more than as a Deſign (for which, the Architect is obliged to ſome Landſcape Painter, frequently) I am apt to imagine myſelf in Vauxhall Gardens, where, the luxuries of Art eclipſe the beauties of Nature; and, inſtead of a Garden, we, in reality, find ourſelves in ſome ſpacious Building; the ſeveral Walks and Avenues of which, reſembling Galleries of communication to various detached Appartments.

SECTION II.

The THEORY of SHADOWS, projected by the Sun.

IN treating on Shadows, it is neceſſary, firſt, to conſider the Situation, Diſtance, and Magnitude of the luminous Body, by which the Shadows are projected. Secondly, the Situation of the Object to the Luminary, and, of the Lines bounding Planes, &c. whoſe Shadows are to be projected; their Poſition in reſpect of ſome other Plane, on which the Shadow is to be projected; and laſtly, the Poſition of the Luminary, in reſpect of the Picture.

The Sun, which is the grand fountain of Light, being placed in the wide Expanſe, diſpenſes its Light, equally, all around, and illumines a concave Sphere, as far as its influence extends; which differs, in degree of Splendor, according to the Diſtance from the luminous Body. If an opake Body be interpoſed, that Body is illumined on its Surface towards the Sun; and conſequently, muſt deprive ſome part of the Expanſe of Light; and, if another opake Body intervene, the Shade, or Shadow of one, is thrown upon, or projected on the other.

A SHADOW, therefore, is the outline projection of one Object on another, Plane or other Superficies, in Right Lines, from ſome luminous Body to the PLANE of PROJECTION; i.e. the Plane on which the Shadow is projected.

By RAYS of LIGHT is to be underſtood Right Lines, from the Luminary, paſſing by the extremes of an Object, and projecting its Shadow.

Fig. 6. To illuſtrate what has been advanced, ſuppoſe the Body, a Globe, at S, to repreſent the Sun, and CD a portion of a concave Sphere, illumined by its Light; let E be ſuppoſed an opake Body, illumined on the Surface towards the Sun.

Now, if this opake Body be alſo a Globe, repreſenting the Earth or any other Planet; the Sun being, in proportion to it, immenſely great, the Space deprived of Light, in the Expanſe, will form a Cone; as ACB.

If a Plane, or any other opake Surface, be placed at FG, within the extent of the Cone, it will receive, thereon, the Shadow of the opake Body, at H; which Projection, if the Plane be direct to the Sun, will be a Circle; but, if the Plane be inclined, as DL, it will form an Ellipſis, at I. Theſe Projections, or Shadows, are leſs than the Object; becauſe the luminous Body is greater. But, if the luminous Body be equal to the Object, or at an infinite Diſtance; the Shadow, on the direct Plane, will be equal to the Object, in diameter, and may be conſidered as its orthographic Projection.

Hence it is evident, that, from the magnitude of the Sun in proportion to the Earth, the Rays of Light, from its extremes to the Earth, are converging; but, when its immenſe Diſtance is conſidered, they are, to all ſenſe, parallel. Then, ſeeing that the largeſt Object, on the Earth, can ſcarce be ſaid to bear any proportion to the whole, we may conſider them as being perfectly parallel.

Next, the Altitude of the Luminary is to be conſidered, ſo as to give the beſt and moſt pleaſing effect to the Picture; this is generally at the diſcretion of the Artiſt. It may, to ſome, be neceſſary to explain what is meant by its Altitude.

Let the Semicircle AFB be ſuppoſed an Arch in the Heavens, in which is the Sun's *apparent* diurnal motion; and let E be the Earth, *ſuppoſed* at reſt. Fig. 7.

If the Sun be at C, the Ark CB is the meaſure of its Altitude; and, the Angle CEB is the Angle of Elevation; EB, being conſidered as the Horizon, and CE the direction of the Rays of Light. If it be at D, then, DEB is the Angle of the Inclination of the Rays of Light to the Horizon, and the ark DB the meaſure of its Altitude; but, when its place is at F, in the middle of the Arch (90 Degrees, each way, from A and B) its Rays have, then, no Inclination to the Horizon, AB; and conſequently, a perpendicular Line, GE, will project no Shadow.

This can only happen to thoſe parts which lie between the Tropics; for although, at the Equinoxes, the Sun appears to deſcribe a Semicircle, at all parts of the Earth, yet, according to the diſtance from the Equator, the Plane of its apparent motion is more or leſs inclined to the Horizon of the place; conſequently, its Altitude, when in the Meridian of any place, is more or leſs elevated above the Horizon, from the Equator to the Poles, where it deſcribes an entire Circle, around the Horizon, having no Elevation, or Meridian.

Laſtly; the Angle of Inclination which a vertical Plane, paſſing through the center of the Luminary, makes with the Picture, is neceſſary to be known, when it is not in the Plane of the Picture.

Now, although the Rays of Light, proceeding from the Sun, are parallel amongſt themſelves, in reſpect of the Earth, yet, their inclination to its Surface varies infinitely.

Suppoſe a Globe illumined by the Sun, at S, whoſe Center only is conſidered*. Fig. 8.

* In projecting Shadows by the Sun, in order to deſcribe the true Contour, or outline of the Shadow, it is neceſſary to conſider it as a luminous Point, only, at an infinite Diſtance; for, if its magnitude be taken into conſideration (ſince every part of its Surface emits Light) the Shadow of a Point, would always be, in proportion to the Sun, as the diſtance of the ſame Point, from the Plane of Projection, is to its diſtance from the Sun. Let S be the Sun, and A, a Point whoſe Shadow is projected, on the Plane X. Fig. 10.

Now, the whole extent of the Shadow of the Point A is a Circle, whoſe Diameter is *ab*; for, ſince it is obvious, that every part of the Sun muſt emit Light; conſequently, a Ray of Light emited from a, and paſſing through the Point A, will project its Shadow to *a*; and a Ray from b will project the ſame Point to *b*, &c. while the Center (S) only, being conſidered, will project it to B, which is the center of the Shade, and its real Shadow. For (ſuppoſing the Point to have ſubſtance) every other part of the Shadow, is more languid, the farther it is from the Center, B; conſequently, at its extremes, it cannot be diſtinguiſhed from the ſurrounding Light; ſeeing that, every Point, in the Sun's ſurface, emits light. Wherefore, ſince the Triangles, aAb, *a*A*b*, are ſimilar, it will conſequently be, as, A*a* is to Aa, ſo is *ab* to ab, that is, as AB to AS. (6. 6. El.)

Hence, it is eaſy to account for the Penumbra of Shadows; which, at a diſtance, appear diſtinctly defined, but on approaching near, we find it otherwiſe; inſomnch that, except where the Lines, in any Object occaſioning the Shade, cut the Surface on which the Shadow is projected, we cannot trace a line at all; and, the farther the Shadow is from that Point, the more penumbral it becomes; that is, the leſs diſtinctly defined; till, at a conſiderable diſtance, it becomes inſenſibly mixed with the Light.

Let

Plate XL. Fig. 8. Let AB be the Horizon of the part at D, to which, the Sun is directly oppoſite. The Ray SC is perpendicular to the Horizon, AB; and conſequently, a perpendicular Line, as CD, to the Horizon of that place, can have no Shadow, but will be projected towards the Center. But, if any other Tangent, as EF, be drawn; the direct Ray SC, to the Globe, is inclined to it, in the Angle SDE; and therefore, Lines perpendicular to EF will project Shadows. As CE to D.

Fig. 9. To illuſtrate this more clearly; ſuppoſe the Plane AB, horizontal, a portion of the Earth's Surface, to which the Rays are inclined, and SE the direction of a Ray of Light. The Angle of Inclination, to the Plane AB, is SED; and, the length of the Shadow of the Perpendicular CD, is DE.

Hence it is manifeſt, that the Shadow of a Right Line, on a Plane, is always a Right Line (1. 7. El.) for it is projected by a *Plane of Shade*, occaſioned by the Line; as CED, which cuts another Plane, whoſe common Section is the Shadow.

And, it is evident, that, the greater the angle of Elevation is, the ſhorter is the Shadow; for, FC projects the ſame Point, C, to G; conſequently, being elevated perpendicularly over C, the Line CD will project no Shadow.

In projecting Shadows by the Sun, there are three CASES to be conſidered, or ſituations of the Luminary; *viz.* it muſt be either on this ſide, or on the other ſide, or in the Plane of the Picture.

In order to determine the Shadows of Objects, delineated on the Picture; having firſt conſidered and determined on the ſituation and altitude of the Luminary, the next thing requiſite, if the luminous Body, or Point (for it is always conſidered as ſuch) be not in the Picture, is to find the Vaniſhing Point of the Rays of Light; but, when it is in the Plane of the Picture, the Rays are parallel on the Picture, and conſequently they have no Vaniſhing Point; the Angle of Elevation is, then, only neceſſary to be conſidered.

Fig. 11. Let FGHI be the Picture, and EC the Diſtance of the Picture. Alſo, let SE be ſuppoſed a Ray of Light.

Imagine a Plane (ESB) to paſs through the Luminary, and through the Eye, at E, cuting the Picture in FG; in which Line, whether the Sun be on this, or on the other ſide of the Picture, as at S, its apparent, or tranſprojected place muſt neceſſarily be; as at F or G.

For, being on the other ſide, at S (ſuppoſed at an immenſe Diſtance) and being in the Plane ESB, conſequently ES is in that Plane, and will cut the Picture in their common Interſection (FG) at F. Wherefore, F repreſents the Sun on the Picture, and is, conſequently, the Vaniſhing Point of the Rays of Light.

When the Sun is on this ſide, at S^1, or S^2, being ſtill in the ſame Plane, produced, conſequently S^1, or S^2E, being produced, will alſo cut the Picture in FG, as at G; and, becauſe all the Rays are parallel amongſt themſelves, and all Lines which are parallel have the ſame Vaniſhing Point, conſequently, G is their Vaniſhing Point. (Def. 22.) for S^1.G is a Ray of Light, to which all other Rays are parallel; or, it is a Right Line, paſſing through the Eye, parallel to them.

N. B. In the former Caſe (when the Sun is on the other ſide of the Picture) it is obvious, that its place, in the Picture, muſt neceſſarily be above the Horizontal Line, as it cannot be ſeen till it is above the Horizon; ſo, in this Caſe (ſeeing it is tranſprojected) being above the Horizon, its tranſprojected place muſt neceſſarily be below the Horizontal Line; from which (in either Caſe) it is farther removed, as the Luminary is more elevated above the Horizon. Alſo, whether its real place be on the right hand, or on the left, its apparent place is the ſame, in the former Caſe, but reverſed in the other. (See TRANSPROJECTION, in the Introduction; Page 52.)

Hitherto I have proceeded introductorily, which, I have called the Theory of Shadows; ſeeing that, all which has been ſaid is theoretic. I ſhall now proceed to practice, and lie down ſuch Rules as are neceſſary, for projecting the Shadows of all regular Objects, particularly ſuch as are right lined; which may be deſcribed on plane Surfaces, with facility and certainty, by adhering to the following Rules.

First. The indefinite projection of the Shadow of a Right Line, on any Surface whatever, is the Interſection of that Surface by a Plane, paſſing through the luminous Point and the Line; which, for diſtinction ſake, I ſhall call the Plane of Shade. This is obvious in itſelf.

Second. The Vaniſhing Line of the *Plane of Shade*, projecting the Shadow of any Right Line, is a Right Line drawn through the Vaniſhing Point of the Rays of Light, and the Vaniſhing Point of the Line whoſe Shadow is required (Th. 10.)

Becauſe, the Vaniſhing Point of the Line, whoſe Shadow is projected, and the Vaniſhing Point of the Rays; projecting the Shadow, are in the Plane of Shade.

Third. The Vaniſhing Point of the Shadow of any Right Line, on a Plane, is the interſecting Point of the Vaniſhing Line of that Plane, and the Vaniſhing Line of the Plane of Shade. (Cor. 2. Theo. 6.)

Becauſe, the common Interſection of thoſe Planes is the Shadow required.

N. B. When the Sun is in the Plane of the Picture, there being no Vaniſhing Point of its Rays, a Right Line, drawn through the Vaniſhing Point of any Line whoſe Shadow is required, parallel to the given Ray, cuts the Vaniſhing Line of the Plane of projection, in the Vaniſhing Point of the Shadow.

Alſo, if the Original Line be parallel to the Picture, and conſequently has no Vaniſhing Point; (the Sun being on either ſide of the Picture) then, a Right Line drawn through the Vaniſhing Point of the Rays, parallel to the Line whoſe Shadow is required, cuts the Vaniſhing Line of the Plane of projection, in the Vaniſhing Point of the Shadow.

But, when the Luminary is in the Plane of the Picture, and the Line, whoſe Shadow is required, parallel to it, the Shadow has no Vaniſhing Point; for it is, in ſuch Caſe, neceſſarily parallel to the Picture; therefore, parallel to the Vaniſhing Line of the Plane of projection.

SECTION III.

Of the Projection of rectilinear Shadows.

PROBLEM I.

Having the Angle of the Sun's Elevation (X) *and the Inclination, to the Picture, of a vertical Plane paſſing through its Center* (*equal* Z) *with the Center and Diſtance of the Picture given; to find the Sun's place on the Picture, or the Vaniſhing Point of the Rays of Light.*

Let CE be the Horizontal Vaniſhing Line, and C the Center of the Picture. Fig. 12.

Make CB equal to the Diſtance, perpendicular to CE, and draw AB parallel to CE. Draw BD, making the Angle ABD equal to Z, cuting CE, at D; and through D, draw FG, perpendicular to CE. FG is the Interſection of a vertical Plane paſſing through the Sun, with the Picture, whoſe inclination to it is equal Z.

Make DE equal to DB, and the Angle DEF, or DEG, equal to X, the Angle of ths Sun's Altitude; F or G is the Vaniſhing Point ſought.

Dem. Turn up the Triangle CBD, perpendicular, and turn over the Plane FEG, or FG, till DE coincides with DB.

Then, if the Sun be on the other ſide of the Picture, EF is parallel to the Sun's Rays; becauſe the Luminary is in the Plane EFG produced, and, the Angle DEF is equal to its Elevation; conſequently, EF produced would paſs through the Center of the Luminary, wherefore, F is the place of the Sun on the Picture; i. e. F repreſents the Sun; conſequently all the Rays center there, and therefore it is their Vaniſhing Point. For its Diſtance is infinite, to all ſenſe.

Case 2d. When the Sun is on this ſide the Picture, being behind the Spectator, it cannot appear in the Picture, but its place is tranſprojected to G; for, DEG is the Angle of its Elevation, and it is in a continuation of the Plane FEG; wherefore, EG is parallel to the Rays of Light; conſequently, G, the tranſprojected place of the Luminary, is their Vaniſhing Point.

Or, as Brook Taylor, very pertinently, calls it, the Shadow of the Spectator's Eye, on the Picture; for E is the Eye, wherefore GE, being produced, would paſs through the Sun's Center.

Plate XLI.

PROBLEM II.

The Vaniſhing Point of the Sun's Rays being given, and the repreſentation of a Line perpendicular to ſome Plane, whoſe Vaniſhing Line is given; together with the Center and Diſtance of the Picture; to find the repreſentation of the Shadow of that Line, on the Plane, and its Vaniſhing Point.

Fig. 13. Let S be the Vaniſhing Point of the Rays, VL the Vaniſhing Line of the Plane, and AB the given repreſentation of a Line perpendicular to that Plane. C is the Center of the Picture, whoſe Diſtance is known.

Find F, the Vaniſhing Point of Lines perpendicular to the Plane, whoſe Vaniſhing Line is VL (Prob. 2. Sect. 12. B. 3.) conſequently, AB vaniſhes in F; draw SF, cuting VL at V. V is the Vaniſhing Point of the Shadow.

Draw VB and SA, interſecting at D; BD is the Shadow of AB, required.

Dem. Whether S be conſidered as the Image of the Sun, on the Picture, or *S* its tranſprojected place, SF is the Vaniſhing Line of the Plane of Shade, for all Lines perpendicular to that Plane; ſeeing that, F is the Vaniſhing Point of all ſuch Lines; wherefore, V, its interſection with VL, is the Vaniſhing Point of the Shadow, of all Lines perpendicular to the Plane whoſe Vaniſhing Line is VL.

For, SV and AB repreſent parallel Lines (Cor. to Theo. 3.) wherefore, a Plane may paſs through both Lines (Ax. 5.) and conſequently, SA, VB, AF, and SF are all in that Plane.

But, F is the Vaniſhing Point of Lines perpendicular to the Plane; and, becauſe S repreſents a Point at an infinite Diſtance in that Plane, SF is the Vaniſhing Line of a Plane paſſing through AB.

But, S repreſents the Sun, the Vaniſhing Point of the Rays of Light, and V is its Seat on the Plane (whoſe Diſtance is ſuppoſed infinite) conſequently, V is the Vaniſhing Point of the Shadow, BD; for, the Plane of Shade, SAF, projecting its Shadow, is parallel to a Plane paſſing through the Eye and the Points S and F, ſeeing they are at an infinite diſtance; and the Shadow, BD, is the interſection of that Plane with the Plane of Projection; therefore its Vaniſhing Point is V, the interſection of their Vaniſhing Lines, VL, and SF. (Cor. 2. Theo. 6.)

This Problem is univerſal; I ſhall, next, apply it to Planes which are perpendicular to the Picture.

No. 2. Secondly. Let AB be a Line perpendicular to the Horizon, whoſe Shadow is required, and S the repreſentation of the Sun, on the Picture; or *S* its tranſprojected Image, the Sun being ſuppoſed on either Side.

Draw SV perpendicular to the Vaniſhing Line; then, V is the Seat of the Luminary, its Diſtance being ſuppoſed infinite.

Draw VB and SA, as before, interſecting at D; BD is the Shadow required.

For, becauſe AB, whoſe Shadow is to be projected, is perpendicular to the Horizon, and the Picture is ſuppoſed vertical; it is parallel to the Picture, conſequently SV, parallel to AB, is the Vaniſhing Line of the Plane of Shade; wherefore, V is the Seat of the Luminary, the Vaniſhing Point of the Shadow; and, SD is a Ray of Light, projecting the Shadow of the Point A, which determines its length, BD.

This is univerſally applicable to all Planes which are perpendicular to the Picture, whether they be horizontal, vertical, or inclined to the Horizon.

Thirdly. When the Luminary is in the Plane of the Picture.

In this Case, the Rays having no Vaniſhing Point, the inclination of the Rays to the Plane of projection, being determined, they are all parallel on the Picture; ſeeing, the diſtance of the Vaniſhing Point is infinite.

No. 3. Draw BD parallel to the Vaniſhing Line, and SA, cuting BD at D, making the Angle SDB equal to the Angle of the Sun's Elevation; BD is the Shadow required.

For, becauſe the Luminary is ſuppoſed in the Plane of the Picture, it is conſequently in every Plane parallel to the Picture; wherefore, the Plane of Shade, SDB, is parallel to the Picture, ſeeing the Line AB is parallel; and conſequently, BD, its Shadow, is parallel to the Vaniſhing Line of the Plane of Projection, ſeeing it is parallel to the Picture.

When

When the Plane of projection is parallel to the Picture, there can be no Shadow projected on it, but when the Luminary is on this ſide.

Let AB repreſent a Line perpendicular to the Plane X; and, let S be the tranſ- No. 4.
projected Image of the Sun, on the Picture; C is its Center. Join CS.

Draw BD, parallel to CS, and AS, cuting BD, at D; BD is the Shadow required

Becauſe the Plane (X) is parallel to the Picture, it has no Vaniſhing Line; wherefore, ſeeing the diſtance of the Luminary is ſuppoſed infinite, its Seat, on that Plane, is alſo at an infinite Diſtance.

But CS, produced, is the Seat of a Ray of Light, projecting its Vaniſhing Point, S; for it paſſes through the Eye, which is perpendicularly oppoſite to C; wherefore, the Seat of the Luminary is at an infinite diſtance, in the Line SC, produced. Conſequently, the Shadows BD, *BD*, &c. are all parallel to CS; and S is the Vaniſhing Point of the Rays, which determine the length of the Shadow.

PROBLEM III.

To project the Shadows of Right Lines, on a Plane to which they are parallel; and, in all poſitions to the Picture.

First, when the Lines are parallel to the Picture, and the Luminary in the Plane of the Picture.

The Shadows of Lines, on a Plane to which they are parallel, cannot be determined, conveniently, without having their Seats on that Plane; and is the ſame as finding the Shadows of Lines perpendicular to the Plane; or, the Shadow of one extreme of the Line being found, the other is eaſily determined.

Let AB be a Line parallel to the Picture, and parallel to the Ground Plane, on Fig. 14.
which the Shadow is to be projected. Its Seat is *ab*.

If the Sun be in the Plane of the Picture, and if it was in the Zenith of that place, the Shadow of AB is *ab*. But, if the altitude of the Sun be the Angle SCD; then, drawing AD, parallel to SC, the Shadow of AB is CD; for, the Shadow of a Perpendicular, A*a*, is *a*D; and CD is equal AB, ſeeing, ABCD, the Plane of the Shadow, is a Parallelogram.

After the ſame manner, the Shadow of the inclined Line, EF (being parallel to No. 2.
the Picture) is projected; by producing the Line till it cuts the Plane, at G; then, drawing GI, parallel to the Vaniſhing Line, and SH, SI, through F and E, parallel to SC, cuting GI, at H and I; HI is its Shadow.

Secondly. If the Line AB be either perpendicular or inclined to the Picture, No. 3.
and parallel to the Horizon, its Seat, at leaſt of one extreme (B) muſt be determined; and as the Point B may be in ſome vertical Plane, (as X) its Seat on the Ground is in the Interſection of that Plane, at b; and ſuppoſing the Plane X parallel to the Picture, the Luminary is in that Plane; conſequently, the Shadow of the extreme B will be ſomewhere in the Interſection of the Plane.

Draw SD, through B, making the Angle SDb equal to the altitude of the Sun, giving the Point D, in the Interſection of the Plane X, for the Shadow of B.

Then, becauſe AB is perpendicular to the Picture, C, is its Vaniſhing Point; and, becauſe the Shadow is projected on a Plane, to which AB is parallel, the Shadow is neceſſarily parallel to AB, and conſequently it has the ſame Vaniſhing Point, C.

Wherefore, draw CD and produce it; draw AE parallel to SD, (the given Ray of Light) cuting CD, produced, at E, the Shadow of A.

DE is the Shadow of AB on the Ground; to which it is parallel and equal.

For the Plane of Shade ABDE is a right angled Parallelogram in Perſpective.

FG is its Shadow on an inclined Plane, to which AB is parallel; projected by the ſame Rays of Light, or Plane of Shade.

SCHOL. Becauſe the Sun is in the Plane X, it cannot be ſaid to be illumined; conſequently, no Shadow can be projected on it; otherwiſe, if the Luminary was ever ſo little on this ſide, the Shadow of AB will be firſt projected on it, from B to D, and then projected to E; in which Caſe, BD would not be a Ray of Light, but a Shadow, and conſequently, not parallel to AE.

Plate XLI.

Case the Second and Third.

Fig. 15. When the Luminary is on this side, or on the other side of the Picture.

First. Let AB be parallel to the Picture, and to the Plane, on which its Shadow is to be projected. Let S be the Image of the Sun, on the Picture; or ⊙ its transprojected Image. Let VL be the Vanishing Line of the Plane of Projection.

Now, the Line AB being parallel to the Picture, has no Vanishing Point; consequently, its Shadow, being on a Plane to which it is parallel, is parallel to the Line. Let D be the Seat of A, on the Plane; C is the Center of the Picture.

Because the Plane, on which the Shadow is to be projected, is inclined to the Picture, through C, draw its Vertical Line EF; and because D is the Seat of A, on the Plane, draw AD and produce it, cuting the Vertical Line at F, the Vanishing Point of Lines perpendicular to the Plane.

For the Seat of a Point on any Plane is produced by a perpendicular to that Plane.

Draw SF, or ⊙F, cuting the Vanishing Line at V, or L, the Seat of the Luminary on the Plane, at an infinite Distance.

Draw VD, or LD; and SA, or ⊙A, cuting VD or LD, at a or *a*, the Shadow of the extreme A, on the Plane (Prob. 2.) Draw ab, or *ab*, parallel to the Van. Line, that is to AB; and, through B, draw SB, or ⊙B, cuting ab, or *ab*, at b, or *b*.

ab is the Shadow of AB, by means of S, the Image of the Sun on the other side of the Picture; or, *ab* is its Shadow, projected by the Vanishing Point ⊙, the transprojected Image of the Luminary, on this Side; both which, are Vanishing Points of the Rays of Light. aD, or *a*D, is the Shadow of the Perpendicular AD.

No. 2. Secondly. When the Line AB is perpendicular to the Picture, and parallel to the Plane of Projection; C is the Center.

Because the Line whose Shadow is to be projected is perpendicular to the Picture, and parallel to the Plane of Projection, that Plane is consequently perpendicular to the Picture, and its Vanishing Line VC passes through the Center. (Theo. 4.)

Let D be the Seat of the extreme A, on the Plane, and let S, or ⊙, be the Image of the Luminary; projected or transprojected.

Draw SV, or ⊙V, perpendicular to the Vanishing Line; and, through D, draw VD indefinite. Draw SA, or ⊙A, till it cuts VD, at a, or *a*, the Shadow of the extreme A, on the Plane, in either Case.

For, AD is perpendicular to the Plane, and Da or D*a*, is its Shadow. (Pr. 2.)

Draw Ca, or C*a*, indefinite, and SB, or ⊙B, cuting it at b, or *b*.

ab is the Shadow of AB, the Sun being beyond the Picture; or, *ab* is the Shadow of AB, the Sun being on this side, by means of its transprojected Image, ⊙.

For, AB being parallel to the Plane of Projection, its Shadow is parallel to AB; consequently, because AB is perpendicular to the Picture, the Center, C, is their common Vanishing Point. And, because the Rays of Light are parallel amongst themselves, they have the same Vanishing Point, S or ⊙.

N. B. When the Line, AB, is inclined to the Picture, and the Plane of Projection is perpendicular to it, there is no difference, in the process, but only in its Vanishing Point.

No. 3. Thirdly. When the Line AB is inclined to the Picture, and the Plane of Projection also inclined to the Picture, let VL be its Vanishing Line.

Find the Vanishing Point (F) of Lines perpendicular to the Plane, (Pr. 2. S. 12.) and draw SF, or F⊙, cuting the Vanishing Line at E, the Seat of the Luminary.

Thro' D, the Seat of A, draw ED indefinite, and SA, or ⊙A cuting ED, at a or *a*.

Then, V being the Vanishing Point of the Line AB, and because the Shadow is parallel to it, draw Va, or V*a*, indefinite, and SB, or ⊙B, cuting it, at b, or *b*; ab, or *ab*, is the Shadow of AB, projected on the Plane whose Vanishing Line is VL.

PROBLEM

PROBLEM IV.

To determine the Shadows of Lines inclined to the Plane of Projection and to the Picture; in any Angle, whatever.

Let AB be inclined to the Plane Z, on which the Shadow is to be projected; and, let ST be a given Ray of Light, the Luminary being ſuppoſed in the Picture. Fig. 16.

Through V, the Vaniſhing Point of AB, draw VC parallel to the given Ray, cuting the Vaniſhing Line of the Plane Z, at C, the Van. Point of the Shadow.

For, the Luminary being in the Plane of the Picture, its Rays have no Vaniſhing Point; conſequently, a Right Line drawn through V, the Vaniſhing Point of any Line, parallel to the Rays, is the Vaniſhing Line of the Plane of Shade, projecting the Shadow of that Line.

Therefore, draw AC, the indefinite Shadow of AB; and BD parallel to the given Ray; that is, parallel to VC, the Vaniſhing Line of the Plane of Shade.

AD is the Shadow of AB on the Plane Z.

If the Line EF, whoſe Shadow is required, does not cut the Plane, on which it is to be projected, let it be produced till it cuts the Plane, at G, and draw GC.

Or, if the Seat of either extreme be given, or found, as *a* of the Point E, or *b* of the extreme F; then, having obtained the Shadow of either, e or f (Prob. 2.) draw Ce, or Cf; and Rays, through the extremes, E and F, parallel to the given Ray, ST; by which means the Shadow, ef, of EF is projected.

Case the Second and Third.

When the Luminary is beyond, or on this ſide of the Plane of the Picture.

Let AB be a Line, whoſe Shadow is to be projected. V is its Vaniſhing Point. No. 2.
Let S and ⊙ be the projected and tranſprojected Images of the Sun.

If the Line does not cut the Plane of Projection, let it be produced, to C.

Draw SV or ⊙V, cuting the Vaniſhing Line EF, at F, the Van. Point of the Shadow. Draw CF; and SA, SB, or ⊙A, ⊙B, cuting it at a and b, or *a* and *b*.

ab is the Shadow of AB when the Luminary is beyond the Picture, and *ab*, when it is on this Side; according to its determined place.

Or the Shadow may be found by means of the Seat of either extreme, given or found, on the Plane of Projection; as in the firſt Caſe.

SCHOL. Any Line, whoſe Shadow is to be projected, cuting the Plane of projection (whoſe Vaniſhing Line is given or found) the proceſs is the ſame, whether the Plane of projection be perpendicular or inclined to the Picture; for, if the Vaniſhing Point of that Line be obtained, the Vaniſhing Line of the Plane of Shade is alſo; whoſe Interſection with the Vaniſhing Line of the Plane of projection is the Vaniſhing Point of the Shadow; which, it muſt be obvious, always paſſes through that Point in which the Line cuts or would cut the Plane.

2. When the Plane of projection is parallel to the Picture, there can be no Shadow projected on it, but when the Luminary is on this ſide; in which Caſe, whether the Line be perpendicular or inclined to the Plane, it is the ſame to the Picture, and, its Vaniſhing Point being determined, the proceſs is the ſame. In this Caſe, the Plane of projection having no Vaniſhing Line, the Shadow, of any Line thereon, is parallel to the Vaniſhing Line of the Plane of Shade.

3. All Lines which are parallel to ſuch Planes are alſo parallel to the Picture; and their Shadows, being parallel to the Lines, are always ſo repreſented; which is not the Caſe on other Planes, but only, when the Lines are parallel to the Picture, as well as to the Plane of projection.

4. The Seats of all ſuch Lines, on the Plane, muſt be had, or the Shadow cannot be determined; in which Caſe, it is but finding the Shadows of two equal Lines, perpendicular to the Plane (as in Fig. 14.) and, joining their extremes, the Shadow of the parallel Line is obtained, as in the ſecond Problem.

Having, in theſe Problems, given every Rule which I conceive neceſſary for the projection of right lined Shadows, I ſhall, next, give ſome Examples of their utility, in the application of them to the Shadows of Objects.

Plate XLII.

As every Object is compoſed either of plane or curved Surfaces, or both, ſo likewiſe, thoſe Surfaces are bounded by right Lines or curved; and, as the outline of the Shadow only is required, we ſhould carefully obſerve on what part of the Object the Light falls, that we do not give the Shadows of ſuch Lines as can (from their ſituation to the Light) have no Shadow; for, according to the altitude of the Luminary, or its ſituation to the Object, different Lines in the Object define its Shadow. Wherefore, ſince it is from the linear perſpective repreſentation of the Object, that the Shadow is projected, it is ſometimes difficult to determine the Line which caſts the Shadow, becauſe they are frequently out of ſight; wherefore, they ought to be determined with caution, before we begin the proceſs.

The conſideration of the Luminary being beyond, or on this ſide, or in the Plane of the Picture, is alſo a circumſtance which ſhould be maturely conſidered. The beſt effects, are (in my opinion) produced, by conſidering the Sun to be on this ſide; becauſe it is obvious, that, when it is beyond the Picture, though ever ſo little, that is, though ever ſo much inclined, it is alſo beyond every Object repreſented on the Picture; but, if the Sun itſelf be repreſented, that is, if it be ſo little elevated and inclined to the Picture, that the Sun's apparent place is within its limits, it muſt then be entirely behind the Objects; conſequently, the whole of every Object, on that ſide, is immerſed in Shade; and the Shadow, of each, is projected towards the Eye, greatly diſtorted, and larger than the Object. Whereas, the Sun being on this ſide, towards either hand, as is moſt ſuitable to the Objects (according to what Faces we would have illumined) the Shadows are projected forwards, from the Eye, diminiſhed, and great part of it hid by the Object; which, with the various effects of Light and Shade, on the Objects, cannot fail rendering the whole agreeable, and moſt pleaſing to the Eye.

EXAMPLE I.

To repreſent the Shadow of a Plane Figure, on a Plane.

Fig. 17. First. Let the Plane ABCDE be perpendicular to the Ground Plane, on which the Shadow is to be projected. Let S and ⊙ be the Image and tranſprojected Image of the Sun; VL is the Vaniſhing Line of the Plane ABCDE, and FG of the Plane of projection.

Draw SF, or ⊙F, perpendicular to the Horizontal Line, cuting it at F; the Vaniſhing Point of Lines perpendicular to the Horizon. (Prob. 2.)

Through A, draw Fb, indefinite; and, if the Shadow be projected by the Image, S, through B, draw Sb, cuting the former, at b. Or, draw B⊙, cuting it at *b*.

Then, for the Shadow of BC (on this ſide) join SV; if it be parallel to the Horizon, draw bc alſo parallel; if it be not parallel, then, SV produced will give its Vaniſhing Point. For the ſame Line on the other Side, draw V⊙, cuting the Horizontal Line at H, and draw *b*H. SC, and C⊙, cuts them at c and *c*, reſpectively.

CD is parallel to the Plane of projection, and G its Vaniſhing Point; therefore, draw cG, or *c*G, and SDd, or D⊙, giving d, or *d*, for the Shadow of D.

Laſtly; for the Line DE (on this ſide) draw SL, cuting the Horizontal Line at I, and draw Id. For the other Side, draw ⊙L; which being produced to the Horizontal Line, will give the Vaniſhing Point of *d*E, and compleats that Shadow. Or, join dE, or *d*E, only.

AbcdE is the Shadow of ABCDE projected on the Ground Plane, by the Image (S) of the Sun, on the other ſide of the Picture; and, A*bcd*E is its Shadow, by the tranſprojected Image, ⊙, being conſidered on this Side.

Fig. 18. Secondly. Let the Plane ABCDE be either perpendicular or inclined to a vertical Plane, whoſe Vaniſhing Line is VL, on which the Shadow is projected.

Whether

Whether the Plane, whoſe Shadow is required, be perpendicular or inclined to the Picture, the proceſs is the ſame. HF is the Van. Line of the Plane ABCDE.

FIRST. Let S be the Image of the Sun, on the other ſide of the Picture.

The Line, AB being parallel to the Picture, draw SV parallel to it; V is the Vaniſhing Point, of its Shadow; wherefore, through A draw Vb, indefinite, and, through B, draw Sb, cuting it at b, which determines its length, Ab.

H being the Vaniſhing Point of the contiguous Line BC, draw SH, and produce it to I, the Vaniſhing Point of its Shadow.

Draw bI, and Sc, through C, cuting it at c. bc is the Shadow of BC.

Then, becauſe CD is parallel to the Plane of projection, and G is its Vaniſhing Point, draw cG, and SD, cuting it at d, the Shadow of D.

Laſtly, draw SF, giving the Vaniſhing Point J if it was neceſſary; but it is obvious, that, dE, being joined, compleats the Shadow, and tends to J.

If the Shadow was obſtructed by another Plane, cuting it at MN, the Vaniſhing Line of which is FN; SV, being produced, would cut that Vaniſhing Line in the Vaniſhing Point of the Shadow of AB, on that Plane; from which, draw M*b*. SH produced, cuts the Vaniſhing Line at P; draw *b*P, cuting the Ray SCc at *e*; and, SG produced cuts it at O; draw *e*O; or join *ef* only, which compleats it.

SECOND. Let ⊙ be the tranſprojected Image of the Sun, on this ſide.

AB being parallel to the Picture, draw ⊙K parallel to it, cuting the Vaniſhing Line, VK, at K, the Vaniſhing Point of its Shadow.

Draw AK, and B⊙, cuting it at *b*, the Shadow of B. ⊙H cuts the Vaniſhing Line at L; draw *b*L, and C⊙, cuting it at *c*. Then, draw *c*G, and D⊙, cuting it at *d*. *cd* is the Shadow of CD, being parallel to the Plane of projection.

If ⊙F be parallel to the Vaniſhing Line, VK; then, *d*E is alſo parallel to it. Or, it will tend to that Point, in which ⊙F, being produced, would cut VK.

SCHOL. If the Plane of projection be much inclined to the Picture, there can be no Shadow caſt on it, the Sun being beyond the Picture; or they will be drag'd out to an immoderate length. For, if the Image of the Sun appear in the Picture, it will be behind the Plane of projection, being much inclined.

By obſerving the Rules, given above, the Shadow of any plane Object may be projected on any Plane, or on various Planes, of which I ſhall give ſome Examples.

EXAMPLE II.

To project the Shadows of right angled Parallelopipeds, on the Ground Plane.

FIRST. When a Face of the Parallelopiped is parallel to the Picture, and, the Luminary in the Plane of the Picture.

X is the Object whoſe Shadow is to be projected. Fig. 19.

In this CASE, the Luminary being in the Picture, the Shadow of the whole Solid is no more than of the Plane ABDE. For, the Plane X being parallel to the Picture, cannot be illumined; ſeeing that the Sun is in the Picture, it is, conſequently, in every other Plane parallel to the Picture. The whole Shadow of that Plane is, therefore, but a Line, generated by producing the Plane.

Draw Ab, indefinite, parallel to the Vaniſhing Line, HL, and, if SB be a given Ray of Light, produce it, till it cuts Ab, at b, the Shadow of B.

Then becauſe BD is perpendicular to the Picture, C, the Center, is its aniſhing Point; and, being parallel to the Plane of projection, its Shadow conſequently parallel, and has, therefore, the ſame Vaniſhing Point.

Draw bC; and through the angle D, draw Dd parallel to Sb, cuting bC at d; or, draw Ed parallel to Ab, which compleats the Shadow, AbdE, required.

SCHOL. If a Line (FG) projected from the Plane X, its whole Shadow will be projected on the Ground. For, ſince the Luminary is in the Plane X, no part of its Shadow can be projected on that Plane; wherefore, fg, its Shadow, is wholly on the Ground Plane.

Plate XLII. Fig. 20.

SECONDLY. When its Planes are inclined to the Picture.

Let AD be a right angled Parallelopiped, the Vaniſhing Points of its Sides, are H and L; and let ST be a given Ray of Light.

Draw Ab parallel to the Horizon; and through B, draw Bb parallel to ST.

Then, L being the Vaniſhing Point of BC, draw bL; and through C, draw Cc, parallel to ST, cuting it at c. Laſtly, draw cH; and Dd parallel to ST; and, through d parallel to Ab. Abcd is the Shadow required.

For the Rays, projecting the Shadow, are all parallel amongſt themſelves; and the Perpendicular AB, being parallel to the Picture, has its Shadow (Ab) parallel to the Vaniſhing Line; alſo, the Shadows of BC and CD have the ſame Vaniſhing Points, reſpectively; becauſe, BC and CD are parallel to the Plane of projection.

CASE 2d. When the Luminary is on the other ſide of the Picture.

Fig. 21.

Firſt; let the Plane AD be parallel to the Picture, and, let S be the Image of the Sun on the Picture, being ſituated beyond it.

Draw SV, perpendicular to the Horizon, V is the Vaniſhing Point of the Shadows of Perpendiculars, on the Ground Plane. (Prob. 1.) Draw VA and SB, and produce them, cuting at b, the Shadow of B. Then, draw bd parallel to BD; and, through D, draw Sd, cuting it at d; bd is the Shadow of BD.

Laſtly; becauſe DE is perpendicular to the Picture, and C is its Center, draw dC; and, through F, draw VF, or SE, cuting it at e, and compleats the Shadow, AbdeF; which, as it is projected back, towards the Eye, is larger than the Object.

SCHOL. When the Object is inclined to the Picture, as in No. 2, the proceſs is the ſame in every reſpect, having regard to the Vaniſhing Points of the Lines BC and CD, to which their Shadows, bc and cd, are reſpectively parallel, and conſequently have the ſame Vaniſhing Points, G and F.

No. 2.

CASE 3d. When the Luminary is ſituated on this ſide the Picture.

Let AD be a Parallelopiped, right angled, as before; and let ⊙ be the tranſ-projected place of the Luminary.

Draw ⊙V perpendicular, and, draw AV and B⊙, cuting at *b*, the Shadow of B. Then, draw *b*G and C⊙, cuting at *c*; and laſtly, draw *c*F, which compleats the Shadow A*bcd*. The Shadow of the Angle D is not ſeen.

EXAMPLE III.

Plate XLIII.

To project the Shadow of a plain Building, on the Ground.

Fig. 22.

Let ADF repreſent a Building, obliquely ſituated to the Picture; and let the Sun be in the Plane of the Picture. The direction of its Rays is SC.

Draw Ab parallel to the Vaniſhing Line, and Bb to the given Ray, cuting Ab, and determining the Shadow of AB.

Through V, the Vaniſhing Point of BC, draw VG parallel to the given Ray of Light; and draw bG; SC produced, cuts it at c, the Shadow of C.

Becauſe the Vaniſhing Point of CD is below the Horizontal Line, equal VH, the Vaniſhing Point (I) of its Shadow, will be on the other ſide VH, equal GH.

Make HI equal GH, and draw cI; and Dd parallel to SC, cuting it at d.

Becauſe of the Sun's altitude, and the ſituation of the Building, the other ſide of the Roof is illumined; and conſequently, the Shadow of CF (the Ridge) is not projected, which is evident; becauſe, being parallel to the Plane of projection, its Shadow is parallel to itſelf, and conſequently has the ſame Vaniſhing Point, L. Wherefore, if from c, the Shadow of C, a Line be drawn to L, it will fall on this Side cd, the Shadow of CD; conſequently it has no Shadow, here.

Therefore, draw dL, cuting DE; de is the Shadow of a Line from D, parallel to CF (the Eaves of the Roof) on the other ſide of the Building.

SECONDLY.

SECONDLY. Let the Sun be ſuppoſed beyond the Picture, and let S be its Image.

Draw ST, perpendicular; and, through A, draw T b indefinite; through B, draw Sb, giving Ab, for the Shadow of AB. Fig. 23.

Then, BC being parallel to the Horizon, and L its Vaniſhing Point, through b, draw L c, and through C draw S c, giving bc, the Shadow of B C.

V being the Vaniſhing Point of CD, draw S V. If SV, be parallel to the Horizon, draw c d, alſo parallel; if SV, be not parallel, produce it, till it cuts the Horizontal Line, on one ſide or the other, and draw cd, tending to that Point; alſo, through D, draw Sd, cuting it at d, the Shadow of D.

Again. Y being the Vaniſhing Point of DE, draw S Y, cuting the Horizontal Line at I, the Vaniſhing Point of the Shadow of DE.

Draw dI, and SE till it cuts dI, at e; and laſtly, join eF, which will tend to its Vaniſhing Point T, and compleats the Shadow, AbcdeF.

In this Caſe, it is obvious that the whole Building, ſave the Plane DB, is in Shade

CASE 3d. When the Luminary is on this ſide of the Picture.

Let ⊙ be its tranſprojected Image. ⊙K being drawn, perpendicular, K is the Vaniſhing Point of the Shadows of perpendicular Lines. (Prob. 2.)

Draw AK and B⊙, interſecting at *b*, the Shadow of B.

Then draw ⊙V, cuting the Van. Line at I, the Van. Point, of the Shadow of BG.

Draw *b*I, and G⊙, cuting it at *g*, the Shadow of G.

Join ⊙ Y; and if it be not parallel, produce it to the Vaniſhing Line.

Through *g*, draw parallel, or tending to the Point in which ⊙Y, cuts the Horizontal Line; which compleats the Shadow, as much as can be ſeen.

I preſume, it is obvious, from theſe Examples, that, the moſt judicious choice is to ſuppoſe the Luminary on this ſide of the Picture, as in this laſt Caſe. Firſt, becauſe the object is more advantageouſly ſituated to the Luminary, and conſequently, will have a more agreeable effect of Light and Shade. Alſo, becauſe the Shadow is projected from the Eye, ſo that, it does not appear diſtorted and prepoſterous, as in the ſecond Caſe; and it is far from being pleaſing to have the Shadows of perpendicular Lines all parallel amongſt themſelves; as when the Luminary is in the Plane of the Picture. Therefore, as I have given Rules and Examples in each, I ſhall, hereafter, conſider the Luminary only on this ſide of the Picture, unleſs it be otherwiſe determined.

EXAMPLE IV.

To project the Shadow of an upright priſmatic Object, whoſe upper Face is inclined to the Horizon, part on the Ground, and part on a vertical Plane, inclined to the Picture, whoſe Interſection with the Ground is given.

Let X be the Object, whoſe Shadow is to be projected; let ⊙ be the tranſprojected Image of the Sun, and JI the Interſection of a vertical Plane with the Ground; KL is its Vaniſhing Line. (Prob. 3. Sect. 3. Book III.) Fig. 24.

Draw ⊙H, perpendicular to the Horizontal Line. Draw AH, and B⊙ cuting it at *b*, the Shadow of the Angle B on the Ground.

But, A*b* cuts the Interſection JI, at a; wherefore, becauſe the Plane Y is vertical, AB is parallel to the Plane; and, being alſo parallel to the Picture, its Shadow is conſequently parallel; draw ab, parallel to AB, till it cuts the Ray B⊙, at b, the Shadow of B on the vertical Plane; as *b* is, on the Ground.

The Vaniſhing Point of BC is F; which, being inclined to the Horizon is not in the Horizontal Line. Draw ⊙F, and produce it, till it cuts KL, the vertical Vaniſhing Line, at K, the Vaniſhing Point of its Shadow; then, draw bK, and C⊙ cuting it at c, the Shadow of C on the vertical Plane.

Plate XLIII.

Then, G being the Vaniſhing Point of CD, a Line on the other ſide, parallel to BE, which projects a Shadow, draw ⊙G, cuting the vertical Vaniſhing Line at L. Draw cL and D⊙ interſecting at d.

If DM be a perpendicular Line, draw de parallel to it, cuting the Interſection at e, and join eM, which tends to the Vaniſhing Point, H, of Perpendiculars, and compleats the Shadow; Aaef on the Ground, and abcde on the vertical Plane, as it was propoſed.

If the vertical Plane was out of the way, its Shadow on the Ground is A*bcdf*; in which, the Shadow of BC tends to N, and CD to O, in the Horizontal Line.

EXAMPLE V.

To project the Shadow of a Croſs, perpendicular to the Horizon, part on the Ground; alſo, on a Plane inclined to the Horizon, and to the Picture, whoſe Vaniſhing Line, and Interſection with the Ground Plane are given.

Fig. 25. Let AFBK be the Object, inclined caſually to the Picture; whoſe Vaniſhing Points are M and N, in the Horizontal Line. Let QR be the Interſection of the inclined Plane with the Ground, whoſe Vaniſhing Line is OP. ST is a given Ray of Light, the Luminary being in the Picture.

Becauſe the Object is inclined to the Picture, and the Luminary is in the Picture, the Side AB and its oppoſite, IJ, will project Shadows.

Draw Aa, and Jb parallel to the Horizon, cuting the Interſection, QR, at a and b. Draw a*b*, and b*i*, parallel to the Vaniſhing Line OP, indefinite; and draw B*b*, parallel to ST, cuting a*b*, at *b*, the Shadow of B.

For, the Line AB being parallel to the Picture, the Plane of Shade occaſioned by that Line is, in this Caſe, parallel to the Picture; conſequently, its ſection with every Surface, on which the Shadow of AB falls, is parallel to the Picture; therefore, Aa is parallel to the Vaniſhing Line of the Ground Plane (MN) and *ab* to OP, of the inclined Plane.

Draw NO and MP, parallel to the Ray of Light, ST; O is the Vaniſhing Point of the Shadows, of all Lines parallel to CD; and P, of all that are parallel to EF, on the inclined Plane.

Draw Cc parallel to ST, cuting the Shadow of AB at c. Draw Oc indefinite, and D*d* parallel to the Rays, cuting it at *d*, the Shadow of D.

DE being parallel to the Picture, draw *de*, its Shadow, parallel to the Vaniſhing Line, OP, meeting the Ray of Light from E, at *e*.

P being the Vaniſhing Point of the Shadow of EF, draw *e*P, meeting the Ray from F, at *f*; and draw *f*O, till it cuts the Interſection QR, at *j*; from which it tends to N, the Vaniſhing Point of FG; being parallel to the Ground.

*d*O cuts the Interſection at *a*, from which Point it alſo tends to N, till it meets the Ray, from K, at *k*. Draw *k*M, cuting the Ray from L; and LG being parallel to the Picture, draw *lg*, parallel to the Horizon, which compleats the Shadow of that Arm, on the Ground.

The Shadow, *bbi*, of the Top, having the ſame Vaniſhing Points, reſpectively, it would be unneceſſary to deſcribe, being obvious, from the Figure.

Secondly. *When the Luminary is on this ſide of the Picture.*

In the foregoing Caſe (as in other preceding) the Vaniſhing Point of the Shadow of every Right Line, on the inclined Plane, was produced, by drawing a Right Line, parallel to the given Ray, from the Vaniſhing Point of the Line whoſe Shadow was required, to the Vaniſhing Line of the inclined Plane; becauſe the Luminary being ſuppoſed in the Plane of the Picture, all its Rays are conſequently parallel amongſt themſelves.

In this, the Rays of Light having a Vaniſhing Point, a Right Line joining the Vaniſhing Point of the Line, and the Vaniſhing Point of the Rays, cuts the Vaniſhing Line, or being produced, would (if it be not parallel) cut the Vaniſhing Line of the Plane of projection, in the Vaniſhing Point of the Shadow (Prob. 4.) or, the Rays, inſtead of being parallel amongſt themſelves (as in the former Caſe) converge in a Point, in which conſiſts the difference.

⊙ is the Vaniſhing Point of the Rays, MN the Vaniſhing Line of the Ground, and OP of the inclined Plane, of which, QR, is its Interſection with the Ground. M and N are the Van. Points of hor. Lines in the Object, as before. Fig. 26.

S being the Seat of the Luminary, on the Ground, draw AS, cuting the Interſection, QR, at a. Then, becauſe AB is parallel to the Picture, produce ⊙S, (being parallel to AB) till it cuts the Vaniſhing Line of the inclined Plane, at T.

Draw aT; B⊙, a Ray of Light, cuts aT at *b*, the Shadow of B; and C⊙ cuts it at *c*; through which, a Line drawn from O gives the Shadow of DK, on the inclined Plane; from the Interſection QR it tends to N. Rays from D and K, tending to ⊙, determine the extremes of the Shadow.

It would be uſeleſs to deſcribe the whole, the ſame Letters of reference, as in the foregoing Caſe, ſhew how the Shadow of every Line is produced, by adhering to the former Leſſons, in this Caſe.

EXAMPLE VI.

To project the Shadow of a Right Line, inclined to the Horizon and to the Picture, on ſeveral Planes, variouſly inclined to both. Plate XLIV.

AC repreſents a Ladder, leaning againſt a Building, on which its Shadow is to be projected. ⊙ is the tranſprojected place of the Luminary; H and I are the Vaniſhing Points of horizontal Lines in the Building, and E of the Ladder. FI is the Vaniſhing Line of the Roof of the Building; and GI of the adjoining Shed; on which, the Shadow is alſo projected. Fig. 27.

Draw E⊙, and produce it to the vertical Vaniſhing Line (KI) of the Sides of the Building, cuting it at K; the horizontal Vaniſhing Line is cut at L, and the inclined Vaniſhing Lines, at M and N.

K, L, M, N, and E, are all Vaniſhing Points of the Shadow. (Prob. 4.)

From the foot of the Ladder, draw Lines to the Vaniſhing Point L, cuting the Interſection of the Plane V, with the Ground, at a and b; AabB, is its Shadow on the Ground.

K being the Vaniſhing Point of the Shadow, on the vertical Planes, draw Lines from K, through a and b, till they cut the common Interſection of the Planes V and U, at c; from which, draw Lines to E, the Vaniſhing Point of the Ladder; becauſe it is parallel to the Plane U.

M being the Vaniſhing Point of the Shadow, on the Plane X, draw dM, cuting the Interſection of that Plane with the next, at e; from which Points draw Lines to K; for the Shadow on the Plane Y, being parallel to V, has the ſame Van. Point.

From the Points *ff* (where the Ladder touches the Eaves of the Roof) draw to N; and, where they cut the Interſection of the Chimney with the Roof, draw to K, and E; from C and D the ends of the Ladder, draw to ⊙, cuting them, which compleats the Shadow of the ſides of the Ladder.

If from each Step, on either Side, Lines are drawn to ⊙, cuting the Shadow of that Side, at *a*, *b*, *c*, &c. and from thoſe Points, Lines are drawn to I, the Vaniſhing Point of the Steps, their Shadows will be compleated, on the Planes V, X, Y, and Z, as well as on the Ground; becauſe the Steps are parallel to all thoſe Planes; being parallel to the Horizon.

On the vertical Plane U, their Vaniſhing Point is where a Right Line drawn from I, through ⊙, would cut the vertical Vaniſhing Line EH, of that End.

The Chimney being perpendicular, ⊙S, drawn perpendicular, cuting the Vaniſhing Line (GI) of the Roof, at R, gives its Vaniſhing Point; J⊙ determines its length.

OPQ repreſents a Sign Poſt, whoſe Shadow, Oo, on the Ground, and opQ, on the Wall, is projected by the ſame Point (⊙) whoſe Seat is S; which, being ſo near the Vaniſhing Point I, conſequently, the Luminary is nearly in the Plane OPQ; wherefore, the Shadow of both, on the Wall, are almoſt in one Line.

In

Plate XLIV.

In this situation of this Object, the Shadow is more pleasing supposing the Sun on the other side of the Picture; as O*o*, on the Ground, *op* of the Post, on the Wall, and *p*Q of the Bearer. The Sun being on the left hand, V (at a proper distance) may be the Vanishing Point, of the Shadow *p*Q; and, the Seat of the Luminary, is the Vanishing Point of O*o*, on the Ground (Prob. 2.) OP being perpendicular. The Rays of Light, P*p*, &c. indicate the rest.

EXAMPE VII.

To project the Shadow of a Building, and the several parts of it on the other; the Luminary being on this side of the Picture, having its Altitude and Inclination given.

Fig. 28.

Let the Angle of the Sun's Elevation be equal X; and the Inclination to the Picture, of a vertical Plane passing through its Center, equal Z.

Find ⊙, the transprojected place of the Luminary; by Prob. I.

Draw ⊙S perpendicular, giving S, the Seat of the Luminary, on the Ground.

Then, draw AS, cuting the Intersection of the Plane of the Arches, on the Ground, at *a*; from which, draw *a*a, perpend. the Shadow of AB on that Plane.

Produce ⊙S, till it cuts VO, the Vanishing Line of the inclined Plane X, at O; the Vanishing Point of the Shadow of AB, on that Plane.

Draw aO indefinite; and, because of the projecture of the Cornice at C, casting a Shadow (JB) on the Plane W, draw B⊙, cuting aO at b, the Shadow of B; from which, draw bc, to the Vanishing Point V; because, *b*C, projecting that Shadow is parallel to the Plane X.

Then, M being the Vanishing Point of CD, draw ⊙M, and produce it, till it cuts the Vanishing Line of the Planes V and W; and, VO being also cut at P, draw cP, cuting the next plane at e. and draw eQ, to the Vanishing Point, in VE.

L being the Vanishing Point of DE, draw ⊙L, and produce it to the Vanishing Line VM, of the Roof, cuting it at N. Draw NE, and D⊙ cuting it at d, and draw Md, to f, which compleats the Shadow of that part. CD, projecting the Shadow df, is parallel to the Plane Y, and has, therefore, the same Vanishing Point, M.

As the Luminary is not much elevated, the Shadow of the angle of the Cornice falls on the Wall, at h. F being its Seat on the Ground, draw FS, cuting the intersection of the Wall with the Ground at *f*; from which draw *f*h, perpendicular, and H⊙, cuting it at h.

RT is the Vanishing Line of the Wall Z, and M being the Vanishing Point of HI, ⊙M, cuting RT gives the Vanishing Point T, of hi, the Shadow of HI, on the Plane Z. From i, the Shadow of IK tends to K, in RT; K⊙ cuts it at k; from which, the Shadow of K*E* (the Ridge, which is not seen) tends to R, till it cuts the Intersection of the Wall with the Ground, whence it tends to V. *E*⊙ cuts it at *e*; from which Point, *ed* is the Shadow of the Hip, at the farther corner.

To determine the Shadow of the projecture of the Cornice. Let m n be the seat of the first Truss, on the top of the Cornice, and m its utmost projecture, which projects the Shadow; or, take any other Point, in mY.

L being the Vanishing Point of the Truss, i. e. of mn, draw from ⊙ through L, till it cuts the Vanishing Line of the Planes W; from which Point, draw no, and m⊙, cuting it at o, the Shadow of m, on the Plane W, produced.

Draw oV, cuting both Planes W, in Gr, and BJ, the Shadow of the extreme projecture of the Cornice, on those Planes.

Having, by the same means, obtained the Shadow of the lower edge of the fronts of the Trusses, their places are obtained by drawing Lines, from each to ⊙. The first falls on the Wall, at *p*, and the first on the other side, at *q*, on the low Roof.

The Shadows of the Piers of the Arcade, against the Wall, within, are projected by means of the Vanishing Point S, till they meet the wall, where they are upright. Lines drawn to ⊙, determine their height. (See Sect. IV. for the Arches.)

Thus, the Shadow of the whole Building is compleated; in which are all the variety of Examples, necessary for projecting the Shadows of right lined Objects.

SECTION IV.

Of the Shadows of curved Lines and curve-lined Objects, on plane and curved Surfaces.

AS the Perſpective of curve-lined Objects is more difficult and liable to error than right-lined, on account of the continual bending, and varying in the direction of curve Lines; ſo, their Shadows are with more difficulty projected. As there are no vaniſhing Points of the Original Lines, ſo neither can there be vaniſhing Points of the Shadows; alſo, when the Shadows, either of right or curved lines, are projected on a curved Surface, there being no Vaniſhing Line, the figure of the Shadow cannot, ſo certainly, be deſcribed thereon. Nevertheleſs, Rules may be preſcribed, for projecting Shadows, in ſuch caſes; which, if followed, will produce the true projection of as many Points, in the Curve, as are neceſſary, for deſcribing the contour of the Shadow, with tolerable accuracy. The principal Shadow being determined, the minutiæ, of Mouldings, &c. may, by a Perſon who has judgment, be done by Hand, and in many caſes they muſt; for, I fairly own, that it is more than human patience can bear, nor indeed is it poſſible, to project Shadows, in all Caſes, with mathematical exactneſs.

The Shadow of a Circle, projected upon any Plane, to which the Circle is parallel, is a Circle, of equal dimenſions; in all ſituations of the Luminary, whatever.

For, whether its ſituation be on this, or on the other ſide, or in the Plane of the Picture; whether it be more or leſs elevated, 'tis ſtill the ſame; the projecting Rays, being parallel amongſt themſelves, generate a Cylinder, for the moſt part oblique; conſequently, being cut by a Plane parallel to the Circle, its projection muſt alſo be a Circle, ſeeing it is the oppoſite Baſe of the Cylinder; but, being cut oblique it is an Ellipſis. So that, in Arches, &c. parallel to the Picture, the Luminary being on this ſide, and projecting the Shadow on a vertical Plane, parallel to the Arches, the Center of each being determined, the repreſentation of the Shadow of each Arch, may be deſcribed with Compaſſes; which will be in proportion to the Arches, as the diſtance of the Plane of the Arches, to the diſtance of the Plane of projection. But, being projected on the Floor, or any other Plane, not parallel to the Arches, their Repreſentations are Ellipſes; except when the Cylinder, projecting the Shadow, is cut ſubcontrary*; in which Caſe, the Shadow will be a Circle.

In all other poſitions of the Arches, to the Picture, and in all ſituations of the Luminary, being projected on a parallel Plane, the Shadows being Circles, their Repreſentations are Ellipſes, deſcribed as the Arches, themſelves; and being projected on any other Plane, the Shadows being Ellipſes, their repreſentations are alſo Ellipſes, more excentric, generally, though not always ſo.

The Shadow of a Sphere or Globe (as well as its perſpective Projection) on a Plane, is an Ellipſis, in all Caſes, whatever; except when a Right Line paſſing through the Center of the Luminary, and of the Sphere, is perpendicular to the Plane of Projection.

* An oblique Cylinder or Cone is cut ſubcontrary, when the Plane of ſection is ſo ſituated, that the Axe of the Cylinder or Cone has the ſame inclination to it, contrarywiſe. As, if the Sun be elevated 45 Degrees, and be ſituated ſo, that a vertical Plane, paſſing through its Center, is perpendicular to the Plane of the Arches, *i. e.* to a vertical Circle; then, if the ſhadow be projected on the Floor, or any horizontal Plane, it will be a Circle; for the Rays cut the Floor, in the ſame Angle, *viz.* 45 Degrees.

EXAMPLE VIII.

Plate XLV. *To project the Shadows of Cylinders, on a horizontal Plane, lying along; another upright, so situated, that its Shadow crosses one of the former.*

Fig. 29. The Bases of the Cylinder (X and Y) being inclined to the Picture, their representations are Ellipses, whose Shadows, may be thus projected.

No. 2. Let A B C D be the representation of a Square, circumscribing a Circle, whose sides are parallel and perpendicular to the Horizon. Draw the Diagonals A C and B D, also the two Diameters a b and c d, parallel to the sides of the Square; and, from the Points e and f, draw perpendiculars to the Base, A D.

Find the Shadow of the Square A B C D (by Prob. 2. and 3.) and draw the Diagonals, A*C* and D*B*, draw d S, also g S and h S, cuting the Diagonals; and through their Intersection, draw *b a*, to the Vanishing Point of A D, &c.

Then, if a Curve be described through the eight Points, *a*, *e*, d, *f*, *b*, *g*, *c*, and *d*, it will be the true representation of the Shadow of the Circle, a c b d.

Or, because there is but half its Shadow required, proceed thus.

Draw a b perpendicular, through its Center; also, c d and e f parallel to a b.

Find the Shadows of a, c, e, &c. (Prob. 2. Case 3.) and describe the Curve *a e b*.

Then, the Sides of the Cylinder being parallel to the Plane of projection, draw *c* V, a Tangent to the Curve; and, the Shadow of the other End being described by the same means, compleats the Shadow of that Cylinder.

Fig. 30. Second. The Shadow of the Cylinder Z, on account of its situation, is but little seen. Its Base is turned from the Light, in the other it was illumined, so that, 'tis the other edge of the Circle which projects the Shadow.

Third. The Sides of the upright Cylinder being perpendicular to the Horizon, draw A S, a Tangent to its Base, at A, cuting a V, the seat of the Side be, on the Ground, of the Cylinder Z, at *A*; from which Point, draw *A b* perpendicular, cuting be (the greatest projecture of the Cylinder Z) at *b*.

But, on account of the inclination of the Rays of Light, the Shadow of the Side A E will be continued on the Ground, till it falls into that of the other, at *a*; and it is continued down the side, from *b* towards *a*, till it is lost, insensibly, in the shade of the other Cylinder.

Take as many Points, b, c, d, in the curve of the Base, of the Cylinder Z, as are necessary; and others, answering to them, that is, of equal height, perspectively, at B, C, D, on the Side, A E, of the upright Cylinder; from all which, draw Lines to S, cuting others, drawn from the corresponding Points b, c, d, to the Vanishing Point V, at *b*, *c*, *d*, &c. through which, a Curve being described, will be the Shadow of the Side A E, on the Cylinder Z.

For, the Side of the Cylinder (A E) projecting the Shadow, is perpendicular to the Plane of projection; wherefore, the Plane of Shade, occasioned by that Line, vanishes in ⊙ S; and, all horizontal Lines (in that Plane) from the Points A, B, C, D, &c. vanish at S; and, consequently, they must cut others, b*b*, &c. drawn in the Side of the Cylinder Z, being of equal height. Therefore, *a b c d* is the section of the Plane of Shade, with the surface of the Cylinder Z, made by the Side, A E, of the upright Cylinder; and consequently, *a b c d* is its Shadow on that Surface.

The upper Base (E F) of the upright Cylinder, is above the Eye, and, being parallel to the Plane of projection, its Shadow is a Circle; wherefore, having obtained the Shadow of any Diameter, as e f, of E F, by drawing E ⊙ and F ⊙; the former cuting A S, at e, and drawing e f to the Vanishing Point of E F; then, the Shadow being a Circle, and projected on a Plane, not parallel to the Picture, its representation is an Ellipsis (Theo. 2. Sect. 5. Book I.)

Describe the representation of a Circle, whose Diameter found is e f (Prob. 2. Sect. 8.) and from G, draw a Tangent, tending to S, the Shadow of the other Side of the upright Cylinder, which also cuts the other in the same manner, and compleats the Shadow, as much as can be seen. Let it be observed, that the Shadow, on the Cylinder Z, is darkest, where the Light is strongest.

EXAMPLE IX.

To project the Shadows of Right Lines, on a convex cylindrical Surface.

Let AK be a Right Line, any how ſituated, in reſpect of the Cylinder X; its Vaniſhing Point is V. Its Shadow on the Cylinder is required; ⊙ being the tranſprojected Image of the Luminary. Fig. 31.

Draw V⊙, cuting the Horizontal Line at J, the Vaniſhing Point of the Shadow on the Ground. Draw AJ, cuting the Baſe of the Cylinder at a; and, through a, draw ⊙*A*, cuting AK at *A*.

Aa, on the Ground, is the Shadow of A*A*, part of the Line AK; beyond which, its Shadow is projected on the Cylinder.

Having obtained S, the Seat of the Luminary, (Prob. 2.) alſo, R, the Vaniſhing Point of the Seat of AK, on the Ground, draw AR; and, from various Points, B, C, &c. (in AK) at diſcretion, beyond *A*, draw B*B*, C*C*, &c. perpendicular; from all which Points, *B*, *C*, &c. ſo obtained, draw Lines to S, cuting the Baſe of the Cylinder, at b, c, &c. from which, draw perpendicular up the Cylinder; and from the ſeveral Points, B, C, &c. draw lines to ⊙, cuting the correſponding Perpendiculars, at *b*, *c*, *d*, &c. through all which, a Curve, a*bcdef*, being deſcribed is the Shadow of *AF*, part of AK, on the cylindrical Surface.

The remaining part of its Shadow, falls on the Ground, behind the Cylinder.

SECOND. FH, on the top of the Cylinder, is a Square, which may repreſent the Abacus of the Tuſcan Capital; to project its Shadow on the Cylinder and Floor.

Take as many Points f, g, h, &c. as are neceſſary, in the lower Lines, which caſt their Shadows on the Cylinder; from all which, draw to S; alſo, from the Angle G, cuting the upper Baſe, or Curve, at 1, 2, 3, &c.

Draw perpendiculars from each Point, down the Cylinder, which are cut, by Rays from each Point, f, g, &c. to ⊙, at *f*, *g*, &c. the Angle G is projected to *G*. The Ray k⊙, touches the Cylinder, at *k*, whence, it is projected to *K*, on the Floor.

Curve Lines, drawn through *Gfgh*, and *Gik*, are the Shadows of the Lines Gk and Gl on the Cylinder; kF is projected on the Floor.

Draw Sa, a Tangent at the Point a, cuting the Ray, from k at *K*; a*K* is the Shadow of the ſide of the Cylinder on the Floor.

Draw *KF* to the Vaniſhing Point of GF, and F⊙, cuting it at *F*, the Shadow of the Angle F; draw *F*L, to the Vaniſhing Point of GH, and FI, the upper Line, on the other ſide, which caſts the Shadow; I⊙ cuts it at *I*; through which, draw *IM*, tending to the vaniſhing Point of HI, and compleats the Shadow on the Floor.

THIRD. NO is a Line projecting from the Cylinder, at pleaſure; let P be its Vaniſhing Point; to project its Shadow, on the Cylinder.

Draw PQ, perpendicular, cuting the horizontal vaniſhing Line, in the vaniſhing Point of the Seat of NO, on any horizontal Plane, imagined at diſcretion (above or below the Vaniſhing Line) cuting the Cylinder. Let *N*u be a ſection of ſuch a Plane with the Cylinder.

Draw Oo, perpendicular, cuting *N*u at o, and, through o, draw Qn, indefinite; from the extreme N, and from various other Points, as r, s, in NO, draw perpendicular, cuting Qn, at n, t, and u, from which, draw to S, cuting *N*u, at *N*, *t*, and *u*. Draw *Nn*, *tr*, and *us*, perpendicular, and N⊙, &c. cuting them at *n*, *r*, and *s*; through which Points, the Shadow of NO is deſcribed. The reſt is obvious, from the Figure.

EXAMPLE

Plate XLV. Fig. 32.

EXAMPLE X.

To project the Shadow of a Tuscan Base on the Floor, casually situated to the Picture.

⊙ is the transprojected Image, and S the Seat of the Luminary.

Draw AS and B⊙, cuting at *b*, and draw *b*V; C⊙, cuting *b*V, determines the Shadow of the edge of the Plinth, BC, provided nothing else interfered. But, the Torus, &c. and Shaft of the Column unite their Shadows with it.

For the Column, draw SB and SF tangents to its Base, on the Ground; from the points of contract, draw BD and FE, perpendicular; which determine where the total Shade, on the Shaft, begins. BD, and EF, are the Sides of the Column which project the Shadow, on the Ground.

Draw D⊙, and E⊙, cuting BS, and FS, at d and e, the Shadows of D and E. DE, is a Diameter of the Column; and de being drawn, tending to the same Vanishing Point, may be considered as its Shadow; on which, as a Diameter found, describe the representation of a Circle (Prob 3. Sect. 8.) or, of a Semicircle, dfe, only, which terminates the Shadow. The Column being supposed cut, by a horizontal Plane, through DE, its Shadow is, consequently, a Circle, being projected on a Plane to which it is parallel.

The Shadow of the Fillet g h, may be projected after the same manner; but, as so little of it is seen, it would be unnecessary trouble.

Draw a Tangent to its Base, at b, from S, and draw b g perpendicular, and g⊙, cuting bS, at g, the Shadow of the Point g. By the same means the Shadows of other Points, may be obtained.

The Shadow of the Torus is not easy to describe; seeing its Surface is continually varying, it is scarce possible to determine what part of it projects the Shadow; except the Point *d*, of the greatest Swell, determined, as the Fillet, by a perpendicular from c. No other Point in that Circle projects a Shadow, on this side; to determine, absolutely, what other Points do, is what I shall not attempt, being persuaded, my Readers will excuse me that trouble.

N. B. The Shadow of the Shaft falls, first, on the Cavetto, before it reaches the Ground; the Fillet on the Torus, and the Torus on the upper Plane of the Plinth; which the Figure describes.

The Shade on the Torus, and on the Column, &c. is more hard and edgy when the Sun shines on them, than otherwise; yet there is no edge defined; also, the Light is broader, and gradates more suddenly into the Shade; and the Reflections are much stronger; but that, I shall reserve to another Section.

EXAMPLE XI.

Fig. 33.

To project the Shadow of a Doric Capital, on a vertical Plane, whose Vanishing Line (V Y) *and Intersection* (A B) *with the Ground Plane are given.*

CD cuts the Plane at a; find its Shadow, c d (Prob. 4.) ⊙ᵛ being the Vanishing Point of the Rays, and G, of the Line; which, being joined and produced, cuts the Vanishing Line, V Y, at V, the Vanishing Point of the Shadow.

After the same manner, the Shadow d e, of D E, &c. may be obtained; and the whole Shadow, c d e f, of the square of the top of the Abacus, in which draw the Diagonals; and let the lesser Trapezium *a b c d*, be the supposed Shadow of the square of the Column, parallel to the former, and found by the same means; in which, the Ellipsis, inscribed, may be the Shadow of the top of the Column, in the Plane of the Abacus, from which draw Perpendiculars; or, rather (because of the swelling of the Column) somewhat diverging, and gently curved.

The Shadows of the Angles, g and h, are obtained as the Square of the top; and, if great accuracy was requisite, the square of the Astragal, i k may be also projected, in its true place. But, as nothing more of its Shadow is defined than what projects beyond the Column, it may be dispensed with, the Points, *i* and *k*, being determined, are sufficient.

Rays of Light, paſſing by the Ovolo to ⊙', may be ſome guide for deſcribing its Shadow; but, on account of the continual variation of its Surface, every way, 'tis not eaſy to determine at what Points they touch it.

Neither can the curve of the Shadow of fg, be deſcribed on the Ovolo, with certainty, as on a Cylinder, yet ſomewhat may be done towards it; for example; the Shadow of the point b is required.

In this proceſs, it is neceſſary to have the perſpective curve (*a e f*) of the ſeat of the ſmall Fillet, from which the Ovolo projects, on the under Plane (g'h) of the Abacus.

Then, draw bS, cuting it at *a*; draw *ab*, perpendicular, and deſcribe the curve *bc*, a ſection of the Ovolo, by the Plane of Shade; b⊙' being drawn, cuting that Curve, determines the Shadow of b, on the Ovolo, at *c*; which, becauſe it touches the Curve only, is the limit of the Shadow on the Ovolo.

The reſt of the Shadow of that Line, from b to g, falls on the vertical Plane.

After the ſame manner as many Points may be determined as the Student has patience for; and when done, 'tis a great chance if it be more exact than a judicious Perſon would deſcribe by gueſs; nevertheleſs it is uſeful, to give an Idea how ſuch Shadows may be projected, and what kind of curves they deſcribe.

EXAMPLE XII.

A vertical, concave, cylindrical Surface being ſo ſituated, that the Shadow of one part is projected on the other; how to determine that Shadow; alſo, of Right Lines any how ſituated to the Surface, and to the Luminary.

Let ABX be a Wall, incloſing a circular Area. ⊙' is the Vaniſhing Point, of the Rays of Light, and S, of the Shadows of Perpendiculars. Fig. 34.

Draw SC, a tangent to the Curve, which determines the point where the Shadow begins, whatever be the Altitude of the Luminary.

Draw AS, cuting the Curve on the Ground, at a; draw a b perpendicular, and B⊙', cuting it at b, the Shadow of B.

By the ſame means, the Shadows of as many Points (D, E, &c.) as are requiſite, may be found; through which, the Curve, Cdeb, may be deſcribed; which is the true Shadow of the Curve CDEB, on the concave Surface.

SECOND. Let FG be a piece of Timber, projecting perpendicularly from the Wall, whoſe Shadow is required; its Van. Point is where GF would cut SR produced.

Draw Ef perpendicular, either to the top or bottom of the Wall; and, through f, draw a Line to the Vaniſhing Point of FG, cuting a perpendicular from G. fg is the Seat of FG, on the Plane of the top, being horizontal.

From ſeveral Points, h, i, &c. draw perpendicular, cuting fg, at k, l, &c. from g, k, and l, draw lines to S, cuting the Curve of the Top, at m, n, o, from which, draw Perpendiculars, indefinite; and from the Points g, h, i, &c. draw to ⊙', cuting them, reſpectively, in the Points *g*, *h*, *i*, which determine the Shadows of thoſe Points on the Wall; through which, a Curve, being deſcribed, is the Shadow of the lower edge of the Timber. The reſt is evident.

HI is another piece, alſo perpendicular to the Wall, its Vaniſhing Point is Y. Its Shadow is deſcribed after the ſame manner; or by means of a ſection of the Wall (Hi) parallel to the Horizon. Draw IS, cuting it at h; from which, draw perpendicular, and I⊙' interſecting at K, which determines its length; the Shadow being ſo nearly perpendicular, it deviates but little from a Right Line.

If its Vaniſhing Point was at S, the Shadow would be perpendicular.

THIRD. LM is a piece of Timber, inclined to the Wall, its Vaniſhing Point is Q.

Draw Q⊙, cuting the Horizontal Line at P, the Vaniſhing Point of the Shadow on the Ground. Draw LP, cuting the Curve at the bottom, at p; through p, draw ⊙O, cuting LM, at O. Lp is the Shadow of LO, on the Ground.

Plate XLVI.

Draw MN, perpendicular; and LP, being drawn, LN is the Seat of LM, on the Ground. From ſeveral Points *a*, *b*, *c*, (between O and M) draw perpendicular to the Seat LN, at *d*, *e*, *f*; from which, draw to S, cuting the Curve at the bottom, at *g*, *h*, *i*, and from them draw perpendiculars up the Wall; alſo, from *a*, *b*, *c*, draw to ⊙, cuting them, reſpectively, at a, b, c; through which, a Curve being deſcribed is the Shadow of OM, on the concave Wall.

For the Shadow of each Perpendicular *ad*, &c. is projected firſt, from *d* to *g*; and *g*a, &c. is parallel to *ad*, &c. *ad* being parallel to the Wall.

EXAMPLE XIII.

To project the Shadows of Arches; or, the Shadow of the circular outline, on the inſide, cylindrical, and plane Surface.

This Example is the ſame, in every reſpect, as the firſt part of the laſt, except in the poſition of the Cylinder to the Picture; their poſition to the Horizon is of no ſignification.

Fig. 35.

FIRST; the Arch (X) is parallel to the Picture; C is the Center.

The Luminary being on this ſide, and ⊙ its Image; draw C⊙, indefinite.

Then, becauſe the plane of the Circle is parallel to the Picture, draw a Tangent to the Curve, at A, parallel to C⊙^{4}; alſo, from various Points B, D, &c. at diſcretion, draw parallel Ordinates, cuting the oppoſite ſide of the Arch, at b, d, &c. from all which, draw to C; and laſtly, draw B⊙, D⊙, &c. cuting bC, dC, &c. reſpectively, at *b*, *d*, &c. through which, a Curve being deſcribed, is the Shadow of all that part of the Circle, from A (where the Shadow begins) to G, where it falls into the perpendicular.

If from A, to *b*, there be too much ſpace, for, deſcribing the curve accurately, take another point (a) and project its Shadow to *a*, and more if neceſſary.

SCHOL. In this, the affinity to the former Example is obvious. The Right Lines, B*B*, D*D*, &c. tending to C, (the Center of the Picture) repreſent perpendiculars to the Baſes of the Cylinder; as, in the former they are perpendicular, being parallel to the Picture; alſo, the Lines Bb, Dd, &c. which are, in this Caſe, parallel to C⊙, in the former they vaniſh in S; becauſe they repreſent parallel Lines, in a Plane not parallel to the Picture; as, in this Caſe, they are parallel, being in a Plane parallel to the Picture. The Rays of Light, in both Caſes, tend to their Vaniſhing Point.

2. The Shadow, from A to *e*, notwithſtanding its apparent convexity towards the Front, is really concave, the ſame as the other; the convexity ariſes only from its poſition to the Eye, owing to the exterior Curve Abe; which, in the former, is concave, the Eye being between the two Baſes, nearly oppoſite to that part of the Surface on which the Shadow falls; in which poſition of the Eye, the true curve of the Shadow is ſeen.

Fig. 36.

Fig. 36. repreſents the ſame thing, inclined to the Picture; in which, the perpendiculars B*B*, &c. vaniſh in H, in the Horizontal Line HL, as before in the Center; conſequently, b*b*, d*d*, &c. vaniſh in the ſame Point.

Alſo Bb, Dd, &c. vaniſh at S, in the Vaniſhing Line of the Arch, where H⊙^{5} cuts it; H being the Vaniſhing Point of Right Lines perpendicular to the Plane of the Arch (Prob. 2.) The Rays of Light vaniſh at ⊙^{5}; conſequently, HS is the Vaniſhing Line of the Plane of Shade, for Lines perpendicular to the Plane of the Arch, and S is its Seat on that Plane.

Fig. 37.

Fig. 37. repreſents another Arch, with the Light flowing inwardly (as in Example 42. Book 3. of the Piazza.) In which Caſe, it is the exterior Curve, from A to F, which projects the Shadow, the Sun being on the other ſide of the Picture (in this Caſe) at S^2; and, G is the Vaniſhing Point of Perpendiculars, as before.

The Figure, having the ſame Letters of reference, explains the reſt.

Here, the curve of the Shadow is ſeen properly, that is, concave; and it is the ſame in both the other, in the Object, though repreſented convex.

In Example 42, of the Piazza, the Luminary is on this ſide of the Picture, although it may to ſome appear abſurd; but, the direction of the Shadows indicate it. It is indeed very much inclined, nearly approaching to parallel; that is, the Luminary is nearly in the Plane of the Picture.

EXAMPLE XIV.

To project the Shadow of the circular outline of a Nich, on the interior, ſpherical, and cylindrical Surface.

FIRST; let the Front of the Nich be parallel to the Picture.

C being the Center, draw C⊙, indefinite, the Interſection of a Plane paſſing through the Luminary, perpendicular to the Picture. Fig. 38.

Draw a Tangent to the Curve at A, parallel to C⊙, and ſeveral parallel Ordinates, B*B*, D*D*, &c. at diſcretion. On the Diameters B*B*, D*D*, and E*E*, deſcribe the repreſentations of Semicircles, in Planes whoſe Vaniſhing Line is C⊙ (Prob. 2. Sect. 8.) (CI is its Diſtance) which are ſections of the Head, parallel amongſt themſelves, and to their Plane of Shade.

As the Section, through F*F*, cuts the cylindrical part of the Nich, alſo; and that through G*G* entirely, thoſe Sections are not Semicircles; the former, where it cuts the Head, is circular, but where it cuts the Cylinder (which only is wanted) it is elliptical; and that through G*G* is a ſemi-Ellipſes, whoſe tranſverſe Diameter is G*G*, and its Conjugate, the Diameter of the Nich.

Deſcribe the repreſentations of ſuch Ellipſes; which being done, from B, D, &c. draw Rays to ⊙, cuting the Curves, reſpectively, at b, d, &c. through which, deſcribe the Curve A b d e f g, the Shadow of the exterior Curve (from A to G.)

Draw gh, perpendicular, the Shadow of the edge GH, to which it is parallel, and join Hh; which compleats the whole Shadow, in that ſituation of the Sun.

If the Luminary be leſs inclined to the Plane of the Nich, but in the ſame Plane of which CI is the Vaniſhing Line, its place being $\odot^2$; A*abdefgh* is the Shadow, which has but little convexity, being farther removed from the Curve of the Head, A*BE*.

SCHOL. This proceſs, though apparently troubleſome, is the ſhorteſt of any that has yet been publiſhed, and much more ſimple than by vertical Sections, as exhibited in Fig. 39;* in which, the ſeveral Points, B, D, &c. in the Arch, by means of perpendiculars to the right Line IK, at the bottom, have correſponding Points, *B*, *D*, &c.

Then (S, being the Seat of the Luminary) lines are drawn from *B*, *D*, &c. to S, cuting the curve of the bottom of the Nich, at b, d, &c. from which, perpendiculars are drawn to the Curve F G H, where the ſpherical Surface begins; through which, the repreſentations of parallel Sections are made (as in the Figure) a moſt difficult proceſs, as, each Curve is a different portion of a Circle. Right Lines from each Point, B, C, &c. tending to $\odot^3$, (as in the former Figure) give the Shadow of each Point in its reſpective Section; through which, the contour of the Shadow is deſcribed. Fig. 39.

Sir William Chambers (if Fame reports true) was the firſt who took notice, in any publication in England, of this ſeeming paradox, *viz.* a convex Shadow of a concave Line; which, ſince, has been moſt ſervilely copied, by almoſt every Architect, or other Artiſt, who had occaſion to repreſent a Nich, either in external or internal Deſigns. Yea, to ſuch an extravagant exceſs it is carried, that, in geometrical Sections, and where it is impoſſible that the Sun ſhould ever ſhine, almoſt every Nich is ſo ſhaded; which, I do maintain, is, in ſuch Caſe, moſt unnatural; nor is the Shadow really convex, but only apparently ſo; yet it is frequently exaggerated, prepoſterouſly.

But, I preſume that, in Sun-ſhine, it has not always the appearance of convexity; for, I have ſhewn (Fig. 38.) that, the leſs the Inclination of the Sun, to the Plane of the Front, the leſs convex is the Curve. There is, alſo, another circumſtance which occaſions a different appearance. Thoſe Authors, who have favoured the World with the method of deſcribing it, have always given it direct, with the Center of the Picture in the middle of the Nich, as in Fig. 38. But, certainly, in the Front of a Building, perſpectively delineated, the Eye cannot be

* In this Example, Fournier is moſt egregiouſly miſtaken; for, he makes all theſe vertical Sections paſs through B, the Vertex of the Nich, when it is evident, that they are parallel amongſt themſelves, and to the Plane of Shade of perpendicular Lines. By which means, the contour of his Shadow is more convex than it can ever poſſibly appear, in any ſituation of the Luminary, and of the Spectator.

Plate XLVII. oppoſite to Niches at each extreme. Suppoſe, then, the Eye obliquely ſituated to a Nich, in a Plane parallel to the Picture, on the ſame ſide with the Sun; I preſume, the convexity will not be ſo great, as in direct oppoſition: yet, it is cuſtomary to ſhade all alike, which is moſt palpably abſurd.

Fig. 39. repreſents a Nich ſo ſituated and ſhaded; the Luminary being on the left hand, its tranſprojected Image is, conſequently, on the right, at ⊙', the Center of the Picture is at S, and the Diſtance is SO.

Now, although the altitude of the Sun, in this, is leſs, and its Inclination greater, than in Fig. 38. yet, the contour of the Shadow is concave, where the other is convex; and this is ſolely owing to the ſituation of the Eye, in reſpect of the Nich; for, being ſeen directly oppoſite, it would appear more convex than the other.

I think, none have ventured to deſcribe this Shadow, when the Nich is in a Plane inclined to the Picture; which, in delineating, more frequently happens ſo than otherwiſe, if the Picture be properly ſituated. Yet, in ſuch Caſe, many, not thinking properly about it, would give the contour of the Shadow the ſame, which can never poſſibly happen, except when the Rays of Light are very much inclined to the Plane of the Nich.

Fig. 40. Fig. 40. is the repreſentation of a Nich, in a Plane inclined to the Picture; C is the Center, the Diſtance is CH, and ⊙ the Image of the Luminary.

The ſame Letters of reference indicate the proceſs the ſame, as already deſcribed, with only this difference; that, as, in the former, the Ordinates were parallel (being parallel to the Picture) ſo, in this, they vaniſh where C⊙ cuts the Vaniſhing Line of the Plane of the Nich.

The outline of the Nich being delineated, and ſeveral Points, in the hither ſide of the Arch, taken at diſcretion, draw the Ordinates, tending to their Vaniſhing Point, from which Point, draw the Tangent, at A, where the Shadow begins.

Then, repreſentations of Semicircles being deſcribed on thoſe Diameters (Prob. 3. Sect. 8.) and Rays drawn from the ſeveral Points, to ⊙, cuting them reſpectively, give the contour of the Shadow, as before; which, in this Caſe (as in the laſt Example) the Eye being oppoſite to the Surface where the Shadow is defined, is really concave*; and which, in other Points of view, would appear convex.

SCHOL. It muſt have been obſerved, that the Center and Diſtance of the Picture are not neceſſary in the projection of Shadows, except in determining the Vaniſhing Point of the Rays; but, in this Example, as the Vaniſhing Line C⊙ is of Planes perpendicular to the Picture, it neceſſarily paſſes through the Center (Theo. 4.) and, the repreſentations of the Sections, through the head of the Nich, could not be deſcribed without the Diſtance.

I have now, I preſume, furniſhed the Reader with Rules and Examples, for the projection of Shadows, by the Sun, ſufficient for any purpoſe almoſt whatever. The Shadows of ſtreight Mouldings, on Planes, are Right Lines, ſave the contour of the Profile; which, in Mouldings above the Horizon, in Cornices, &c. are almoſt wholly ſhaded by the projecting Fillets, &c. and for circular Mouldings, or others below the Eye, the projection of the Shadows of Circles, on plane or other Surfaces (as in this laſt Section) contain all that can be done in ſuch Subjects.

He who has well digeſted what is there advanced, will never be much at a loſs, in the moſt difficult Caſes (where it is poſſible to project the Shadow, at all, by Rule) in complex, finiſhed Subjects; of which, Examples may be ſeen, throughout the Work, particularly in the Frontiſpiece; that being intended as a general Leſſon. Yet, in that, I have deſignedly deviated from the rigid obſervance of mathematical exactneſs; which, as I have obſerved in the Preface, does not always produce the beſt effect of Light and Shade. For, Example.

According to the general direction of the Rays of Light given, in that Picture, and the Shadows of perpendicular Lines, it muſt be obvious, that, as the Columns,

* As the Line which projects the Shadow is concave, the Syſtem of Rays, projecting it, form a cylindrical Surface; conſequently, the Shadow is concave, towards the Eye, on whatever Surface it falls.

in the Front, ſtand off, detached, their Shadows would cover more than the whole ſpace between them, on the Wall; which, I am perſuaded, would make that part appear heavy, and too hollow, as receding more than it really does. A little Light is alſo introduced on the circular Colonade, on the right-hand, in order to relieve it from the Building; although it would be wholly in Shade from the Building, according to the ſituation and altitude of the Luminary, the height, magnitude, and diſtance of the Object, occaſioning the Shade. Such liberties may freely be taken, where it manifeſtly produces a better general Effect; and, when it does not claſh with, and evidently contradict the invariable Law of Nature.

In that Piece, as in moſt other in this Work, the Luminary is on this ſide of the Picture; and though, in general, it is productive of the beſt effects, yet there may ſometimes be reaſons for ſuppoſing it on the other ſide, or in the Plane of the Picture, as in Plate 27, of Chelſea College. If that Piece had been ſhaded on the ſuppoſition of the Sun being on this ſide, either it muſt be much inclined, and conſequently the Vaniſhing Point at a great Diſtance, or every Face, in that Object, would be illumined, being on the right-hand; or, being on the left, the whole Front would be immerſed in Shade; with very little Light, in the Picture; but on the Roof, and end of the Building. Beſides, from the ſituation of that Building, the Sun can never ſhine on both, the Front and hither End; and I think that circumſtance ſhould always be attended to, when a real Object is repreſented, which is the ſole reaſon why the Queen's Palace (Plate 28.) is ſhaded from the left; for I am of opinion, that it would have a better Effect being ſhaded from the other hand; but, as the Sun cannot poſſibly be on the right-hand, it has certainly the moſt natural Effect.

Although there be no Face of that Object, as there repreſented, but what receives Light, immediately from the Sun, ſave one Face of the Library; yet, as ſome Faces are more oppoſed to the Luminary than others, there are various Teints, which diſtinguiſh one Plane from another, though not ſo ſtrongly. For, according as the Rays of Light are leſs or more inclined to the Face of any Object, it may have all the variety of Teints, from the ſtrongeſt Light to total Shade; and till the inclination is ſuch, that they are nearly parallel to the Plane, it is ſtill illumined, though in a very ſmall degree; and, when they become parallel, that is, when the Sun is in the continuation of any Plane, it is as much deprived of Light, as if it was, in reality, on the other ſide; and perhaps more ſo, as it then receives the leaſt advantage from Reflection.

SECTION V.

Of SHADOWS projected by a Torch or Candle.

IF the Theory of Shadows, in general, (Sect. 2nd.) and the Practice by Sunſhine be well underſtood, little need be ſaid reſpecting Candle-light Shadows, for in reality there is no difference, except in the diſtance of the luminous Body, or Point; which, on account of the ſhort Diſtance, and being ſituated amidſt various Objects, its Light is diffuſed in all directions; as it is from the Sun, reſpecting the whole Syſtem of Planets: alſo, in reſpect of any ſingle Object, of the ſame magnitude, or nearly, as the Flame, the Rays may be conſidered as parallel.

In projecting Shadows by Candle-light, there is but one Caſe, or ſituation of the luminous Point, in reſpect of the Picture; for, it is always ſuppoſed on the other ſide. In reality, it is an Object delineated in the Picture; and, when determined,

Plate XLVI.

mined, it is the Center of the Rays of Light; from which, the Shadows of other Objects are projected all around, in every reſpect the ſame as by the Sun, in that Caſe. Therefore, the Theory being the ſame, and the Practice having but little variation, few Examples will ſuffice.

I ſhall not ſuppoſe it neceſſary to ſhew how to find the repreſentation of the Flame of the Candle, the common Center of the Rays of Light, which is the ſame as finding any other Point, in any Object whatever.

The Shadows of Right Lines, perpendiculars, to any Plane, tend to the Seat of the luminous Point on that Plane; which, in this Caſe, is finite. In reſpect of the Sun, being ſuppoſed at an infinite Diſtance, it is, conſequently, in the Vaniſhing Line of the Plane; but in this Caſe, it muſt be found on the Plane of projection; which being found, the Shadows are readily determined.

Shadows projected by a luminous Point, at a ſhort Diſtance, it is manifeſt muſt neceſſarily be larger than the Object; ſeeing that, all the Rays diverge from that Point, the ſame as from the Sun, when it appears in the Picture; which, on account of its immenſe Diſtance, the Rays have not ſo much divergency. But, the Objects whoſe Shadows are projected by them, are generally of much greater magnitude; the one being confined to ſmall Objects only, as Chairs, Tables, &c. within a Room, the other is of Buildings, &c. external.

PROBLEM I.

The repreſentation of a luminous Point, being given, and its Seat on the Ground, or other horizontal Plane, together with the Interſections of other Planes, with that Plane, whether they be perpendicular, parallel, or inclined to the Picture, or to the Horizon, or to both; to determine its Seat on the other Planes, and the Shadows of Right Lines perpendicular to them.

Fig. 41. Let AB be the Interſection of a vertical Plane with the Ground Plane, parallel to the Picture, and BD of a vertical Plane perpendicular to the Picture. Alſo, let BD be the Interſection of the two vertical Planes; and DE, DH, EI, EF, &c. Interſections of various Planes with each other.

In ſhort, let LABG, repreſent a Floor, GBDH a vertical Plane, HDEI the Cieling, and IEFK an inclined Cieling, as in a Garret; AFEDB is the farther End of the Room, parallel to the Picture, all the other Planes are perpendicular to it. O is the repreſentation of the luminous Point (as the Flame of a Candle) and S is its Seat on the Floor.

First. To find the Seat of the luminous Point, on any of the other Planes.

OS, being perpendicular to the Floor, conſequently parallel to the Picture, imagine a Plane paſſing through OS, parallel to the Picture, cuting all the Planes (ſave AFEDB) perpendicularly; the Section of which, with them, is GHIKLG.

Now, becauſe the luminous Point is in that Plane, and it is perpendicular to the Planes BDHG, &c. a Perpendicular from O, to each Plane, muſt neceſſarily be in that imaginary Plane; and conſequently, muſt cut each Plane, in its Interſection with the Plane; wherefore, Perpendiculars (Of) to each Interſection, GH, HI, IK, &c. give the Seat (f) of the luminous Point, on each Plane.

The proceſs is ſo very obvious, that I ſhall avoid a further deſcription.

To find its Seat, on the Plane AFEDB; draw SC, cuting the Interſection, AB, at r; draw rs, perpendicular, and OC, cuting it at s, the Seat of O on that Plane. For, OC repreſents a perpendicular to the Plane, † cuting it at s.

† Cor. to Theo. 4.

Secondly. To find the Shadows of Right Lines perpendicular to thoſe Planes.

Through the Point *B*, or b, where the Perpendicular *AB*, or ab, cuts the Plane, draw S*B*, indefinite; and, through the extreme *A*, or a, draw O*C*, or Oc, cuting it at *C*, or c, the Shadow of the Point *A*, or a.

Conſequently, *BC*, or bc, is the Shadow of *AB*, or a b.

This proceſs, it is obvious, is the ſame in each Plane; and, changing S for ſ, the deſcription, given, ſerves for all alike.

The Parallelopiped, on which the Candle ſtands, is right angled; wherefore, *AB*, &c. are perpendicular to the Floor; by which means, the Shadows, *C* and c of the Angles, *A*, &c. being obtained, *Cc* and *Cc* are the Shadows of Lines parallel to the Floor, as in the 3rd Problem.

The Seat of the Light, on the upper Face of the Parallelopiped, is in the middle of the foot of the Candleſtick, which directs the Shadow of *ab*.

In the other Block, the ſide *AB*, perpendicular to the Floor, projects a Shadow on the Floor, to *D*, in the Interſection B*D*; from which it is caſt on the Wall, parallel to itſelf. The other Side, which is perpendicular to the Wall, is produced till it cuts the Wall, at *B*, through which Point, draw ſ*C*, and O*C* through A, interſecting at *C*; or, till it cuts the perpendicular Shadow, *DC*.

The Shadow of the piece of Timber (X) projecting perpendicularly from the Wall, is managed as the Lines, ſimply; in which it is obvious, that, as the Shadows of the Sides tend to ſ, they are diverging from that Point. The Shadows of ad and de are parallel to them, reſpectively; wherefore, *de* has the ſame Vaniſhing Point as the Original Line (de) projecting the Shadow.

Perpendiculars to the horizontal Cieling are in the ſame poſition as on the Floor; to the vertical Planes they are horizontal Lines; and to the inclined Cieling, IEFK, they are (as in all other) perpendicular to any Right Line, drawn in the Plane, through the Point, in which the Perpendicular cuts the Plane.

The Perpendicular *ab*, to that Plane, is ſo ſituated, that the Eye is in the Plane of Shade; wherefore, its Shadow cannot be determined as the reſt, being in a continuation of the Line, but may be thus found.

Through the Seat, ſ, of the Light on that Plane, draw ſ*B*, at pleaſure; and, through *b*, where *ab* cuts the Plane, draw *bB*, perpendicular to *ab*, cuting ſ*B*, at *B*, and draw *Bd*, parallel to *ab*, and *ad* to *bB*. Then, through *d*, draw O*e*, cuting ſ*B* produced, at *e*, and draw *ce* parallel to *bB*; which determines, *bc*, for the Shadow of *ab*, which could not be determined by the Ray O*c*.

SECONDLY. When the Plane is vertical, and inclined to the Picture. Fig. 42.

Let AB, be the Interſection of a vertical Plane, with the Floor, and D its Vaniſhing Point. The Center of the Picture is C; O is the luminous Point, and S, its Seat on the Floor.

To determine its Seat on the vertical Plane, draw CE, perpendicular, and equal to the Diſtance of the Picture. Join DE, and draw EF, perpendicular to DE, cuting the Horizontal Line at F; the Van. Point of perpendiculars to the Plane.

Then, draw SF, cuting AB, at r, and rſ, perpendicular, cuting OF at ſ, the Seat of O, on that Plane; for, Oſ, repreſents a Perpendicular to the Plane.

Let ab, be a Line, perpendicular to the vertical Plane, its Vaniſhing Point is F.

To project its Shadow on the Plane; draw ſb, through the Point b, where it cuts the Plane, indefinite; and through a, draw Oc, cuting ſb, produced, at c.

bc, is the Shadow of a b, on the vertical Plane.

THIRDLY. When the Plane is inclined to the Horizon, and to the Picture.

Let AB, be the Interſection of a Plane, inclined to the Horizon and alſo to the Picture; C is the Center, and D the Vaniſhing Point of AB. Fig. 43.

O is the luminous Point, and S its Seat, on the Floor.

Find the Vaniſhing Point, F, of Lines perpendicular to the Interſection AB, (Prob. 4.) and draw SF, cuting it at r, as in the laſt.

Then, the Inclination of the Plane to the Horizon being known, find its Vaniſhing Line, DG, (by 5th of the ſame) and the Vaniſhing Point H, of Lines perpendicular to the Plane, (Prob. 2. Sect. 12.) which will be that Point, where a

Plate XLVII. Perpendicular to the Vaniſhing Line, paſſing through C, the Center of the Picture, is cut by a Perpendicular from F. Alſo, find the Vaniſhing Point G, of Lines, in the Plane, perpendicular to the Interſection AB.

Draw rG, and OH, cuting it at ſ, the Seat of O, (the luminous Point) on that Plane. For, H is the Vaniſhing Point of Perpendiculars to the Plane, and rG repreſents a perpendicular to AB; wherefore OH cuts the Plane at ſ.

Becauſe, OS being perpendicular to the Horizon, and F is the Vaniſhing Point of Sr, perpendicular to AB; conſequently, GH, paſſing through F, alſo perpendicular (that is, parallel to OS) is the Vaniſhing Line of a Plane Ofr S, paſſing through OS, perpendicular to the inclined Plane; and H, being the Vaniſhing Point of Perpendiculars to the Plane, OH muſt cut it in rG, the common Section of the two Planes; therefore, ſ, where OH, cuts rG, is the Seat of the Point O, on the inclined Plane.

The Seat, ſ, of the luminous Point being found, and ab, a given Line perpendicular to the Plane, its Shadow is determined, by drawing ſc, through b, indefinite, and Oc, through a, interſecting at c; giving bc, the Shadow of ab, as in all the foregoing.

In this Problem is contained the whole Theory of projecting Shadows by Candle Light, in which there is not any difference to that of projecting them by the Sun, the Seat being obtained; in reſpect of Lines parallel or inclined to the Plane of projection, there is very little difference in the proceſs.

The chief difficulty in projecting Shadows, by Candle Light, is to find the Seat of the luminous Point; for which, the Rules given will anſwer in all Caſes, and in all poſitions of Planes, whatever, having the Interſection of the Plane, with ſome other, on which, the Seat is already determined, and the Inclination of the Planes, to each other, known.

PROBLEM II.

To project the Shadows of Right Lines, parallel to the Plane of projection, however ſituated to the Picture.

The Shadows of Right Lines, on Planes to which they are parallel, whether they are projected by the Sun or by Candle Light, it is obvious, are always parallel to the Lines projecting the Shadows.

Fig. 44. † 2. 6. El. For, however the luminous Point be ſituated, or at whatever Diſtance, the Rays OA, OB, &c. generating the Plane of Shade, are all cut proportionally by the Line and the Plane of projection †, conſequently, ſeeing the extreme Rays, OA, OB, or any other, are ſo cut, as OA is to AC, ſo is OB to BD, it neceſſarily follows, that CD is parallel to AB (Caſe 2. 2. 6. El.)

Wherefore, if the Line be parallel to the Picture, the Shadow is parallel on the Picture; that is, to the Vaniſhing Line of the Plane of projection; and conſequently, however otherwiſe the Line be ſituated to the Picture, the Shadow, being parallel to the Line, has neceſſarily the ſame Vaniſhing Point (Cor. Th. 3.)

Fig. 45. No. 1. Let ACD be a right angled Parallelopiped, having the Face AC parallel to the Picture; the Sides, CD, &c. are conſequently perpendicular to the Picture, and parallel to the Floor, on which the Shadow is projected.

Let O, be the luminous Point, and S its Seat, on the Floor.

Through A, draw Sb, indefinite, and OB cuting it at b; and, becauſe BC is parallel to the Floor, and to the Picture, its Shadow is parallel to the Line.

Wherefore, draw bc, parallel to BC, and Oc (through C) cuting it at c.

Then, becauſe CD is perpendicular to the Picture, and parallel to the Plane of projection, C (the Center) is the Vaniſhing Point of its Shadow; wherefore, draw cC, and OC, thro' D, which compleats the Shadow, as much as can be ſeen.

No 2. 2. EH is another Parallelopiped, obliquely ſituated to the Picture; having one Face ſo ſituated, that the luminous Point, is in its Plane, produced.

Produce SE, and, through F, draw Of, cuting it at f, the Shadow of f.

Then,

Then, K being the Vaniſhing Point of FG, draw fK, and Og (through G) cuting it at g; fg is the Shadow of FG, to which it is parallel, having the ſame Vaniſhing Point. And, becauſe GH vaniſhes at I, draw gI, and Oh (through H) cuting it; and laſtly, draw hS, which compleats the Shadow Efgh.

To project the Shadows of Right Lines parallel to any other Plane, however ſituated to the Picture, has nothing more of variety in it; the ſeat of the Luminary being obtained on the Plane.

3. The Shadows of the Shelves, LM, &c. at the farther End of the Room, may be thus obtained, being parallel to the Picture. No. 3.

Through the feet of the upright pieces, R, and T, draw right Lines, from S, cuting the Interſection of the Wall with the Floor, at r, and the Door, at t; from which, draw Lines parallel to LR, and MT, indefinite. From O, draw through the extremes of the Shelves, L, N, &c. cuting the Wall, in the Shadow rl, at l, n, &c. from which Points, draw parallel to the Shelves, cuting the edge of the upright piece, at *l*, *n*, &c. and join, N*n*, O*o*, &c. as in the Figure.

4. The Shelves, againſt the ſide Wall, may have their Shadows projected after the ſame manner, with only this difference; that, the Shadows, inſtead of being drawn parallel to the edges of the Shelves, tend to the ſame Vaniſhing Point. No. 4.

As ab, the Shadow of AB, and cd of CD, tending to the Center.

5. For the Shadow of the ſingle Shelf on the other ſide, find the Seat, ſ, of the Light on the Wall (Prob. 1.) then, ab being perpendicular to the Plane, draw ſa, indefinite, and Ob, being produced, cuts it at *b*. Draw *b*C, cuting the Angle of the Room, at *c*, and join *cd*. No. 5.

After the ſame manner, the breadth of the Shadows of the Shelves on the other Side are determined.

6. For, the hanging Frame and Shelves, above, proceed thus. Fig. 45. No. 6.

Find *S*, the Seat of the Light on the Cieling; *AB* and *CD* being perpendicular, draw *SA*, *SC*, indefinite; and, through *B*, and *D*, draw O*B*, O*D*, cuting them at *b*, and *d*. Draw *bd*, the Shadow of *BD*; and, from the Center of the Picture (being the Vaniſhing Point of *BE* and *DF*) draw *be*, and *df*, the Shadows of *BE* and *DF*, on the Cieling.

Laſtly; through *G* or *H*, draw O*g*, or O*h*, cuting *be*, or *df*, at *g* or *h*; through which, draw *gh*, parallel to *GH* (being parallel to the Picture) and, through the extremes of the Shelves, draw Lines from O, cuting *gh* at *a* and *c*.

The reſt, for that Shelf, is too obvious to need further explanation.

The other, notwithſtanding its inclination, is performed by the ſame means; only, inſtead of being parallel, the Shadow tends to the ſame vaniſhing Point, as the Shelf; or, both extremes being determined, the vaniſhing Point is uſeleſs. Their widths are determined after the ſame manner.

7. The Shadow of the Stool, on the Counter and Floor, is thus projected. No. 7.

From S, the Seat of the luminous Point, draw Right Lines through the feet of the Frame, a, b, &c. till they cut the Interſection, AC, of the Counter with the Floor, at *a*, *b*, &c. from which Points, draw Perpendiculars; which are the indefinite Shadows of the Frame, on the front of the Counter; and if, through the Angles, e, f, &c. Lines are drawn from O, cuting the Perpendiculars, at *e*, *f*, &c. reſpectively, their lengths are determined.

The Shadow of the croſs Frame is determined, by drawing Lines, from O, through their extremes, cuting the Shadows of the Legs, at *i*, *k*, &c.

Plate XLVII. No. 8.

8. The Box on which the Candle ſtands has its upper Face, only, illumined, and conſequently its Shadow, only, is projected, on the Floor, &c. by drawing Lines, from S, through the Seat of each Angle, on the Floor; and Om, O , being produced, till they cut them, at *m* and *n*. Op, cuts the other Box, in the perpendicular *r p*, and if *n o* be drawn, to the Vaniſhing Point of n p, till it cuts the Box, at *o*; and the point *q* being obtained, by the ſame means, *op* and *pq*, being joined, compleat its Shadow.

PROBLEM III.

To project the Shadows of Right Lines, any how inclined to the Plane of projection, and to the Picture.

This Problem I ſhall reſolve alſo, by Examples, as the foregoing.

The common Center of the Rays of Light, that is, the place of the luminous Point being determined, the proceſs is the ſame as by-Sun-ſhine, having found the Vaniſhing Line of the Plane of Shade; which paſſes through the Vaniſhing Point of the Line, but not through the luminous Point.

Fig. 46. ABCD repreſents a high pair of Steps, and U, X, Y, Z, a large folding Screen, ſo ſituated, in reſpect of the Light, at O, that the Shadow of the inclined Sides, (AB and CD) of the Steps, fall on the Screen.

Their ſituation, in reſpect of the Picture, and alſo their poſition to each other, together with the Seat of the Light, on the Floor, are determined as in No. 2.

No. 2. Let U, X, &c. be the Seats of the Leaves of the Screen; A, D, E, F, of the feet of the Steps, and, S the Seat of the Light; BC, is the Seat of the top of the Steps, which being contracted, AB and CD, are the Seats of the inclination of the Sides, on the Floor, which caſt their Shadows on the Screen; let their inclination to the Horizon be the Angle CDG, equal BA*G*.

Let V be the Vaniſhing Point of the Side, AB, of the Steps.

Having found S, the Seat of the Light, alſo F, the Vaniſhing Point of the Seat of AB, on the Floor, draw FS, indefinite; VO, being produced, cuts it at G.

Draw GA, and produce it to the Horizontal Line, cuting it at L, and draw VL, the Vaniſhing Line of the Plane of Shade, of AB.

AL cuts the Interſection, of the Leaf U with the Floor, at *a*; through H, the Vaniſhing Point of that Interſection, HI being drawn, perpendicular, is its Vaniſhing Line, which cuts VL at I; draw *a*I, cuting the Interſection of the Leaves, U and X, at b.

Then, where the Vaniſhing Line of the Leaf, X, cuts VL, at J, draw bJ, cuting the Interſection of the Leaves X and Y, at c.

The Leaf Y, is parallel to the Picture; wherefore, draw cd parallel to the Vaniſhing Line, VL, cuting the next Interſection at d; and, laſtly, draw dK, to the Point, K, where the Vaniſhing Line of the Leaf Z, cuts VL, which compleats the Shadow of AB, on the Screen.

The remainder of its Shadow falls behind the Screen, out of ſight.

The Shadow of the other Side, CD, whoſe Vaniſhing Point is R; falls only on the two Leaves, Y, and Z; and on the Wall, to g, moſtly out of ſight.

RO cuts the Floor at *G*, and *G*D determines the Vaniſhing Point, U, of the Shadow of CD, on the Floor; wherefore, RU is the Vaniſhing Line of the Plane of Shade, of CD; to which, the Shadow of that Line, on the Leaf Y is parallel; and, on the Leaf Z it tends to X in the Vaniſhing Line of the Leaf, as the Shadow of AB, on the ſame Leaf, to K; the Points where the Vaniſhing Line of the Leaf is cut, by the Vaniſhing Lines of the Plane of Shade of each Line (AB and CD) reſpectively.

The Shadow of the Supporter, CE, is projected firſt on the Floor, to f, where it cuts the Wall; and, if the Shadow of any other Point, in the Line, be projected on the Wall, as at g, then, fg is its Shadow on the Wall.

For the Steps; from O, Lines are drawn through the extremes *A*, *B*, *C*, &c. on either Side, till they cut the Floor, or leaves of the Screen, at *a*, *b*, *c*, &c. and from, *a*, *b*, &c. to the Vaniſhing Point of the Steps, on the Floor, and on the Leaves, being parallel to them. Otherwiſe, they are drawn to that Point in the Vaniſhing Line, of each Leaf, in which, the Vaniſhing Line of the Plane of Shade of the Step cuts it. Or, drawing through both extremes, of each Step, as *B* and *D*, &c. cuting the Shadows of both Sides, on the ſame Plane, the Vaniſhing Point is unneceſſary; but when it falls within bounds, it is the moſt correct.

In this Caſe (viz. of Lines inclined to the Plane of Projection) the Vaniſhing Line of the Plane of Shade is neceſſary, when the Shadow is projected on various Planes, but it is not ſo eaſily determined, as for Shadows projected by the Sun; nothing more being required than to draw a Right Line through the Vaniſhing Points, of the Line and of the Rays, becauſe its diſtance is ſuppoſed infinite; and conſequently, the ſame Vaniſhing Line of any Plane of Shade, ſerves for all Lines which are parallel. But, here, the Light being at a ſhort diſtance, it cannot poſſibly be in the Vaniſhing Line, which is at an infinite diſtance; except apparently ſo, when it happens to be ſo ſituated, in reſpect of the Eye.

That V L is the Vaniſhing Line of the Plane of Shade of A B, is manifeſt; and is determined either by means of the horizontal Vaniſhing Line (H L) or the vertical Vaniſhing Line (M N) of the Wall, W, on which the Shadow would be projected, the Screen being out of the way. For, A B, cuts the Wall, being prodced, at P, as the Floor at A; and V O repreſents a Line parallel to A B; which, is a Ray of Light in the ſame Plane with A B, cuting the Floor, at G, and the Wall, at Q.

Conſequently, G A, produced, is the Shadow of A B, on the Floor; and, P Q produced, is its Shadow on the Wall; the former cuts the Vaniſhing Line of the Floor, at L, the latter cuts the vertical Vaniſhing Line at M; both which, are in the ſame Right Line paſſing through V; and, ſince L and M are Vaniſhing Points of the Shadow, V L is, conſequently, the Vaniſhing Line of the Plane of Shade, occaſioned by the Line A B (Theorem 10.)

2. The Shadow of the Screen, on the Wall and Cieling, may be thus projected.

Through, *A*, *B*, &c. at the bottom of the Screen, draw S*A*, S*B*, &c. and produce them to the Interſection of the Wall (W) with the Floor, cuting it at 1, 2, &c. from which Points, draw Perpendiculars up the Wall; and, through the Angles *F*, *G*, &c. at the top, draw O*F*, O*G*, &c. cuting the Perpendiculars, correſponding with *AF*, &c. at *f*, *g*, &c. and draw *fg*, *gh*, &c.

But the Ray O*I*, cuts the Cieling; wherefore, having found ſ, the Seat of the Light on the Cieling, draw ſi, to that Point where the Perpendicular, from 4, cuts the Interſection of the Wall with the Cieling; and O*I*, produced, limits the Shadow, at *i*; let it be produced, alſo, till it cuts the Perpendicular at k, and join *h*k, cuting the Interſection at *j*, and join *ji*.

This proceſs would be indiſpenſibly neceſſary, provided the Lines, HI, &c. were not parallel to the Ceiling; but being ſo, they are drawn either parallel to the Line (as for *HI*) or to the ſame Vaniſhing Point (as for *IK*). *A* 1 *fghjikl*, are the extremes of the Shadow of the Screen, on the Floor, Wall, and Cieling.

By the ſame means, the Shadow of A B, may be projected on the Screen, thus.

Draw S*A*, S*B*, &c. cuting A F, the Seat of A B on the Floor, at n, o, p, and q; from which, draw Perpendiculars, cuting A B, at *A*, *B*, &c. through which Points, draw O*A*a, &c. cuting the correſponding angles of the Screen, at a, b, c, &c. which, being joined by Right Lines, give the ſame Shadow as before.

This proceſs, though ſhorter, is by no means ſo correct and maſterly; but, as it is performed in leſs room, it may be applied when the inclination is ſuch, that the Vaniſhing Points are very remote.

3. The Shadow of the Table, on which the Candle ſtands may be thus determined.

Having obtained the Seat (s) of the Light on the Table, and (S) on the Floor, through s, draw a b, parallel to the Horizon, alſo c d, and e f at pleaſure, cuting the Horizontal Line, at H, and, F; and through S, on the Floor, draw H *c*, and F *e*, indefinite; alſo *ab*, parallel to a b.

Then, Rays drawn, from O, through the extremes of thoſe Lines, *viz.* O a, O c, &c. (being produced) will cut the correſponding Lines in the Shadow of each, reſpectively, by which means as many points may be obtained as are neceſſary, for obtaining the true curve of the Shadow, which is a Circle, the Table being circular.

The

Plate XLVII.

The Candle ſtanding towards one edge of the Table, the Rays proceeding from it, around its Circumference, form a ſcalene, or oblique Cone; which being projected to the Floor (to which the Table is ſuppoſed parallel) the Section, thereon (which is its Shadow) is conſequently a Circle; and, being ſeen oblique, its Repreſentation is an Ellipſis (Theo. 2. Sect. 5.)

No. 4. 4. The Shadow of the concave edge of the hollow Cylinder (X) on the interior Surface, may be projected, in the following manner.

Find *s*, the ſeat of the Light on the Plane of its Baſe, and draw *s*a, *s*b, &c. at pleaſure, cuting the circumference on both ſides; from the Points, d, e, &c. draw Right Lines to the Vaniſhing Point of the ſides of the Cylinder, and draw Oa, Ob, &c. cuting them, reſpectively, at *a*, *b*, &c. through which, the contour of the Shadow may be deſcribed.

No. 5. 5. For the Shadow of the edge, of the conical Veſſel (W) on the interior Surface.

If Right Lines be drawn from *S*, the Seat of the Light, on the Plane of its upper Baſe, cuting it on both ſides, and making vertical Sections through them, down the Sides, within; then, draw Oa, &c. cuting the oppoſite, correſponding lines, at *a*, *b*, &c. through which, the Shadow may be deſcribed, on the Side.

Where they fall on the Bottom, join *cd*, &c. Ob, Oc, &c. cuts thoſe Lines, at *f*, *g*, &c. through which, the curve of the Shadow is deſcribed, thereon.

By taking ſeveral Points (g, h, &c.) in the exterior Curve, and finding their Seats on the Floor, its Shadow (*ghi*) may be deſcribed on the Floor.

No. 6. 6. The Shadows of Globes, it is not eaſy to deſcribe, with certainty.

If a Tangent be drawn, from the Light (O) to any part of its Surface (as a) and, through that Point, a Section, by a Plane, be deſcribed, perpendicular to the Axis, (OC), (which, I freely own, is not eaſy to do, being obliquely ſituated*) then, take as many Points in its Circumference as are neceſſary (a, b, d, &c.) and find their Seats, (*A*, *B*, *D*, &c.) on the Floor, or Cieling; and, through S, or *f*, the Seat of the Light, draw S*A*, S*B*. &c. and Oa, Ob, &c. cuting them, reſpectively, at *a*, *b*, *d*, &c. a Curve deſcribed through *a*, *b*, &c. will be an Ellipſis, the true Shadow of the Sphere; being an oblique ſection of the Cone of Rays.

No. 7. 7. The Shadow in the Nich is deſcribed as by Sunſhine, with little variation.

Find the Seat (*f*) of the Light on its Plane, (Prob. 1.) from which, draw the Ordinates, ab, cd, &c. on which Ordinates deſcribe Sections through the head of the Nich, perpendicular to the Plane, and draw Oa, Oc, &c. giving the Points *a*, *c*, &c. a Curve deſcribed through thoſe Points is the true contour of the Shadow, in the Head†. Draw *ef* perpendicular, and join *fg*.

The Shadow of the edge of the Veſſel, def, falls within the Nich; the reſt, with the Stool, falls on the Wall, the Legs being firſt projected on the Floor, through the Feet, by means of S, the Seat of the Light thereon.

* The Shadow of a Globe or Sphere, it is obvious, is no more than the Shadow of a plane Circle, which is not the full diameter of the Globe, becauſe, Tangents to a Sphere, from the ſame Point cannot touch both extremes of the ſame Diameter, ſeeing they would be parallel. The Circle projecting the Shadow is that, whoſe Circumference is the Baſe of the Cone of Rays.

† In this Example, the Light is ſo ſituated as to caſt very little Shadow into the Nich; but, if the Candle was more oblique to it, the contour of the Shadow would be nearly the ſame figure, the Light being below the head of the Nich; as a tangent to the curve of the Head would always touch it on that ſide of the Vertex towards the Light. Whereas, by Sunſhine, the Light being elevated above the Nich, a Tangent touches it on the other ſide, and gives a different figure of the Shadow. And, if the Candle be elevated above it, the contour of the Shadow would be nearly the ſame; the difference ariſing only from the ſeat of the Light, on the Plane of the Nich, which, in this is finite, in the other, at an infinite diſtance; and alſo from the Rays of Light, which in this are diverging, in the other they are converging, the Light being on this ſide of the Picture.

SECTION VI.

Of reflected Light; and of the reflected Images of Objects, on Water, and poliſhed, plane Surfaces; Keeping, &c.

Plate XLVIII.

IN treating on this Subject, it may be neceſſary to conſider, in the firſt place, what is meant by Reflection, ſimply, or conſidered abſtractedly.

REFLECTION, in a phyſical ſenſe, ſignifies a rebounding of Matter, by Percuſſion; i. e. when an elaſtic Body,* in motion, ſtrikes another Body, alſo elaſtic; it rebounds, from that other Body, in a different direction from that in which it was at firſt impelled; making an Angle, with its firſt direction, greater or leſs, according to the obliquity in which it ſtrikes the Surface of the other Body.

If a Globe ſtrikes a Plane, or the Surface of another Globe (or any Surface whatever) perpendicularly, it will rebound from the other Surface, perpendicularly, in direct oppoſition to its firſt, or incident motion; as if A falls, perpendicular, to the Plane EF, ſtriking it at B, it will rebound again, from B, towards A. But, it is found, from experiment, that if the incident motion be from C to B, oblique to the Plane, or other Surface, it will rebound, or be reflected from B towards D, alſo oblique; making the Angle ABD, with a perpendicular, at that Point (called the Angle of Reflection) equal to ABC (called the Angle of Incidence) or, which is the ſame thing, the Angle CBE, in which it inclines to the Plane, in its incident motion, is equal to DBF, in which it reflects from the Plane. And this is the ſame, in all poſitions of the Plane, whether horizontal, vertical, or inclined Hence, Light is ſaid to be reflected, from one Surface to another; and, probably, from that conſideration it is imagined to be material. Fig. 47.

Without taking Matter into conſideration, it is certain, that any Surface (not wholly opake) being oppoſed to a luminous Body, becomes illumined itſelf; and, in an inferior degree, illumines other Objects, in vicinity with it.

The firſt and grand inſtance, of which, is the Moon and other Planets; as they are more or leſs illumined, by the Sun, or rather, the more their illumined Surfaces are towards the Earth, the more the Earth is illumined; even to ſuch a degree (though but reflected) as to project Shadows, ſtrongly defined. The Caſe is perfectly ſimilar in reſpect of other Bodies, on the Earth. For, however any Surface be ſituated to the Sun, being illumined by its Light, that Surface illumines others, which are near it, more or leſs, according as that Surface is ſituated to the Sun, and as they are ſituated in reſpect of each other. Without which, Bodies, or the Surfaces of Bodies, which are not illumined, directly from the Sun or other luminous Body, would be ſo totally immerſed in Shade as not to be viſible, excepting their exterior Figure or Outline.

Now, admiting Rays of Light to be emited from any luminous Body, at S, and falling, directly, on another Body, or plane Surface, at A, they are ſaid to be reflected, directly, again, towards S; but, falling on it oblique, as at B, they are reflected towards D, making the Angle DBE equal SBA. Fig. 48.

Hence ariſes an objection to the Newtonian Syſtem, reſpecting the reflection of Light, from one Body to another, and from that other Body to the Eye; by which means, only, it is conjectured, that Objects, not illumined, directly from the Sun, or other Light, become viſible.

* By Elaſtic, in this place, is meant, ſimply, hard Bodies only, as Stones, Metals, &c. of which, ſome are more elaſtic than others. Lead, or pure Gold, yields to the ſtroke, and therefore, does not rebound like Tin, or Tin like Copper, nor that like Iron, or hard Steel. So Clay will not rebound like Free Stone, nor that like Marble; becauſe, after Percuſſion, the more remote parts of Matter are ſtill in motion, till the Body is compreſſed by the Stroke; which deprives it, either wholly or in part, of Motion; and conſequently, it rebounds with leſs force, or not at all.

Plate XLVIII. Fig. 48.

Let AB be a Plane Surface, directly oppoſed to the Sun; and, ſuppoſe AB the utmoſt limits of the Plane. Let X be an Object, having one plane Face (CD) parallel to AB; and ſuppoſe no other Object, or Body, near.

Now, the diſtance of the Sun being ſuppoſed infinite, its Rays, conſequently, fall perpendicularly on AB, in every part.

Let SA, *S*B, &c. repreſent Rays of Light from the Sun. Then, according to the Maxim laid down, they are reflected, directly back again, towards S (which, being material, is ſomewhat repugnant to our Ideas of Matter; ſeeing that, it is continually flowing, with equal and unremited velocity, from the Sun to AB, how can it return back, in direct oppoſition?) conſequently, the Object X being ſituated ſo, that none of the reflected Rays can poſſibly fall on the Surface CD, that Surface is wholly inviſible, to any Eye at E, or *E*. *Query*; whether it would be viſible or not?

I am of opinion that it would be clearly viſible, to any Eye, on this Side CD; not only in reſpect of its Figure, but that, the Surface, CD, would be illumined, by means of Reflection, from AB; when, according to the general Maxim, it could not, ſeeing no Rays, from AB, are reflected to CD. Yet, I am fully perſuaded that it may be ſeen; not only, directly, at E, but alſo oblique, in any direction, as at *E*; although it is manifeſt, that the Rays, from AB, muſt be reflected oblique, to CD, and again to E, or *E*, in various Angles and Directions; and yet, the original Rays all fall perpendicularly on AB.

Now, if CD be ſeen, at all, it is manifeſt it muſt be illumined; and it is evident, that it cannot be ſo from the Sun, directly, conſequently, it muſt be from Reflection; and ſince no other Body is near, it muſt be from the Surface AB. Hence, then, it is manifeſt, that, Light is reflected in other directions, from one Object to another. How or by what means it is reflected, I will not attempt to enquire; but ſhall only make a few Obſervations, reſpecting the effects it produces, on Objects, ſo eſſential to the perfecting a Picture, or true Portrait of Nature.

It had, formerly, been cuſtomary, with many, to repreſent Objects, immerſed in Shade, ſo very obſcurely, as ſcarce to be diſtinguiſhable; whereas, it is not ſo in Nature. In clear Day-light, when the Sun does not ſhine out, we ſee Objects in their true Colours, and every part is diſtinct; but, when the Sun breaks out, and darts its Rays on thoſe parts which are oppoſite, the Colours are more intenſely vivid, occaſioned by the refulgent luſtre of the Sun-Beams. Nevertheleſs, thoſe parts which are prevented from receiving that additional Light, and appear to be in Shade, cannot poſſibly be deprived of what Light they had before, but muſt rather receive additional Light on them; the difference, then, can only ariſe, from the ſplendor of the ſurrounding Light, which dazzles the Eye, and renders thoſe parts obſcure, which, before, were diſtinctly ſeen. But, when out of the full glow of Sun-ſhine, we perceive every Object or parts of Objects, which are in Shade, as diſtinctly as before; and in the ſame Colours, though greatly different from thoſe on which the Sun ſhines.

Some, again, of late Years, run into the oppoſite extreme, and make too little diſtinction, between the fulleſt Light and the ſtrongeſt Shade; by which means, their Pieces look flat, and do not produce, on the Mind, a juſt Idea of what is intended. For, although it is certain, that, in Sunſhine, every part receives additional Light; yet, the contraſt is ſo ſtrong, from the brilliancy of the Colours, where the Rays fall, that, the other Parts appear as if deprived of Light, in compariſon with theſe; and, ſince that vivid luſtre cannot be given by Art, and Colour, the difference muſt be made by keeping the other parts ſomewhat under.

Reſpecting the degree of reflected Light on Objects, it is not eaſy to determine; that being more or leſs, according to the ſituation of the Object, in reſpect of others; alſo, according as that other Object is ſituated to the Light.

If one Object be ſo ſituated to another, that the Surface of one, being directly oppoſed to the Light, is alſo much oppoſed to the ſhaded ſide of the other Object; the

the reflected Light, on the other, will be ſtronger than when they are more obliquely ſituated; either the one, in reſpect of the Light, or the other, in reſpect of the illumined Surface.

For example. The Cylinder, AB, being ſituated near the Wall X, on which the Light is direct; the ſhaded Edge (AB) has a much ſtronger reflection, than if the Plane (X) was more inclined to the Light; or, being removed, and no other reflecting Surface near, there would be no Reflection on the Column, in this poſition of the Light, ſave what is received obliquely, from the Ground. Fig. 49.

Reſpecting plane Surfaces. We frequently ſee (in Prints and Drawings) the ſhaded face of a Pedeſtal, &c. very dark, at the hither edge, and gradated towards the other, ſo, as to give it the appearance of concavity; which cannot be, when no other Object is near. But, when one part of an Object projects from the other, as the Plane X from Z, the Plane Z, being much oppoſed to a ſtrong Light, and conſequently the Shadow of the Plane X, on Z, is very narrow; then, the Light, reflected from Z to X, will be ſtrongeſt where it joins to Z, and gradually leſſened, towards the other edge; but not in ſo extravagant a degree, as may be ſeen in various Prints; particularly in Mr. Kirby's Perſpective of Architecture. Fig. 50.

Now, the reaſon for this is extremely obvious; becauſe of the vicinity of the part which joins to Z, the Light reflected on it from Z is ſtronger than on thoſe parts which are more remote; and conſequently, the reflected Light is more languid on the remote parts. But, if the Light was very oblique on Z, ſo as to caſt a great breadth of Shadow, on the Plane; in that Caſe, the Light, reflected to the Plane X, will be more faint; inſomuch that, there is barely a diſtinction between the Shadow, and the Object occaſioning that Shadow; which there always ſhould be, in ſome degree, the Shadow being darkeſt; becauſe, the Shadow cannot receive any advantage from Reflection, which the Object does.

In reſpect of Mouldings; the edge of the Facia, AB, being in full Light, caſts its Shadow on the Cavetto below; conſequently, the Mouldings cannot have their effect from the direct Light, but from Reflection, only. On the returning Moulding; the under Facia or Planceer (BC) by means of Reflection, from below, is brighter than the vertical Facia, over it; alſo the vertical Face of each Fillet, is darker than the horizontal; which in other caſes, having the Light directly on them, is conſequently brighter.

In Figure 4, Plate 40, the great projecture of the Corona (X) deprives the Mouldings, below, of Light, entirely, from above, which are diſtinguiſhable only by means of Reflection, from below; conſequently, the Ovolo is brighteſt towards the lower edge, and the Cavetto, towards the upper; and, they are more or leſs ſo, according as they are oppoſed to ſome illumined Body, reflecting Light on them. It may, therefore, be made a general Rule, with very little exception, that, the effect of reflected Light on Objects is reverſe to the direct Light; particular regard being had to the ſituation of the Object which occaſions the reflection. For, certainly, in whatever degree an Object is illumined, or from whatever cauſe, whether directly or reflected, the effect of it, on the Object, is the ſame, in proportion to the degree of Light. But when (as is frequent) various Objects reflect Light on the ſame Object, no one being particularly predominant; the Light by that means is ſcattered and confuſed, ſo that, there is little or no Shade on the Object; inſomuch that, the parts can ſcarce be diſtinguiſhed, one from another. The ſame effect is produced from various luminous Bodies, variouſly ſituated to an Object, or from various inlets of Light, on Objects, each deſtroying the effect occaſioned by the other.

In a Nich (ſee Plate 43.) the Shadow is ſtrongeſt at the edge or outline, and gradually ſoftened towards that ſide of the Nich, which occaſions the Shadow; owing to the ſtrong Reflection from the other Side, on which the Light falls, direct.

Plate XLVIII. The Shade (without Sun-ſhine) on a convex or concave ſegment of a Cylinder, or Sphere, is the ſame; each may be the other, by ſuppoſing the Light on either Hand.

More might be ſaid in reſpect of reflected Light; but, if the Obſervations I have made be well attended to, it will be found ſufficient, for any purpoſe whatever.

Of the reflected Images of Objects on Water, and poliſhed Surfaces.

The Reflections of Objects on the Surface of ſtill Water gives (where Water is repreſented) tranſparency to the Water, and perfection to the Picture; which, if well managed, has a pleaſing effect.

Reflected Images of Objects on Mirrours, except when ſeen direct, is a real Anamorphoſis, or Diſtortion. On a horizontal Mirrour, the Object ſtanding upright, the Image appears the ſame, inverted; yet, the Picture is really on the Surface of the Water; for, an Object on the Water, or being cut by its Surface, always appears double, to an Eye at any diſtance from it, horizontally. For inſtance, the Eye, being at E, ſees the Image of the Object AB inverted, apparently in the Water; but it is manifeſt, that 'tis not ſo, in reality, for, the Repreſentation is on the ſurface of the Water; the part AC being repreſented by Ac, which is drag'd out, to a prepoſterous length, whilſt bc repreſents the upper part, the Cone BC, which is much leſs than the Object; nevertheleſs, I ſhall prove, that the Image is repreſented equal to the Object; but, being ſeen under unequal Angles, they do not appear exactly equal.

Fig. 51. It is obvious, whether by means of Rays of Light, reflected to and again, from the Object to the Water, and from thence to the Eye, or otherwiſe, that, if AD be the Surface of Water (or a poliſhed Mirrour) and AB, an Object, ſtanding erect, we ſhall ſee the Image of the Point B, on the Water, at b (the Eye being at E) where Right Lines, drawn from B and E to the Water, at b, make equal Angles with its Surface. Conſequently, if Eb be produced, meeting BA, produced, at, *B*, A*B*, and, A*B*, equal will be to AB;* is the apparent place of the Object, which appears on the Surface of the Water. Hence it is manifeſt, that the Image of the Object AB, repreſented on the Surface of the Water (and which, in real length, is Ab) is repreſented equal to the Object itſelf. For, the Point A is common to both, and becauſe B, appears at *B*, it appears juſt as much within the Water, as its real place is from the Water, *viz.* A*B* equal AB, and A*C* equal AC, &c.

Hence, we have an unerring Rule, for repreſenting the reflected Images of Objects on poliſhed Surfaces. But it muſt be obſerved, that, if the Object be at ſome diſtance, beyond the Water, the meaſure of the Object muſt be applied from its Seat, on the Plane of the ſurface of the Water, not from the Water edge, as when it is in the Water or cloſe to it. Alſo, that the Image repreſented on the Water, is not ſimilar to the Repreſentation of the Object, on the Picture, which I ſhall ſhew; except when a plane Figure only is repreſented, parallel to the Picture.

EXAMPLES.

Fig. 52. Let X be an Object in the Water, having one Face (X) parallel to the Picture.

No. 1. Produce the Sides, BA and CD, making A*B* and *CD* reſpectively equal to them; alſo, make the perpendicular *E*F equal to EF, and join *BE* and *CE*; i. e. make the Figure A*E*D ſimilar to AED, inverted, from the Line AD where the Surface of the Water cuts the Object. The Image of the Door, &c. is alſo ſimilar.

* For, the Angle BbA=EbF (the Anngle of incidence, to the Angle of reflection) and, Ab*B*=EbF (2. 1. El.) wherefore BbA=Ab*B* (Ax. 3. El.) then, becauſe the Angles at A are right (AB being ſuppoſed perpendicular to the Surface of the Water) and Ab is common to the two Triangles, ABb and A*b*b, conſequently, A*B* is equal to A*B*; for the Triangles are congruous (11. 1. El.)

For

For the Face AG, being perpendicular to the Picture, the horizontal Line, BG, vaniſhes in the Center; wherefore, BG being parallel to the Water, its Image will be parallel to the Line, and conſequently, it has the ſame Vaniſhing Point.

Draw *BC*, meeting GH, produced at *G*, which compleats the Image of that Face, and which, it is obvious, is not ſimilar to the Repreſentation ABGH; ſeeing that, the Angles which are acute, in one, are obtuſe in the other. Likewiſe, a ſide of the Roof is repreſented in the Object, which, it is manifeſt cannot be ſeen in the Water, by Reflection.

The Piles, at U, which are perpendicular, have their Images alſo equal, each to the Repreſentation; likewiſe, the Pile W is reflected equal, and equally inclined, being parallel to the Picture. But, being otherwiſe inclined, as Y and Z, one hanging towards, the other from the Picture, the Seats of their tops, on the ſurface of the Water, being determined, at a, produce the Perpendicular ba, making a*b* equal to a b, and draw from *b*, to the Timbers, where they are cut by the Surface. No. 2.

The Vaniſhing Point of the reflected Image is at the ſame diſtance on the other ſide of the horizontal Line, as that of the Object, whether it be above or below it.

The Image of the Bridge, on the Water, may be obtained after the ſame manner, particularly the Piers. The reflected Images of all horizontal Lines tend to the ſame Point, in the horizontal Vaniſhing Line, as the Repreſentations; for, Right Lines, being parallel to the Surface of the Water, their Images are either parallel to them, or they have the ſame Vaniſhing Points, reſpectively. No. 3.

The horizontal Line, HI, of the Bridge, vaniſhes at O; wherefore, PQ, drawn perpendicular through O, is the Vaniſhing Line of the Plane of the face of the Bridge; IK vaniſhes at P, and the inclined Line at the hither end at Q, equally diſtant from O, being equally inclined. Wherefore, k*I* being made equal to Ik, the horizontal Line has the ſame Vaniſhing Point (O) as its original, and the inclined Line *IK*, tends to Q, repreſenting a parallel to GH, at the hither end.

For the Arches; draw as many perpendicular Lines, ab, cd, &c. as are neceſſary, and produce them in the Water; making each equal to its correſponding one, *a*b equal ab, &c. by which means, as many Points, in the Curve, as you pleaſe, may be obtained, and the image of the Arch deſcribed through them; each being a Semi-Ellipſis, of which, ik is a Diameter.

The Crane, &c. on the Wharf, hanging over the Water, has its Image reflected, by means of perpendicular Lines from each part, and finding their Seats on the ſurface of the Water (as *m*, of M) repeating the ſame meaſure downwards. No. 4.

For the Hogſhead; draw as many Lines (repreſenting lines parallel to th eHorizon) as are neceſſary, tending to the Vaniſhing Point V of the horizontal Diameter ab; then, by means of perpendiculars from each extreme of thoſe Lines, their Images are acquired, and the curve of the Head deſcribed through them. If other Lines are drawn, from one head to the other, to the Vaniſhing Point X, of its Axe, the whole Image may be deſcribed, as in the Figure.

The Warehouſes, &c. ſtanding at ſome diſtance from the Water, have their reflected Images deſcribed as the other Objects; by ſuppoſing the ſurface of the Water produced, and the Seat of each Angle determined thereon; as of A, B, C, the Angles of the Warehouſe W; which are made equal, downward, from their Seats (*a*, *b*, *c*,) reſpectively, and the horizontal Lines have the ſame Vaniſhing Points. No. 5.

The Spire of the Church, at T, is reflected on the Water; becauſe, its Seat (S) on the level of the Water, being obtained, the height (ST) being applied downward reaches the Water, otherwiſe it would not. No. 6.

Some Perſons have expreſſed ſurprize, that, Objects, at a great diſtance, ſhould have their Images reflected on Water which is near. The moſt diſtant Objects, which can be ſeen, will be reflected; provided, that the Eye be properly ſituated, near the Surface, and there be no other Objects intervening, i. e. when Right Lines

Plate XLVIII.

from the Object and from the Eye, making equal Angles with the ſurface of the Water, meet on the Water; otherwiſe, the Objects cannot be reflected. It ſeems indeed rather ſurprizing, that a Surface, very much inclined to the Horizon, and nearly approaching to perpendicular with the Picture, as a gentle Aſſent can be reflected; but, the Eye being ſo ſituated, as above, and the ſurface of the Water being on a level with the foot of the hill, its inclined Surface will be reflected thereon.

It has been objected, that the Reflections are too ſtrong, in the Plate; I grant they are, as a Picture; but my deſign is to ſhew the figure of the Image on the Water, rather than the effect, conſidering the Water not ſimply as Water, but as a Mirrour. Nevertheleſs, in clear Water, perfectly at reſt, the Images of Objects on its Surface will be found as diſtinct as the Objects themſelves appear, according to their Diſtance; ſomewhat differing in Colour, rather darker, in the cleareſt Water.

Thus much may ſuffice, for Reflection on a horizontal Surface; which is the ſame, whether it be a clear Fluid (as Water) or other plane Mirrour.

Secondly. Of Reflection on Mirrours, vertical and inclined.

Fig. 53.

Let W be a Mirrour, parallel to the Picture, whoſe Center is S; and X, an Object, whoſe reflected Image is required.

In this Caſe, it muſt be obſerved (as in the former) that the Image, of any Point in an Object, is reflected to the ſame diſtance from its Seat on the Mirrour, as the original Point from its Seat; ſo, that diſtance is, here, repreſented, and its Seat obtained, perſpectively (in the former it is geometrical) and, becauſe the reflecting Surface is parallel to the Picture, the Center is, conſequently, the Vaniſhing Point of Lines producing its Seat, being perpendicular thereto.

Draw Lines, from the Feet, A, D, &c. of the Chair to S, the Center; and where they cut the Interſection of the Mirrour with the Floor, at a, d, &c. make a*a* repreſent a length equal Aa, &c. (Prob. 8.) and draw *a*b and *d*c perpendicular; then, BS and CS, cuting thoſe Perpendiculars, reſpectively, give their reflected Images. For, as every Object appears to be as far on the other ſide, as it really is on this ſide, from the Mirrour; conſequently, in this Caſe, it muſt be ſo perſpectively.

The reflected Images of horizontal Lines, as the rails of the Chair, &c. (in this Caſe) tend to a Vaniſhing Point in the Horizontal Line, at the ſame diſtance from the Center of the Picture, on the contrary ſide, as the Vaniſhing Point of the Repreſentation. If V be the Vaniſhing Point of BC, the front Rail; then, in SG a diſtance being taken equal to SV, is the Vaniſhing Point of bc, the reflected Image of BC. I is the Vaniſhing Point of the ſide Rail CE; SK being made equal SI, K is the Vaniſhing Point of its reflected Image. After the ſame manner, the middle Point of the top Rail may be obtained, by a Perpendicular, ef to the Floor, and as many other Points as are neceſſary.

On a vertical Mirrour, inclined to the Picture, there is no difference in the proceſs, but only in the Vaniſhing Point of Perpendiculars to the Mirrour; and, if the Vaniſhing Point of Perpendiculars be determined, 'tis the ſame in all inclinations.

Fig. 54.

Ler Z be a Mirrour inclined to the Horizon, and caſually inclined to the Picture. It is required to find the reflected Image of the Chair, on the Mirrour Z.

V being the Vaniſhing Point of the bottom of the Mirrour, which is horizontal, find the Vaniſhing Line (VL) of the Mirrour, its inclination to the Horizon being known (Prob. 5. Sect. 3.) and alſo, the Vaniſhing Point (F) of Lines perpendicular to the Mirrour (Prob. 2. Sect. 12.)

Then, having found the Interſection (*CD*) of the Mirrour with the Floor, produced (its Vaniſhing Point is V) and G, the Vaniſhing Point of Perpendiculars thereto, draw AG, DG, &c. cuting *CD*, at *A*, *D*, &c. and through them, draw *A*b and *D*c, tending to the Vaniſhing Point of the ſides of the Mirrour*, indefinite

* This Vaniſhing Point is the Interſection of the Vaniſhing Line (VL) of the Mirrour, and the vertical Vaniſhing Line RG, produced.

Draw

Draw AF, DF, &c. cuting them, reſpectively, in the correſponding Points, a, d, &c. (their Seats on the Mirrour*) beyond which, their Images are repreſented equal, in Perſpective; by making a *a*, repreſent an equal meaſure as A a, &c. (Prob. 8.)

Find the repreſentation of the Interſection of the plane of the Front, with the Mirrour (which will paſs through V) and the Interſecting Points of AB and DC, therein, by producing them; from which Points, draw Lines through *a* and *d*, indefinite, and BF, CF, &c. cuting them in their reſpective Images on the Mirrour.

Then, having found the Interſection *EB*, of the Plane of the ſide of the Chair, with the Mirrour, and the interſecting Points, *E* and *B*, of *G*A and *F*B, draw *Ea* and *Bb* indefinite; and, from *F* and *G*, draw lines to F, the Vaniſhing Point of Lines perpendicular to the Mirrour, cuting them at *f* and *g*.

Thus, as many Points (as e, at the top) may be determined as occaſion requires, from their Seats on the Floor; by which means, the reflected Image of any Object, whatever, may be repreſented on the Mirrour.

BC is parallel to the Mirrour, wherefore, the Image (*bc*) of BC, has the ſame Vaniſhing Point, in the Horizontal Line; otherwiſe, it would not; for, although all horizontal Lines, whatever, vaniſh in the Horizon of the Picture; in this Caſe, the Mirrour is a Picture, and the repreſentation of the Chair is conſidered as the original Object, in reſpect of that Picture, each Plane having its peculiar Vaniſhing Line thereon.

PQ is the Interſection of the Plane of the Mirrour with the Cieling; by means of which, the Cage, hanging from the Cieling, at N, is reflected. If its Seat, on the Cieling be determined, the Image of each Angle on the Mirrour is repreſented, indefinite, and their repreſentative lengths are determined, by drawing lines from each extreme to F, the Vaniſhing Point of perpendiculars to the Mirrour; as no is the Seat of NO, on the Mirrour, and *no* is its indefinite Image. After the ſame manner, the corners of the Cage may be repreſented on the Mirrour.

Of KEEPING; and the effect of DISTANCE.

The Term, KEEPING, in the Art of Painting, in general, is uſed to ſignify a juſt and proper ſubordination of all the parts of a Picture to the principal Object; in reſpect of Magnitude, Colour, and diſtinctneſs of Parts.

The magnitudes of Objects, in reſpect of each other, perſpectively, are various, according to the Station from which they are viewed; and conſequently, the ſubordination of Colour, &c. is not in proportion to the Objects, but to their Diſtance from the Eye. It is abſolutely impoſſible to lie down Rules, by which the Artiſt may, with certainty, produce the deſired effect; ſeeing that, in hazy or foggy weather, or in a miſty morning, &c. Objects are leſs diſtinct, at a ſhort Diſtance, than, in a clear Day, at a much greater; wherefore, no proportion or degree can be determined. If Objects, of known magnitude, appear not to be far diſtant, in the Picture, and yet, their parts not diſtinctly defined, in compariſon with others, in the Fore-ground; it implies, that the Air is more groſs and hazy, than if the parts were more perfect.

KEEPING is, in a great meaſure, ſynonimous with Aerial Perſpective; which ſignifies a diminution of Light, Colour, and diſtinctneſs of the parts of Objects, in a regular gradation, as linear Perſpective of Magnitude; owing to the effect of Air, between the Eye and the Object; which, being a Medium, obſtructs the ſight, in ſome degree, at any Diſtance; conſequently, as there muſt be a greater quantity of Air between the Eye and diſtant Objects than near ones, ſo their parts are leſs diſcernable; the Lights and Shades are inſenſibly mixed, and, at a great Diſtance, it is all a confuſed jumble, of Light, Shade, and Colour, without diſtinction.

It is cuſtomary with many, in delineating, and may frequently be ſeen in Architectural Deſigns, &c. to make a conſiderable difference, in Teint, between one plane Surface and another; when the one, in the Original Object, recedes but a

* *A*b and *D*c are the Seats of AB and CD on the Plane of the Mirrour, produced.

few

Plate XLVIII.

few Inches, which is abſurd; for, if that ſudden gradation of effect was continued to the diſtance of a few Yards, it would become totally black, before it was poſſible for the Air to have any apparent effect on it. Now it is far from being ſo in Nature, which is obvious to any Eye; for, if the materials are clean, and of an even and uniform Colour (without which no judgment can be made) I will venture to affirm, that two Plane Surfaces (V and Y) parallel between themſelves, at ſeveral Feet diſtance from each other, and having a full Light on them, cannot be diſtinguiſhed one from the other, nor ſo much as the Line or Edge (A D) ſeen, at a proper diſtance for delineating; provided the Light be not (on this ſide) ſo very oblique (as at S, No. 3.) as to ſhade the other, beyond where the Line OC cuts it. I ſay, the Eye being ſo ſituated (at O) that the Angle C, appears to cut the receding Plane, at B, they will appear as one continued Surface. Nevertheleſs, it may be neceſſary, in delineating, to make a diſtinction, in proportion to the Diſtance; but, if the Plane V has Mouldings, &c. on it, being cut, apparently, by the edge of the other, the Line is ſufficiently defined without it.

Fig. 50.

No. 2.

As it muſt be obvious, that no poſitive Rule can be preſcribed, I ſhall juſt make a few obſervations, and conclude the Subject; and, with it, the Book.

In reſpect of Magnitude; it is evident, that Objects mry appear, in Perſpective, to have any proportion to each other, although not greatly diſtant. For Example; in Plate 27, of Chelſea College, the fartheſt Building, with a Pediment, is not above two eleventh parts of the hither one (to which it is equal) in height; and yet it does not offend the Eye, nor appear at too great a Diſtance. But, if the gradation of Light was in proportion, it would be too great; becauſe, not merely the height is to be conſidered, but the ſquare of the height, which would reduce it to the proportion of thirty to one; but where we ſhall fix the ſtandard for unity, I am at a loſs to deviſe.

Now, if we were at twice the Diſtance from the hither Building (the diſtance between the two Objects being ſomewhat more than twice that) the proportion of one to the other, would not be much greater than one to three; the gradation of Light, in that caſe, would be nearly as one to ten, but one third part of the former. Yet, I preſume, that in two Pictures ſo delineated, it would not be adviſable to make that difference in the effect of Aerial Perſpective.

In ſhort, as it is abſolutely impoſſible to fix any criterion to determine the Ratio of the gradation of Light and the effect of Diſtance, but muſt ever be at the diſcretion of the Artiſt, I ſhall only obſerve, that the Objects in the Fore-ground, as the hither end of the Bridge (Plate 48) being ſuppoſed near the Picture, muſt have its parts diſtinct and perfect, with ſtrong Lights and Shades, which gradate to the other End. The Buildings on the Wharf, &c. ſhould be leſs perfect, as they recede from the Eye. The Church, at a Diſtance, the Trees and Hills, one beyond another, muſt conſequently be leſs and leſs diſtinct, according to their Diſtance; till at laſt they are ſcarce diſtinguiſhable from the Sky. For all which, the Artiſt can have no other Rule than, carefully and judiciouſly, to copy Nature (the beſt Miſtreſs) which, in that Caſe, is not always uniform, but infinitely variable.

FINIS.

Plate I.

Fig. 1.

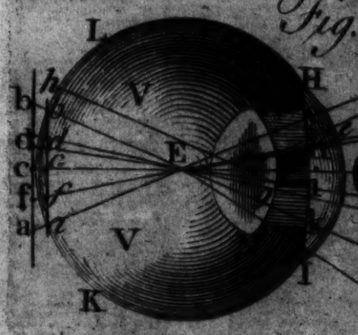


Fig. 2.

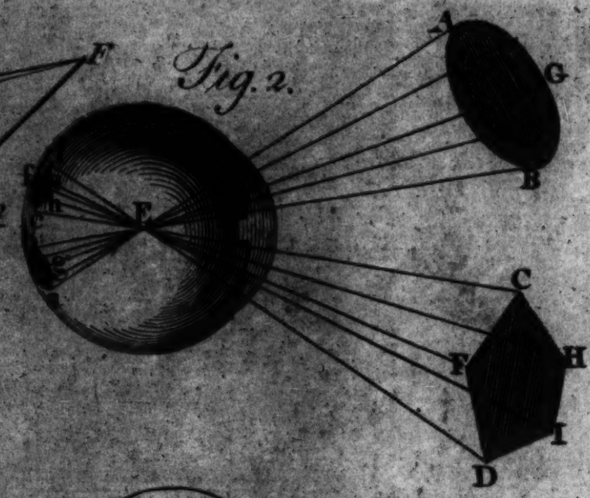
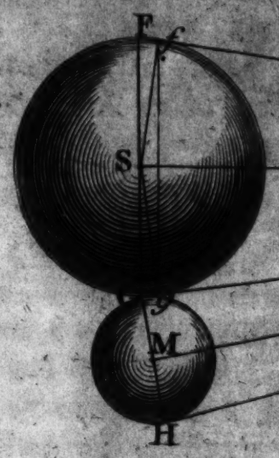


Fig. 3.



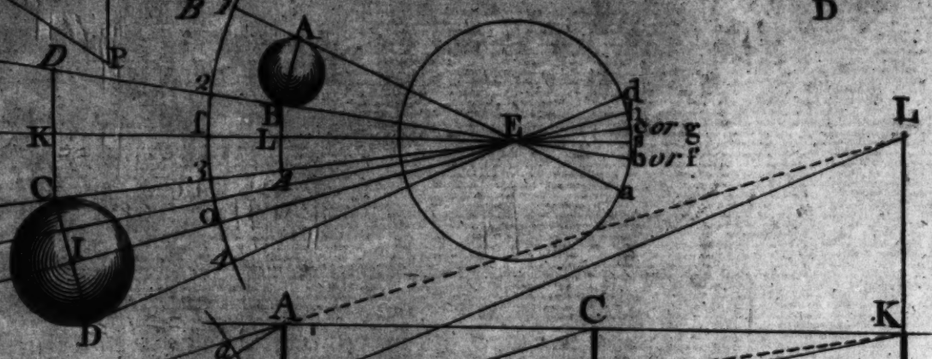


Fig. 4.

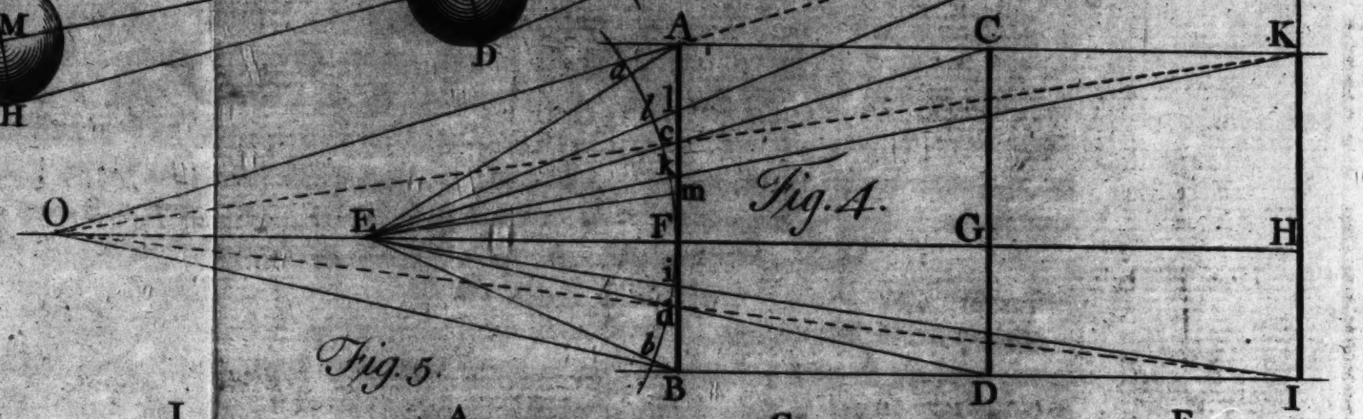


Fig. 5.

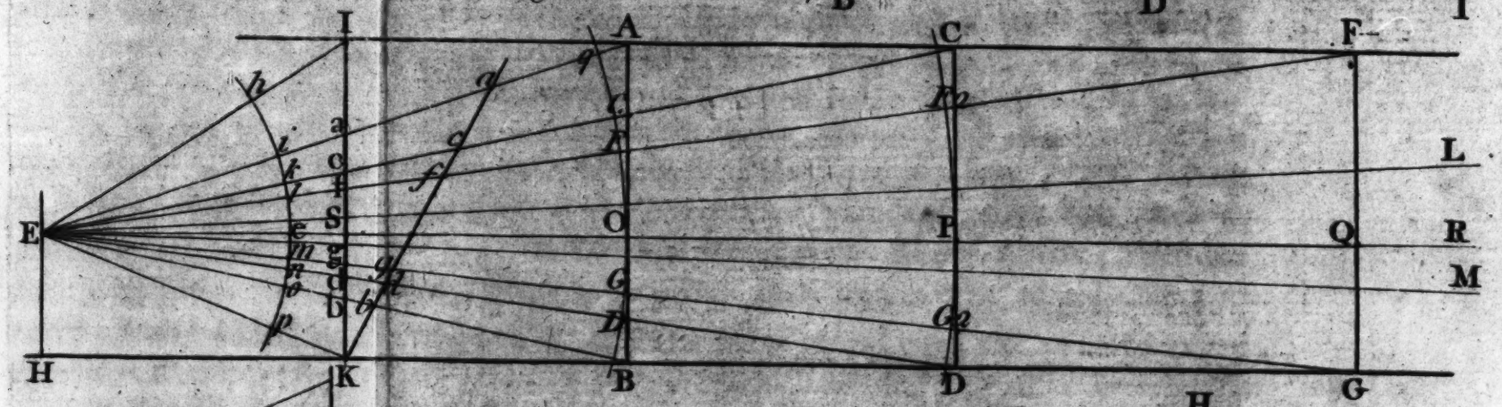


Fig. 6.

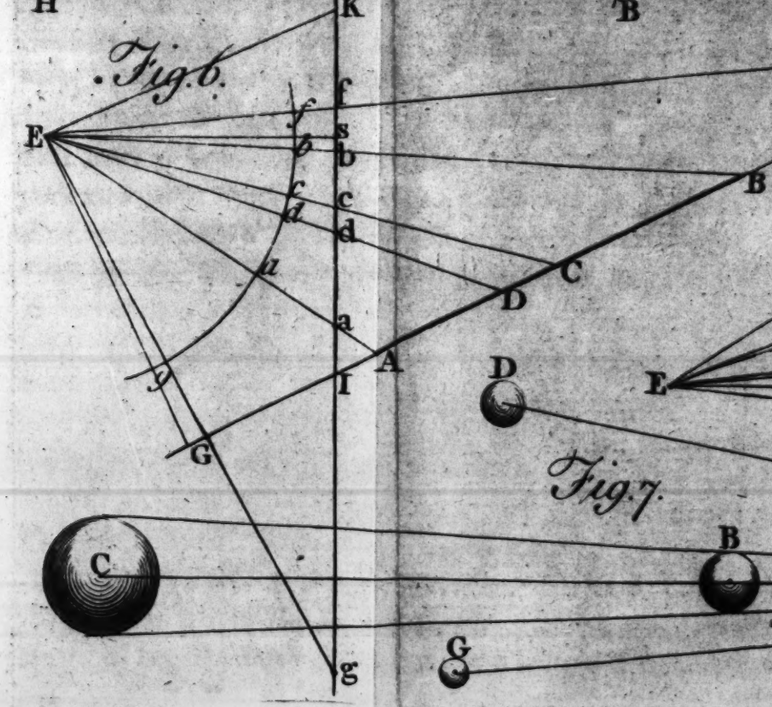


Fig. 7.

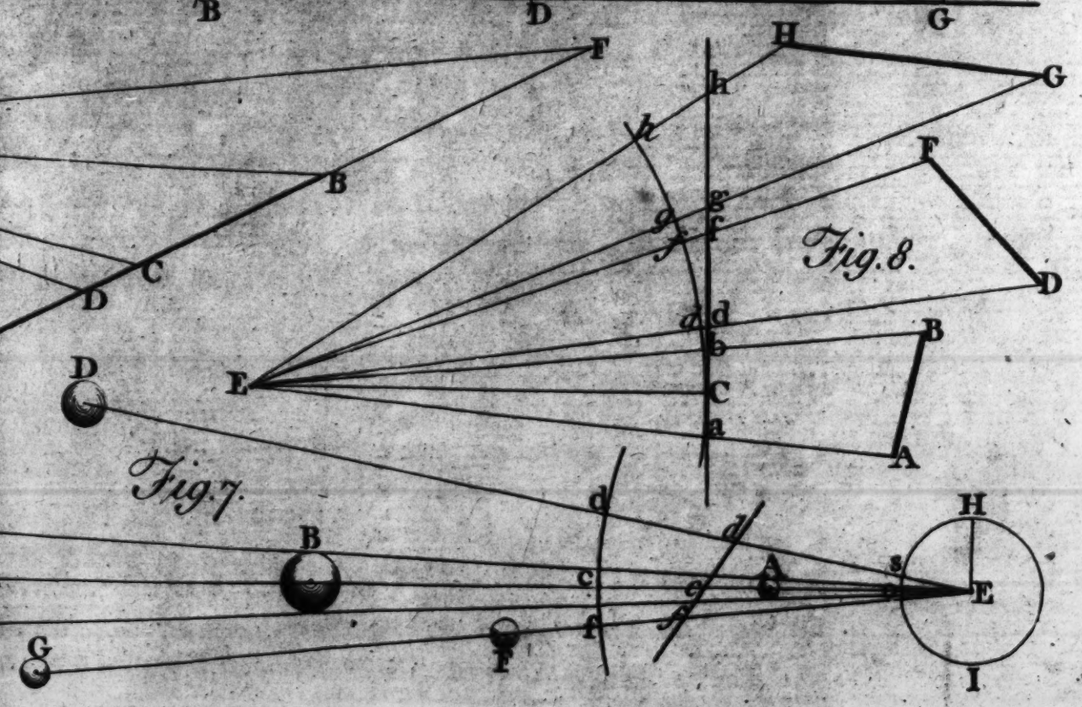


Fig. 8.

Plate II.

Fig: 9.

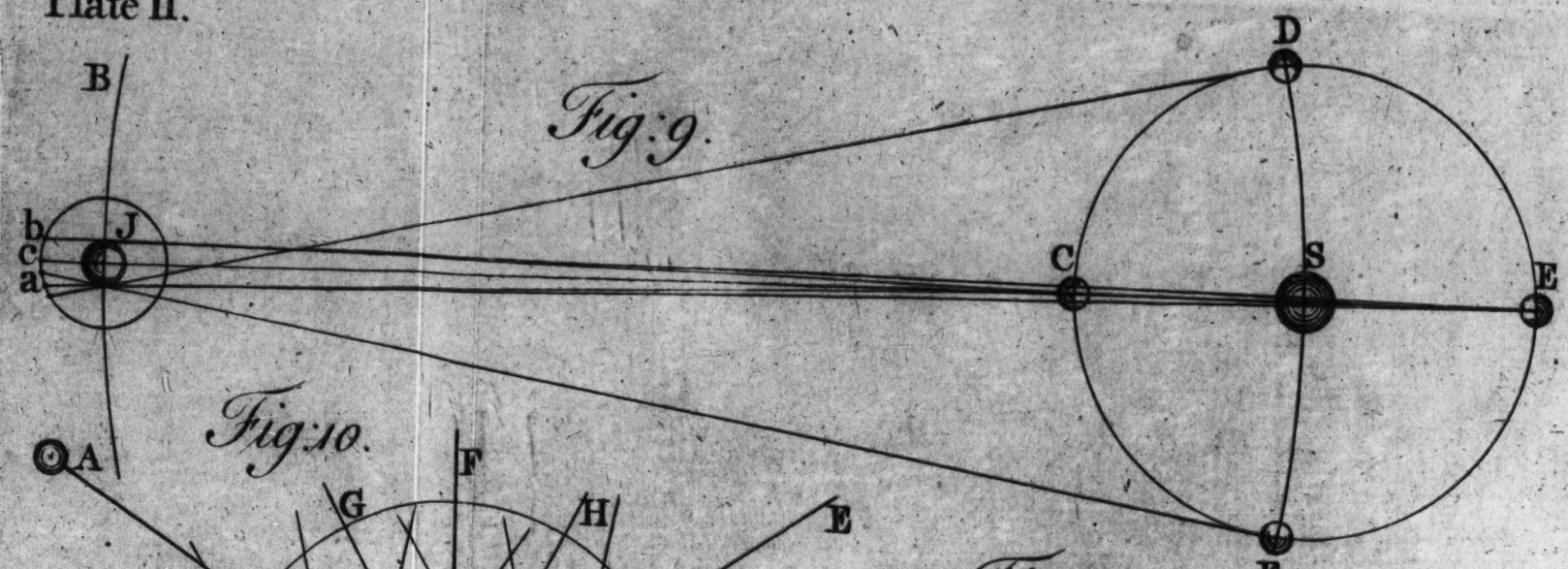


Fig: 10.

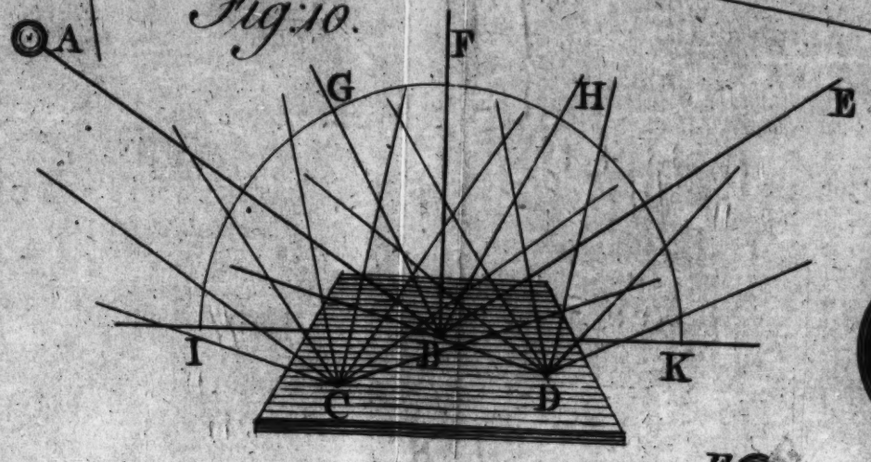


Fig: 11.

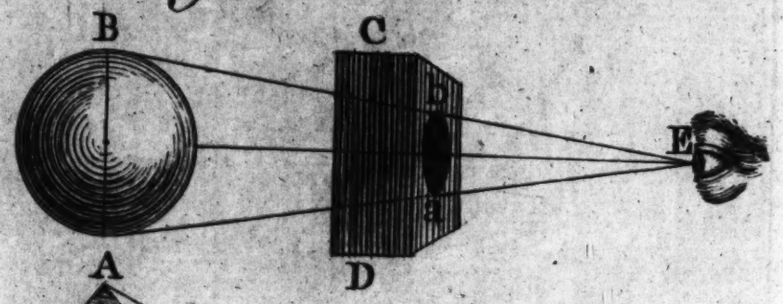


Fig: 12.

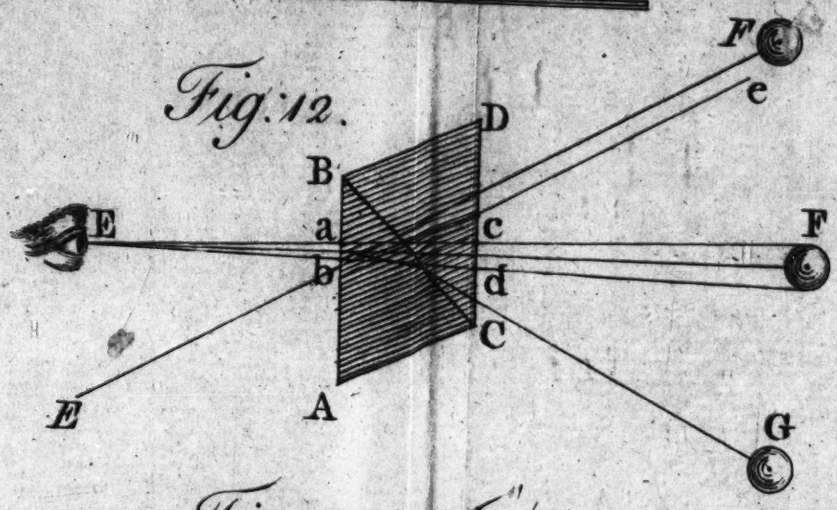


Fig: 13.

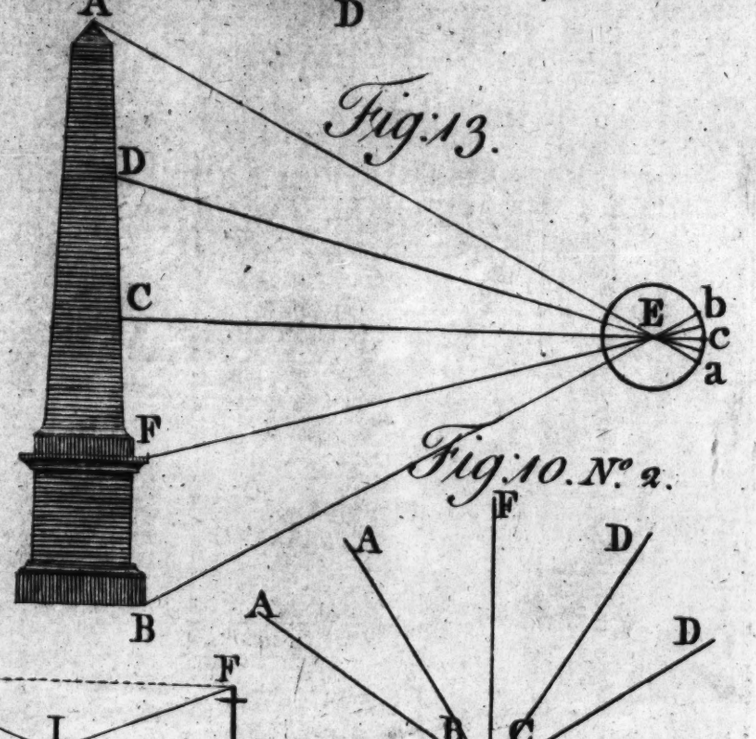


Fig: 14.

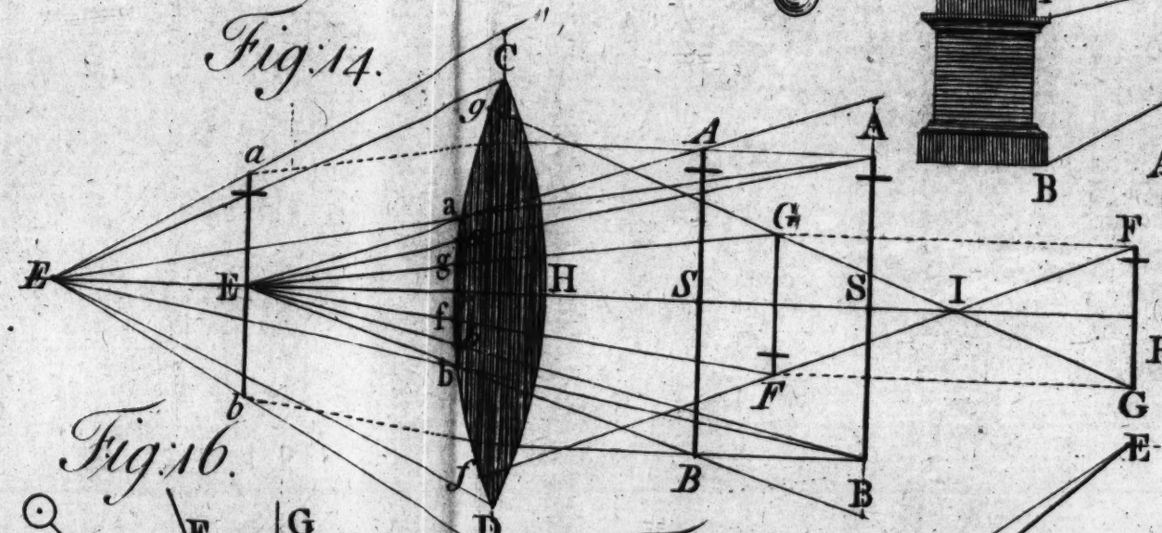


Fig: 10. N.° 2.

Fig: 16.

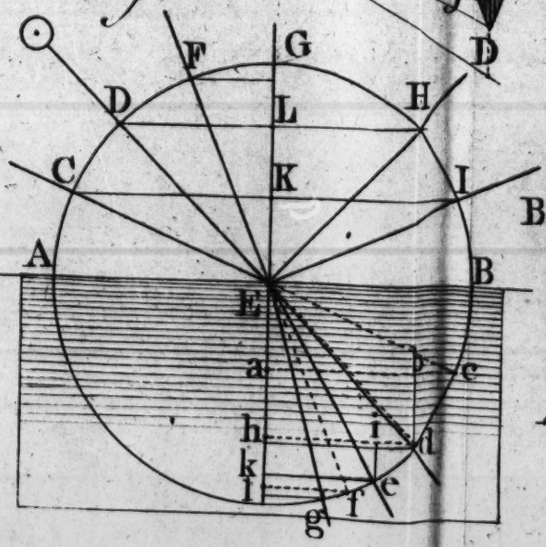


Fig: 15.

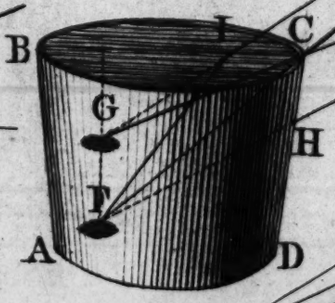
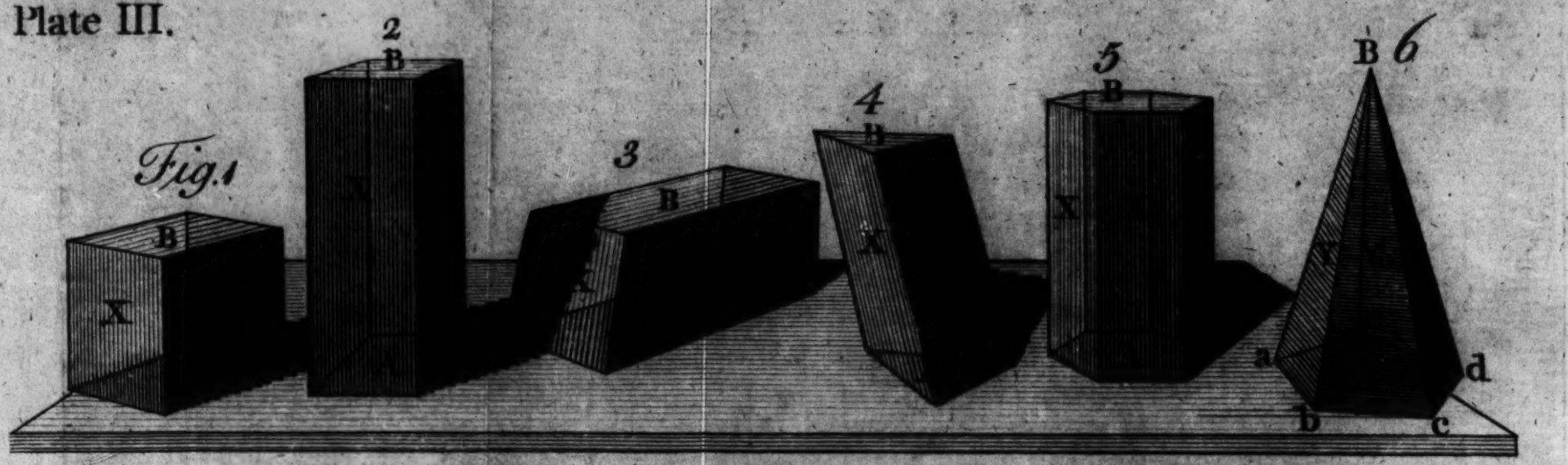
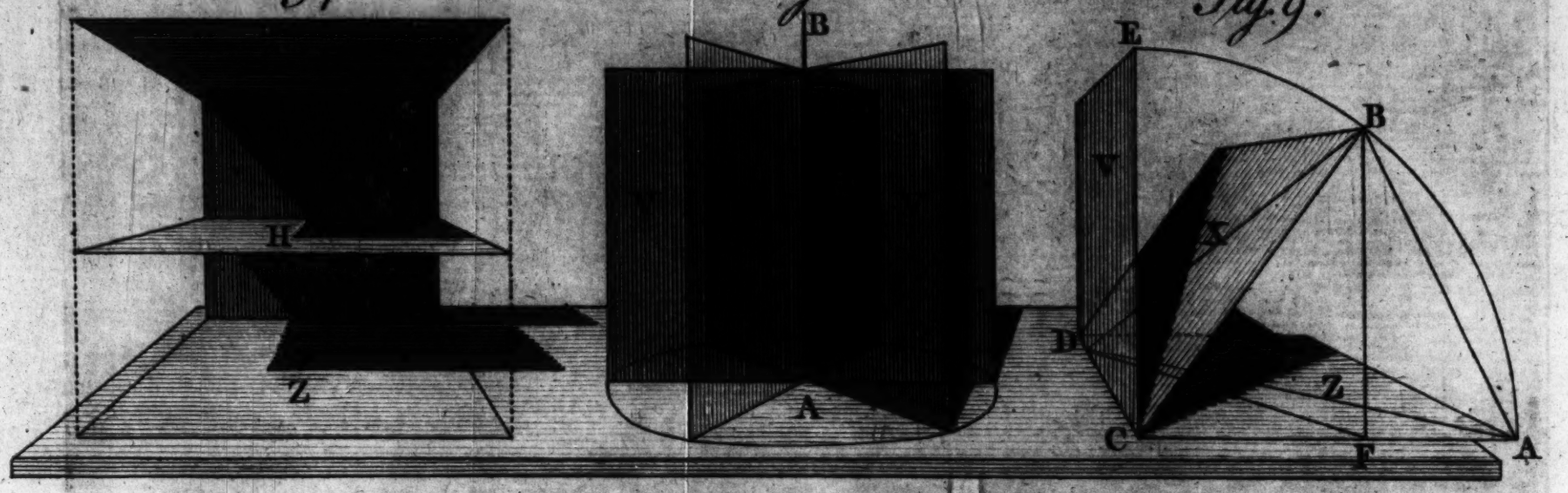


Fig: 17.

Plate III.


Fig. 1
2
B
3
4
5
B 6
X
a
b
c
d


Fig. 7.
H
Z
Fig. 8.
B
A
Fig. 9.
E
B
V
X
D
C
Z
F
A

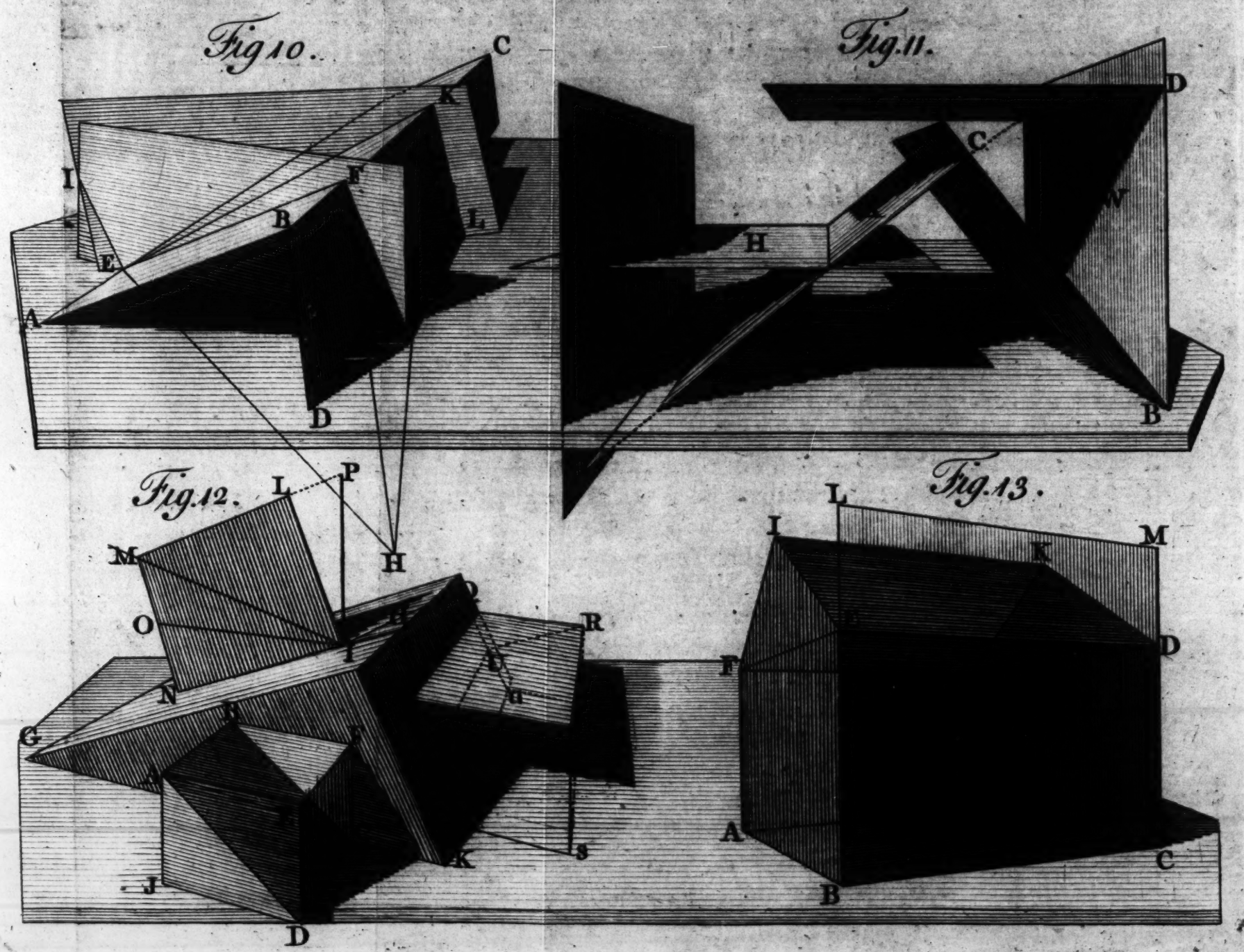
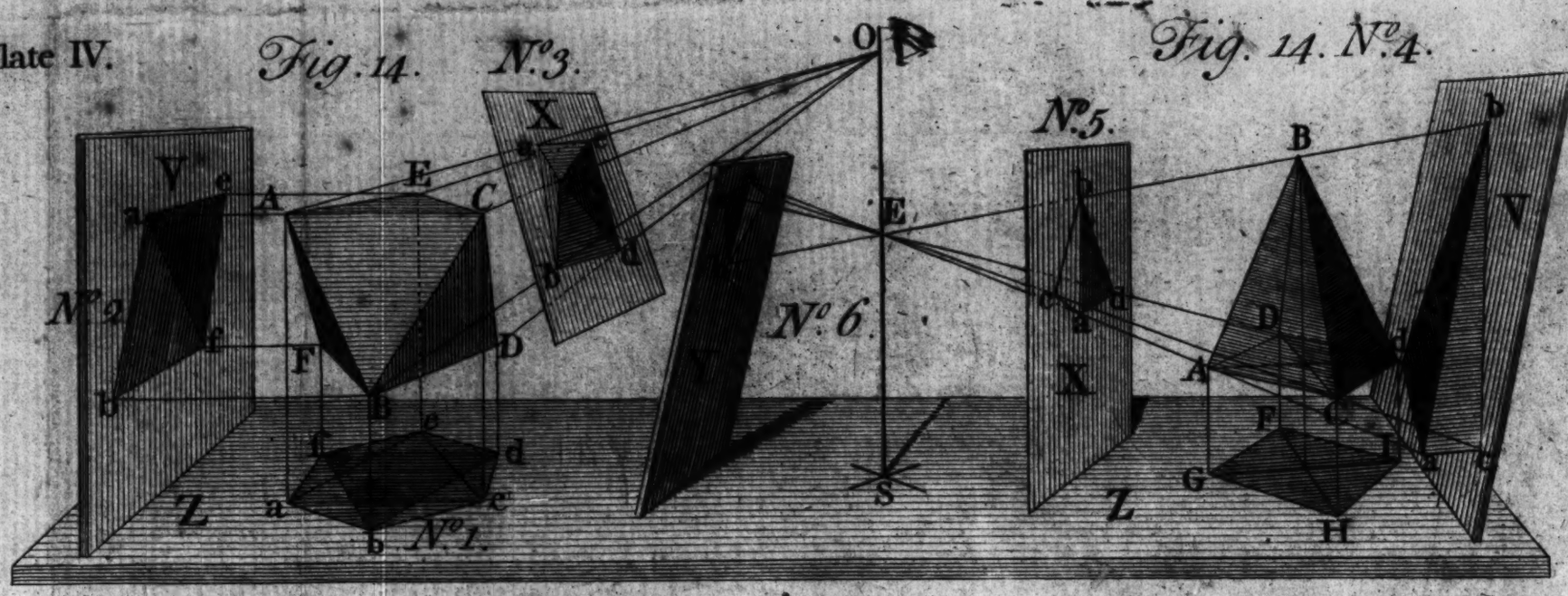

Fig. 10.
C
K
I
F
B
L
E
A
D
Fig. 11.
D
C
W
H
B

Fig. 12.
L
P
M
H
O
R
N
G
A
J
K
S
D
Fig. 13.
L
I
M
K
D
F
A
B
C

Plate IV.

Fig. 14. N.° 3.



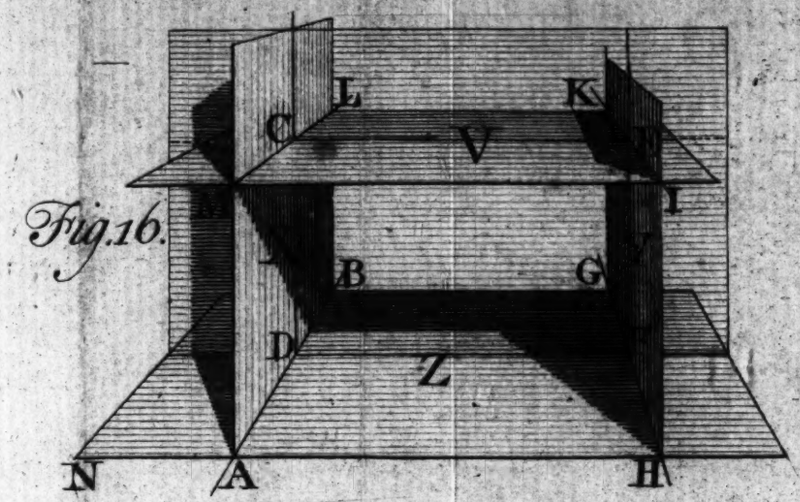


Fig. 16.

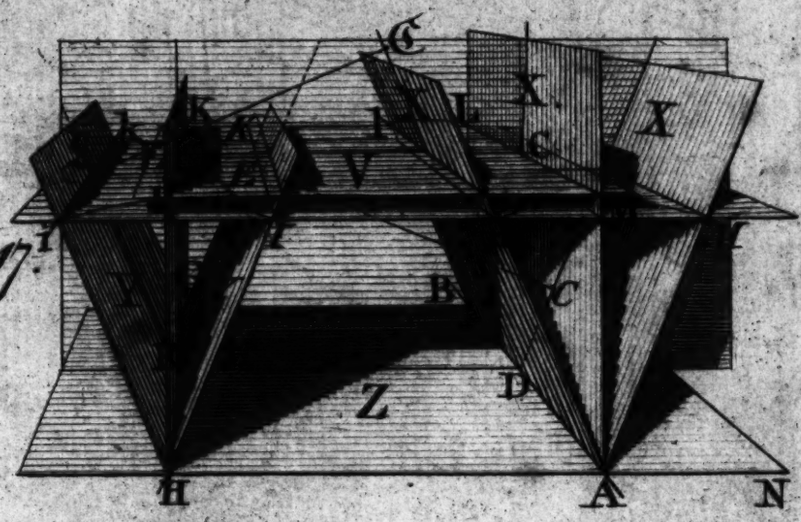


Fig. 17.

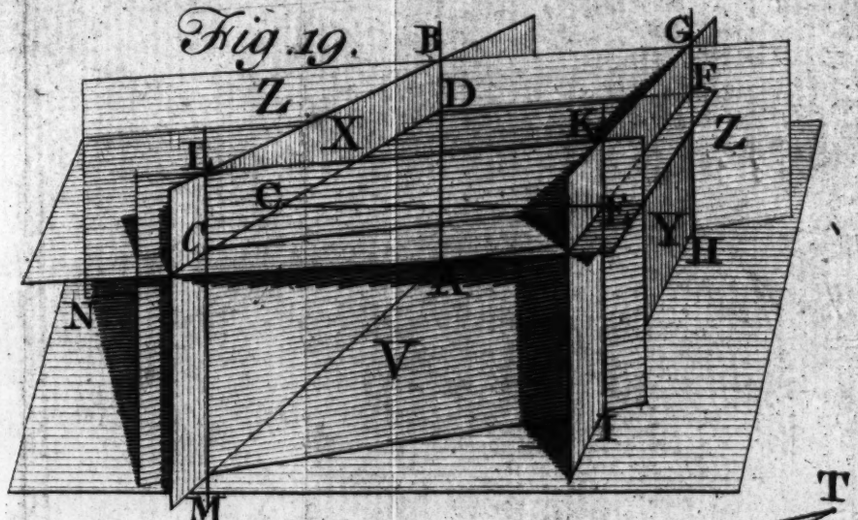


Fig. 19.

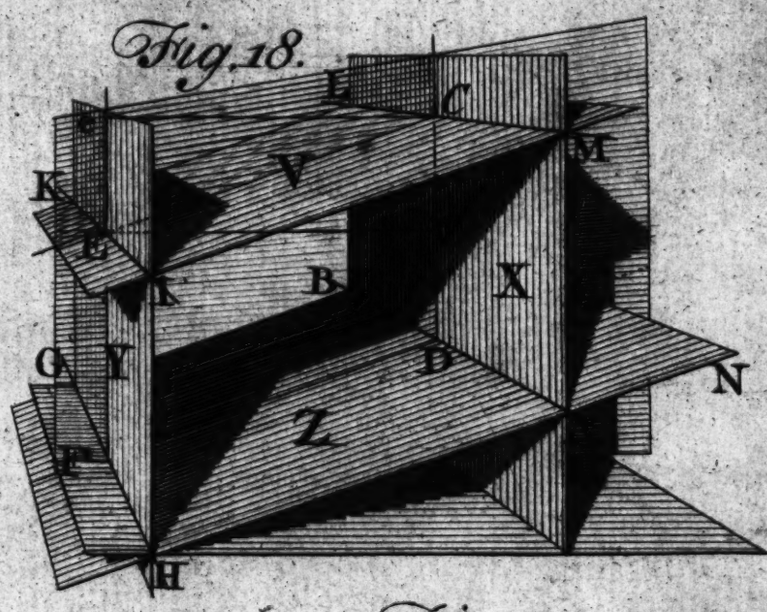


Fig. 18.

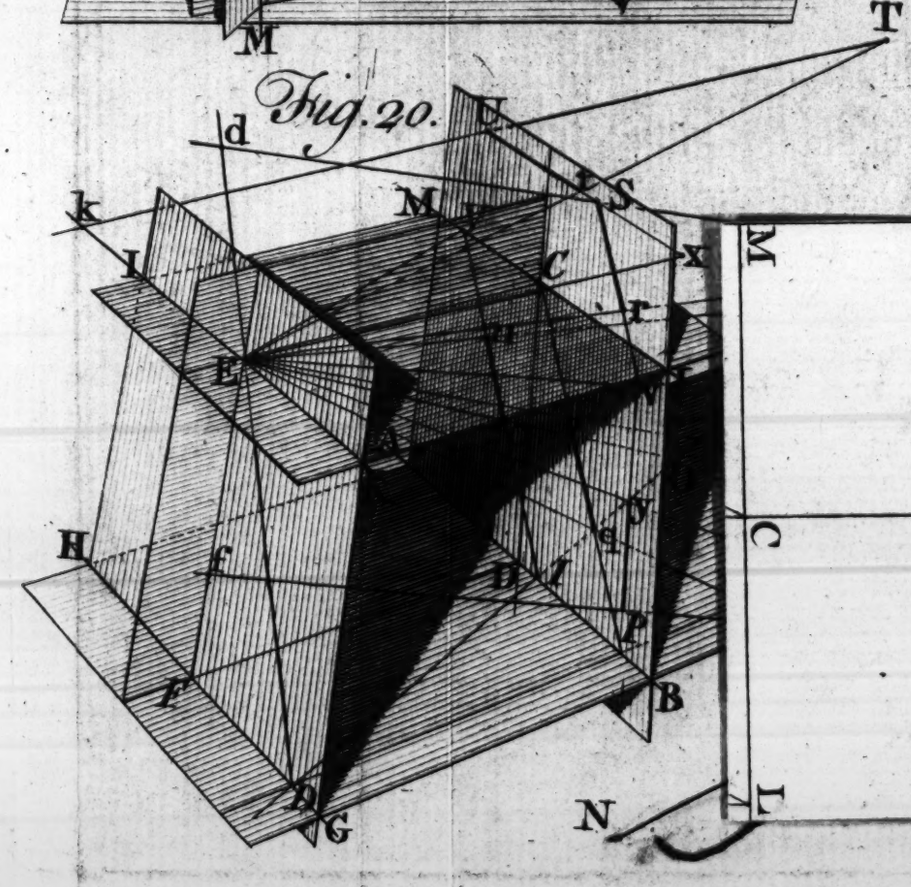


Fig. 20.

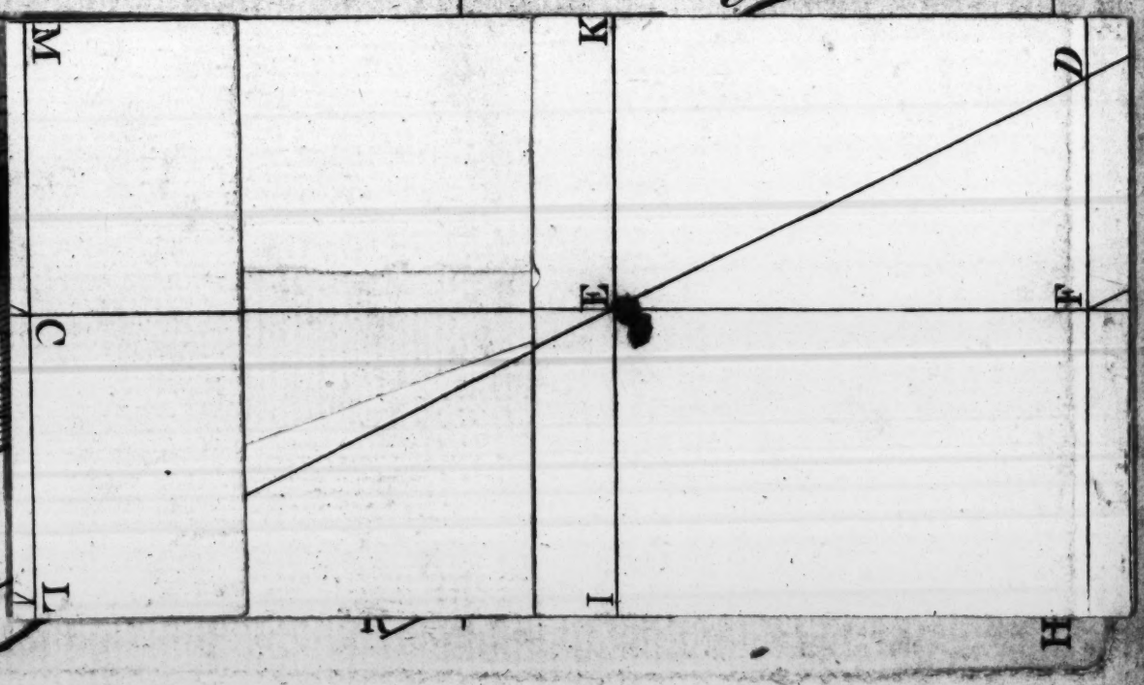


Fig. 21

Plate V.

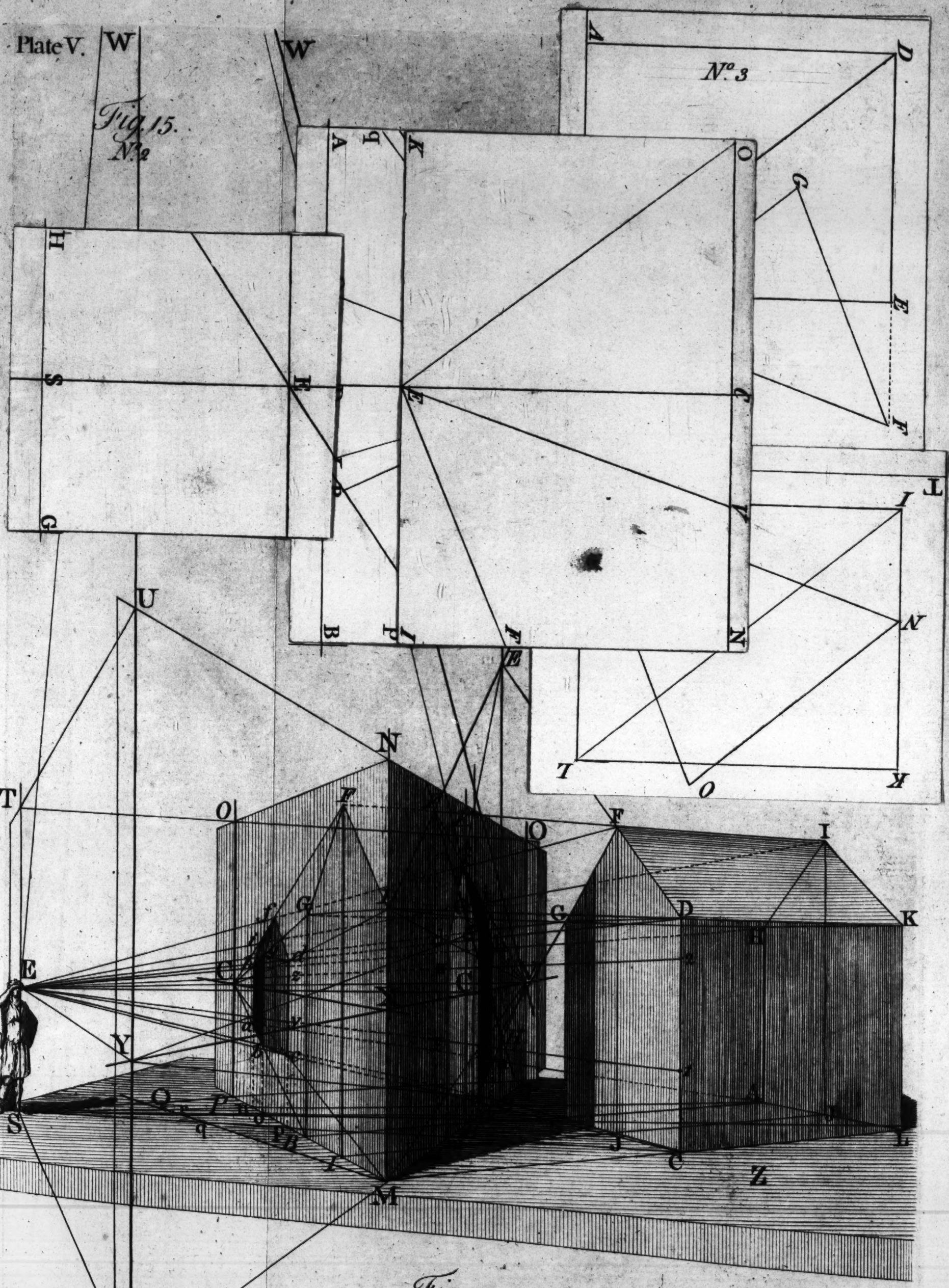


Fig. 15. N°. 2

Fig. 15.

Plate VI

Fig. 22.

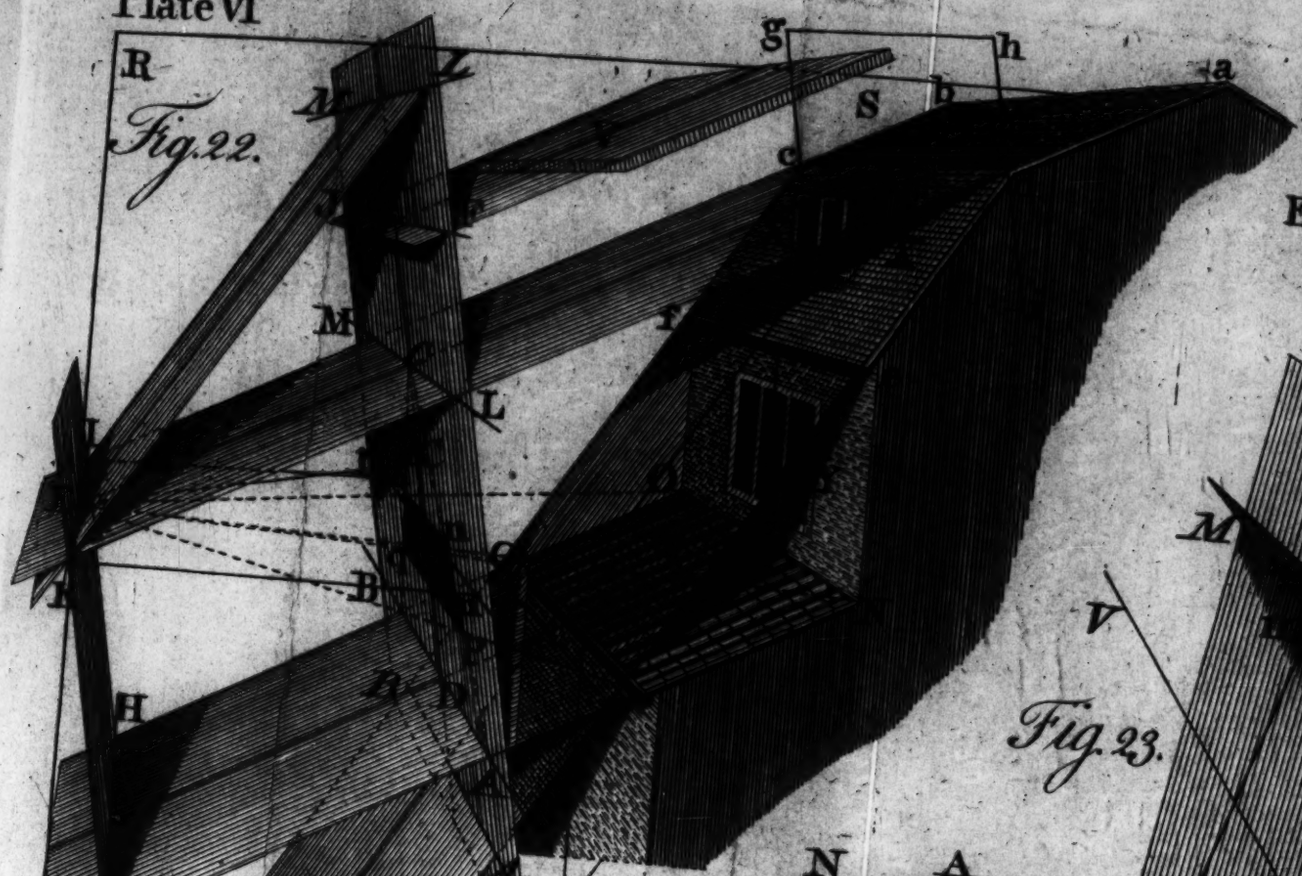


Fig. 24.

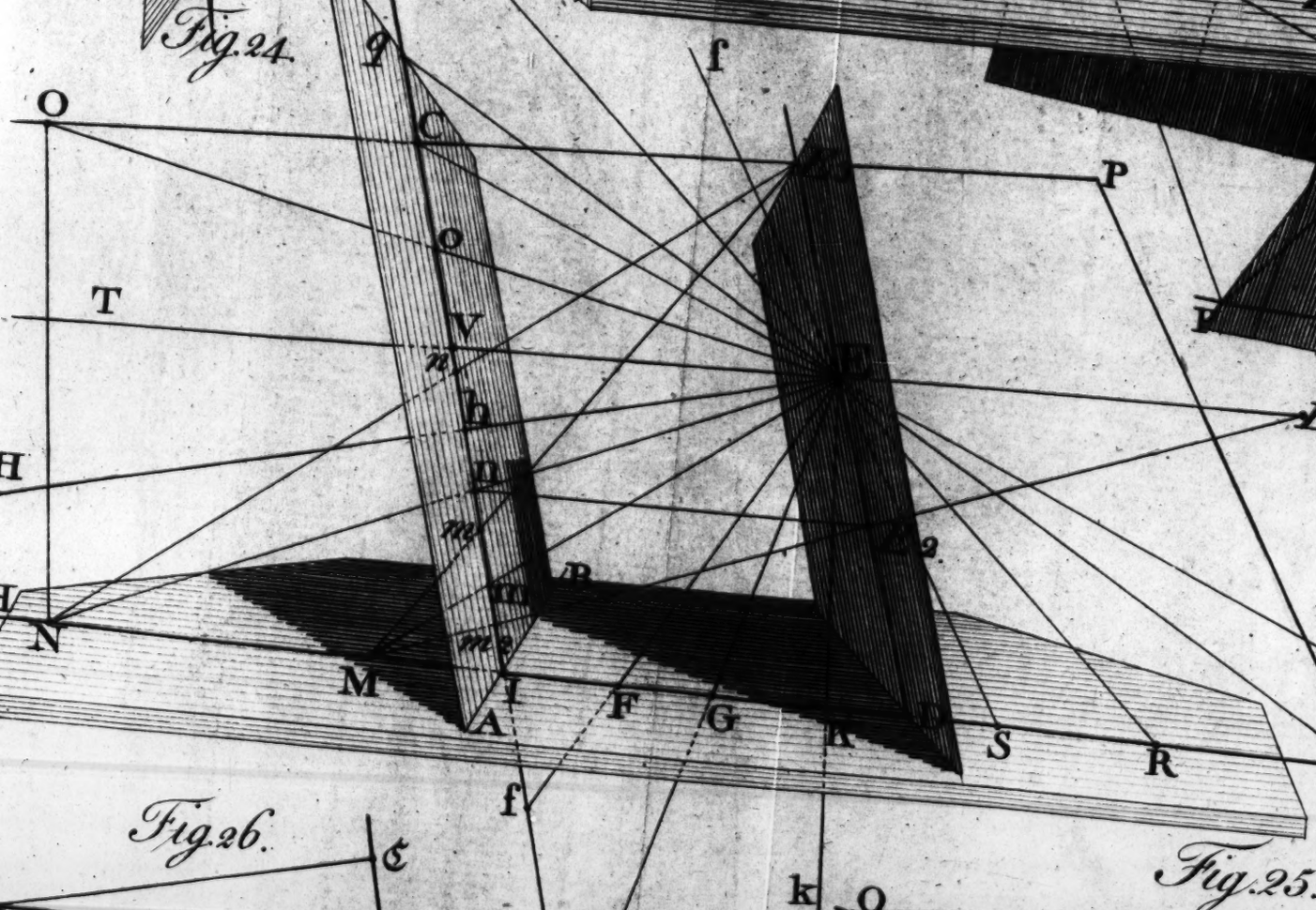
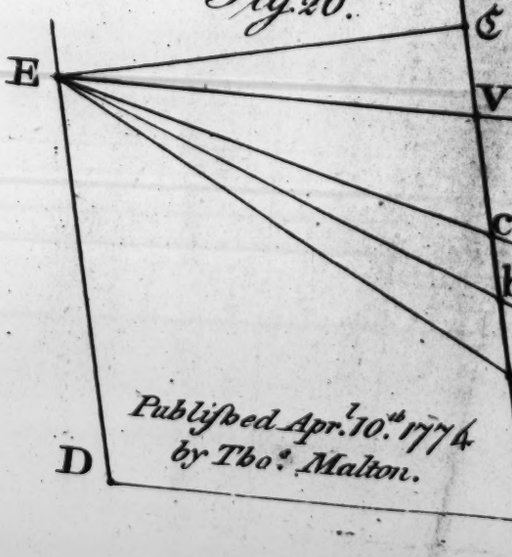


Fig. 26.



Published Apr.l 10.th 1774 by Thos. Malton.

Fig. 22. N.o 2.

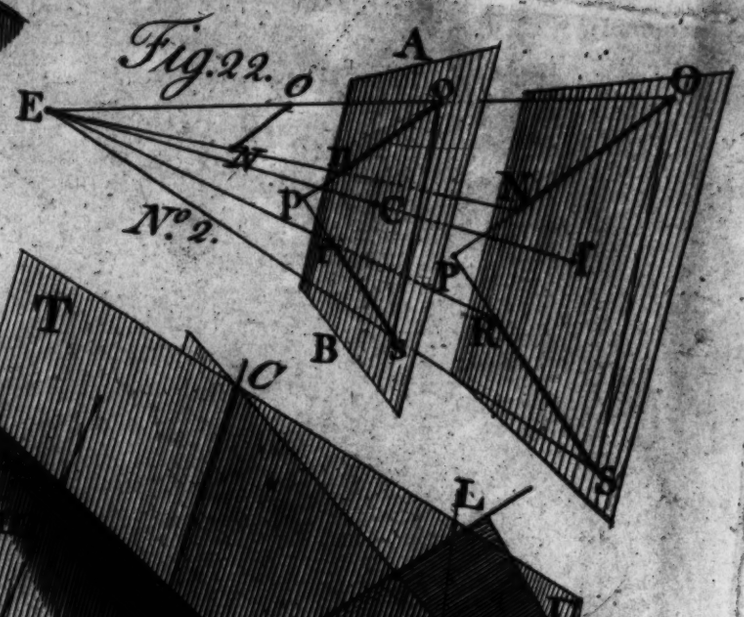


Fig. 23.

Plate VII.

Fig. 27.

N°. 1.

Fig. 28.

Fig. 29.

Fig. 30.

Fig. 31.

Fig. 32.

Fig. 33

Fig. 34.

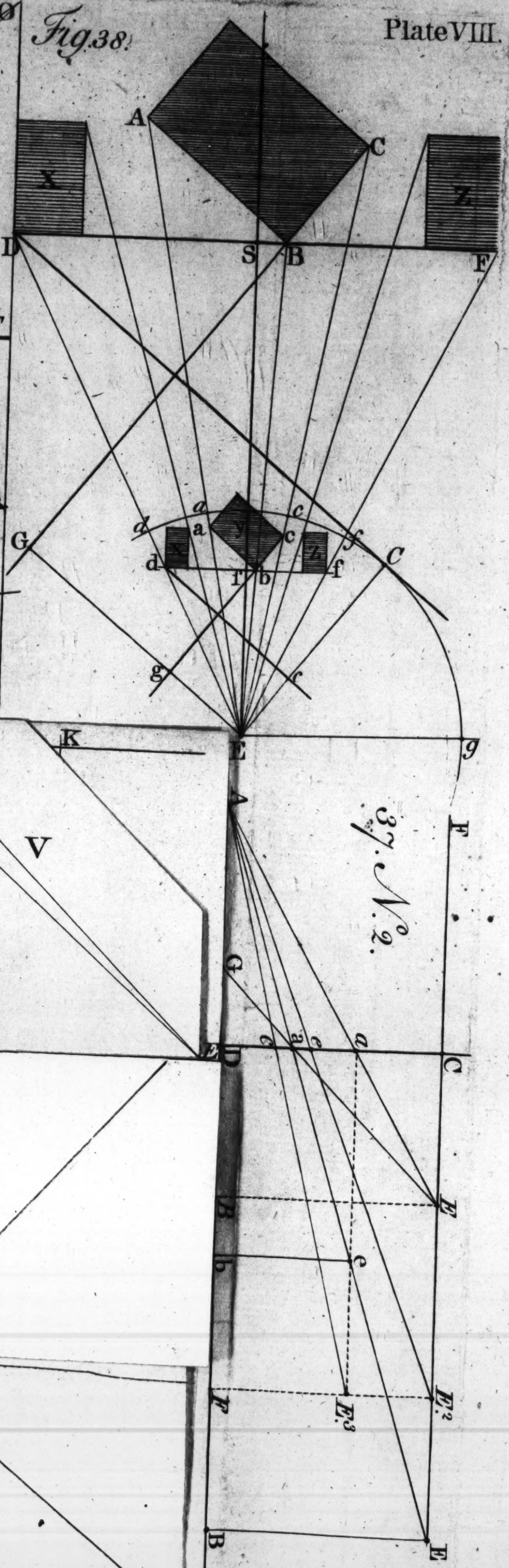
Fig. 35

Fig. 36.

Published Apr. 10th 1774 by Thos. Malton.

Plate VIII.

Fig. 38.



37. N.3.

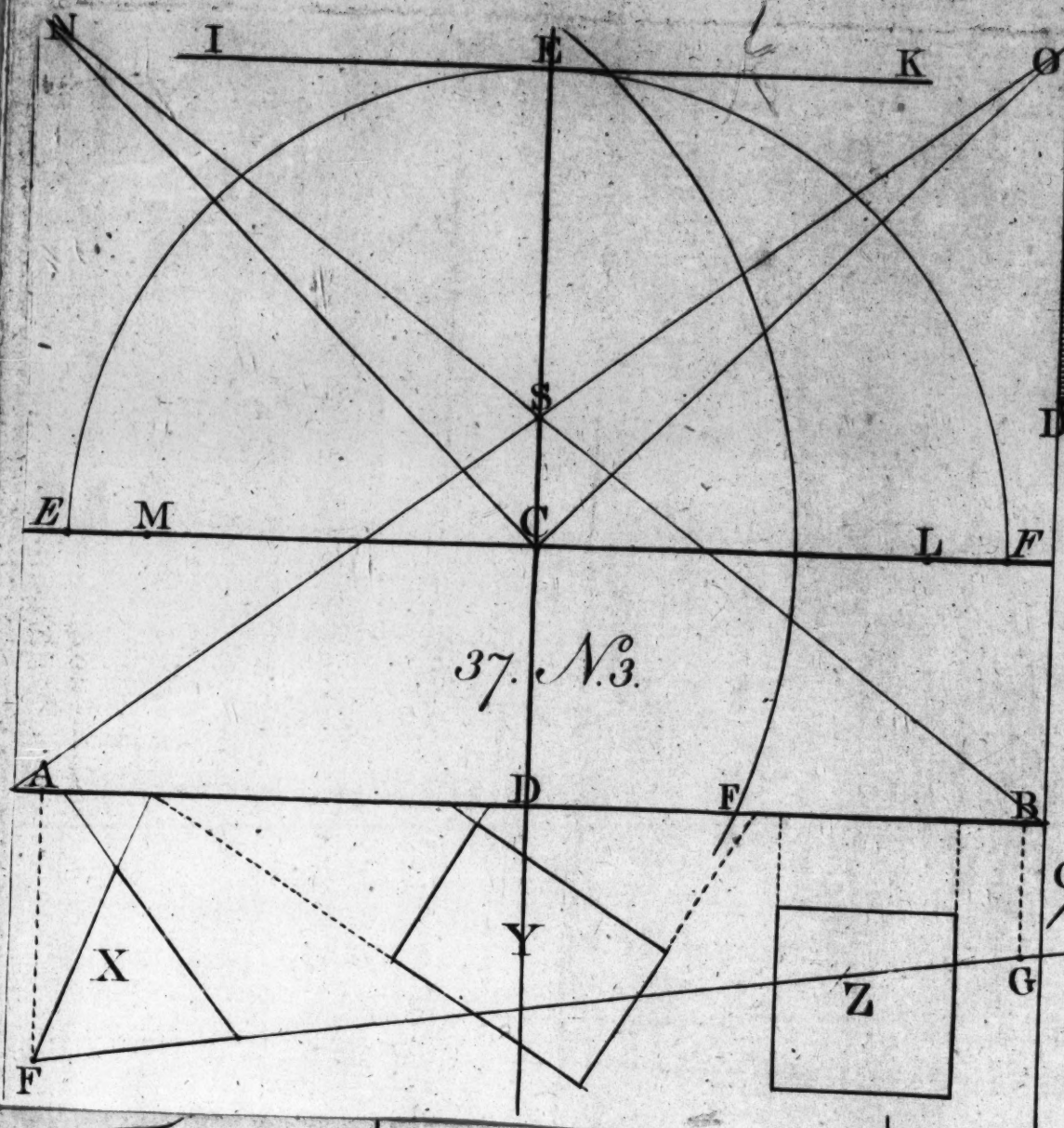
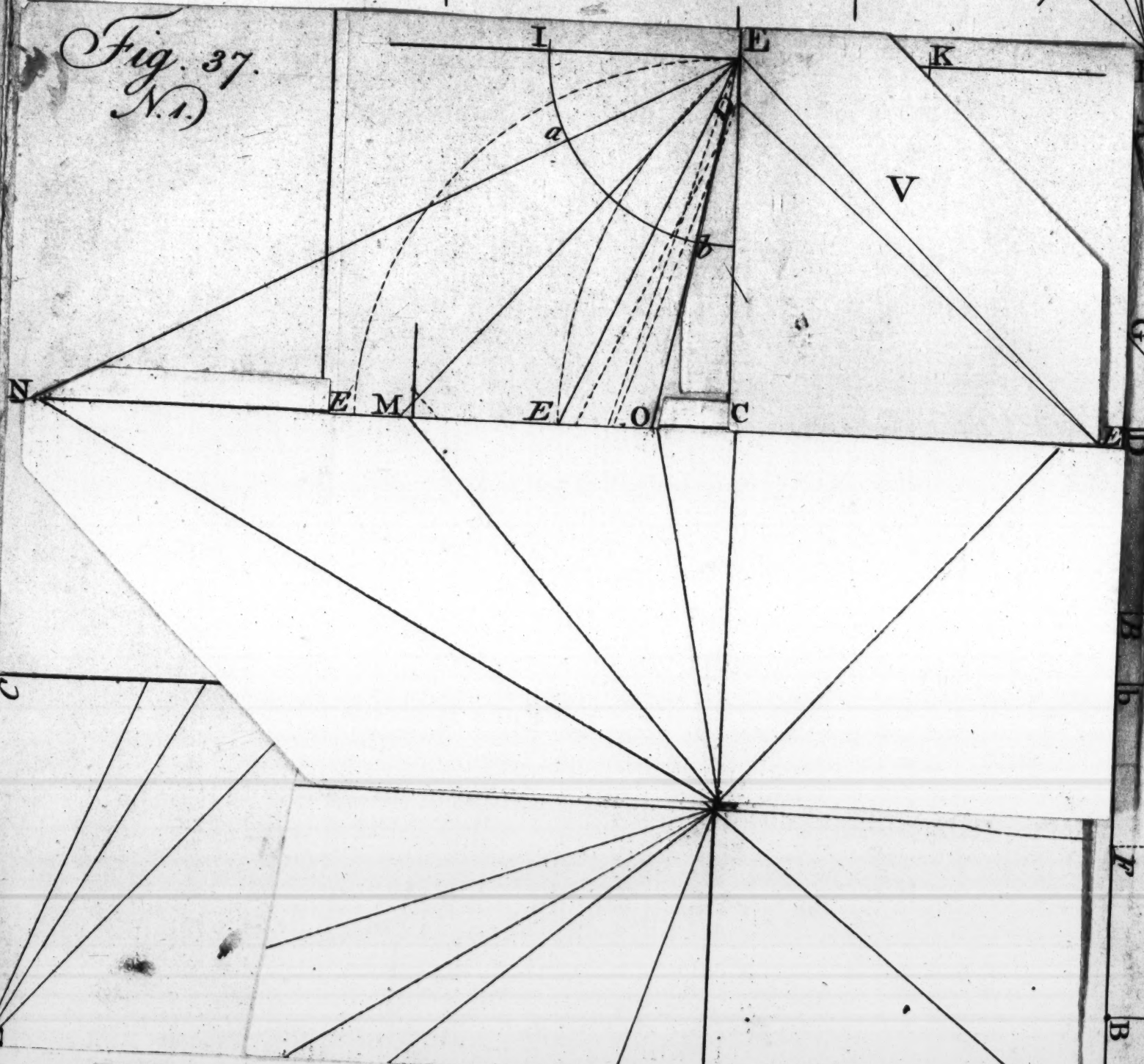


Fig. 37. N.1.



37. N.2.

Plate IX.

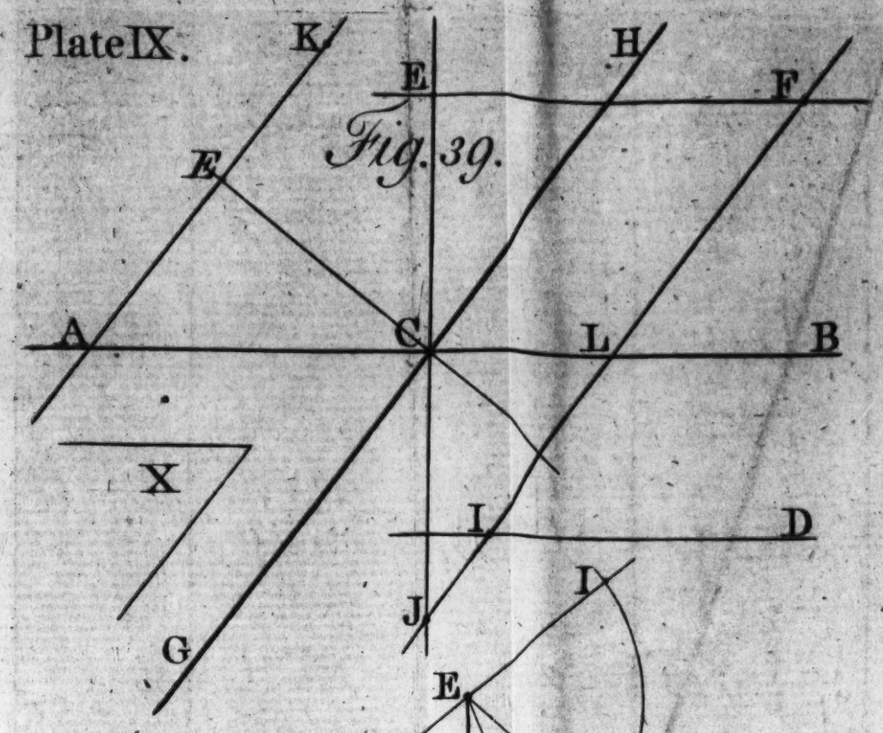


Fig. 39.

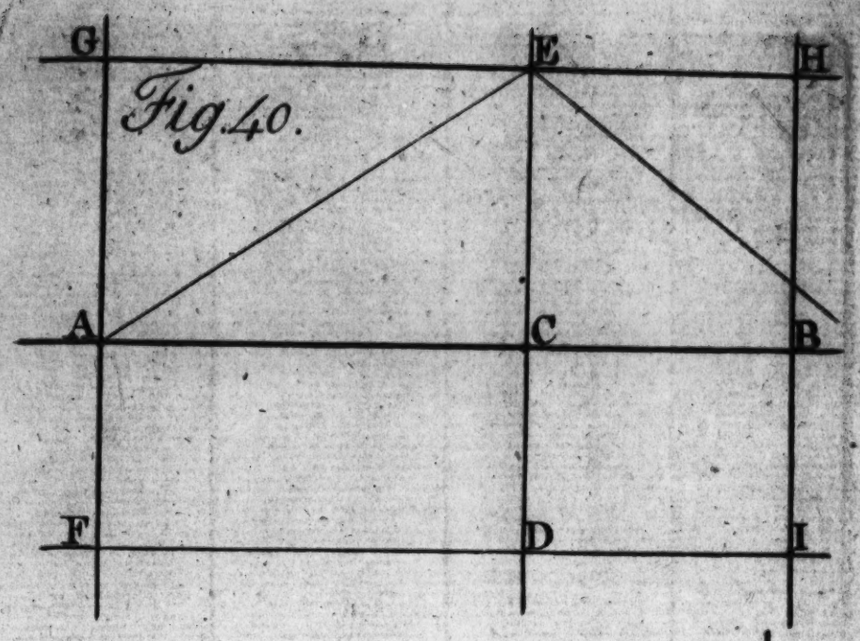


Fig. 40.

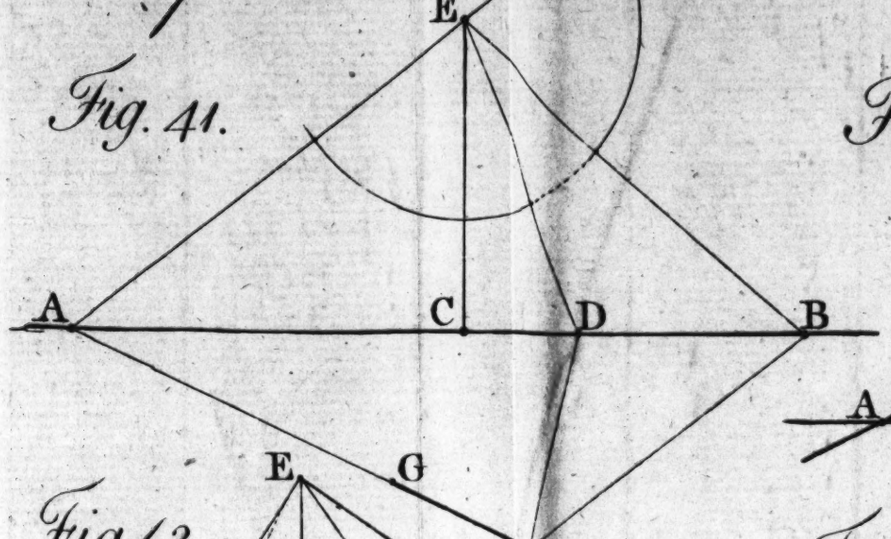


Fig. 41.

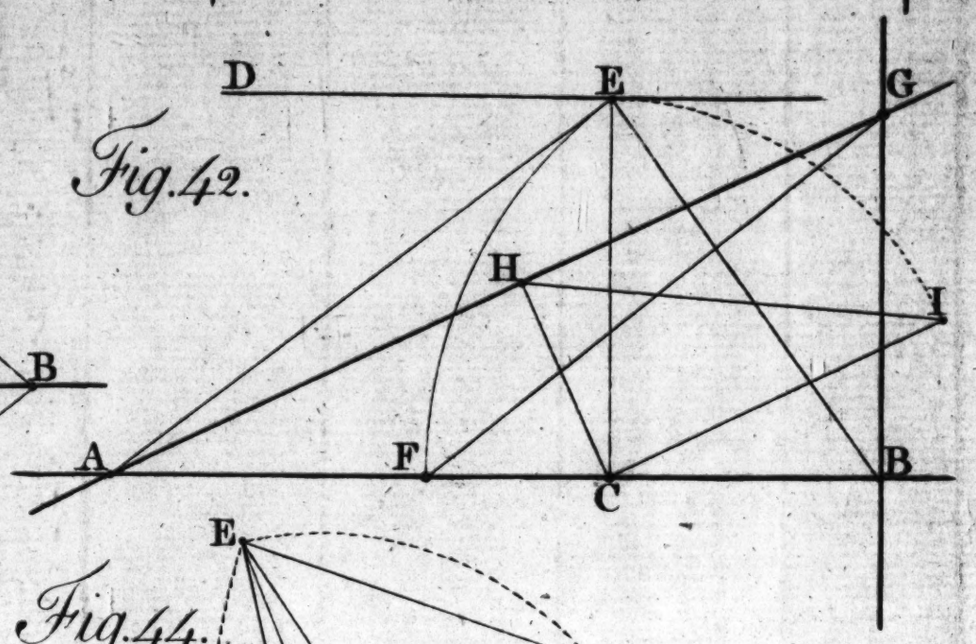


Fig. 42.

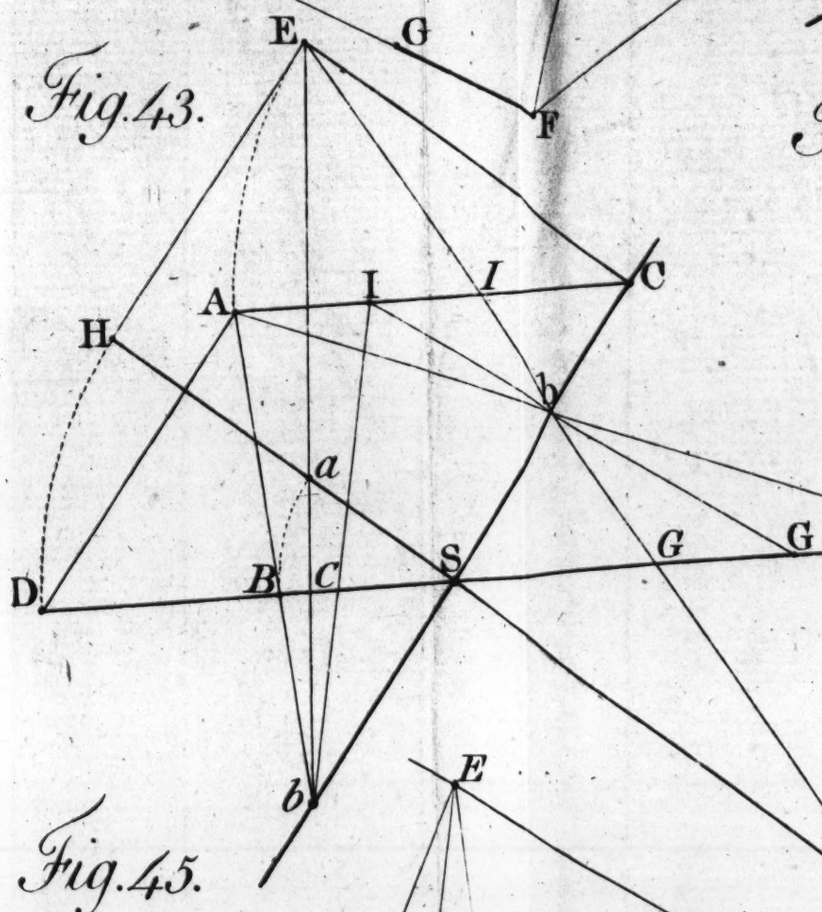


Fig. 43.

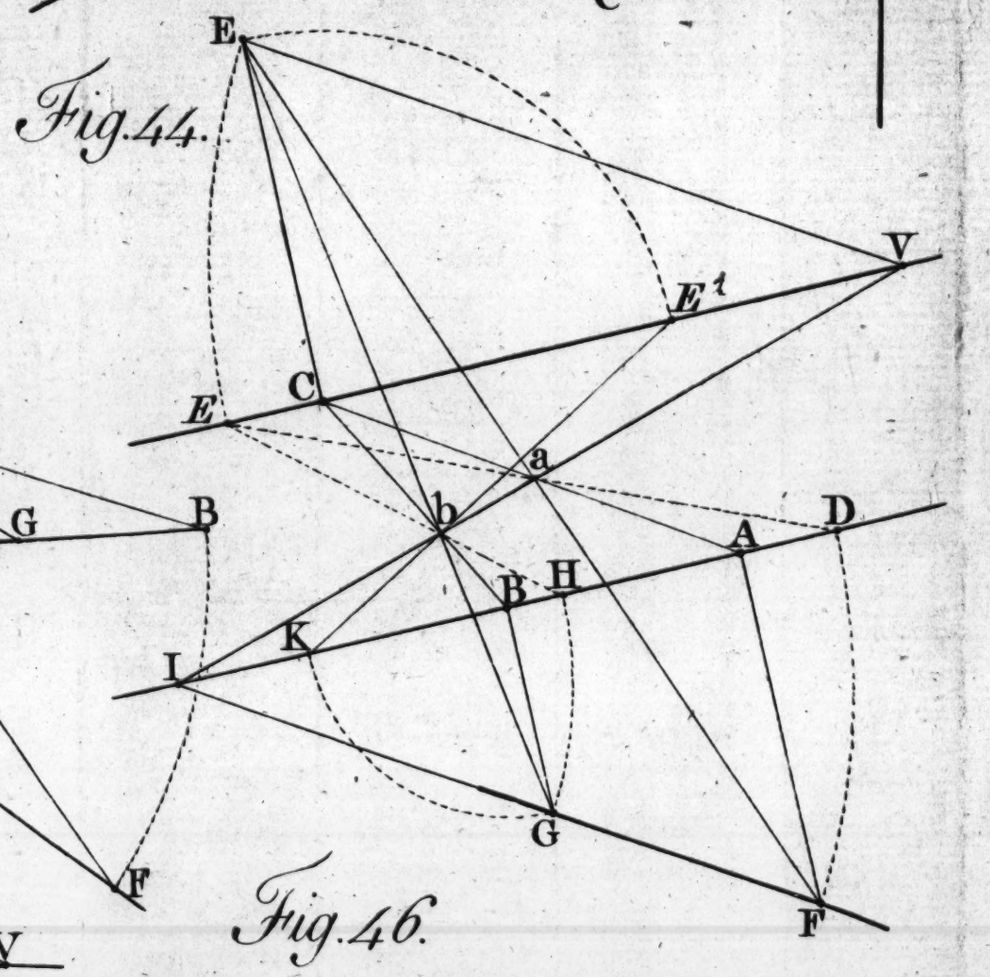


Fig. 44.

Fig. 45.

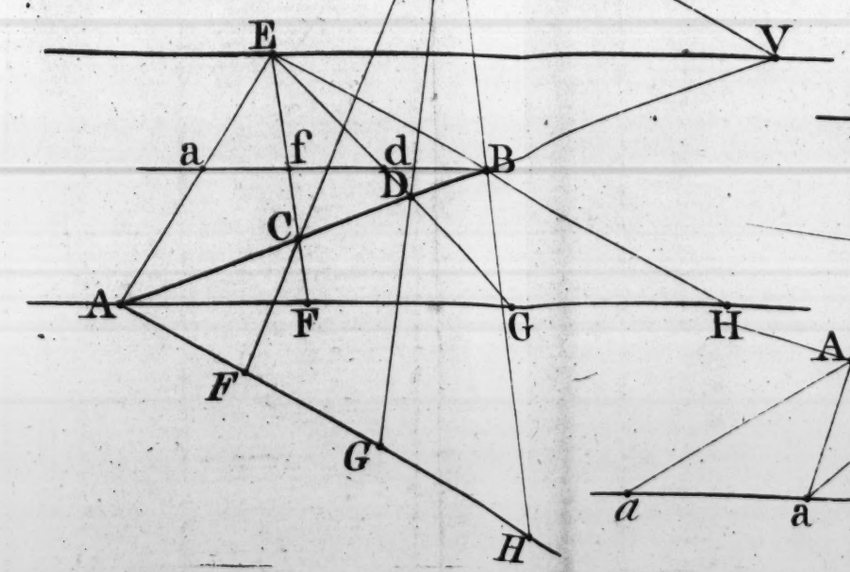


Fig. 46.

Plate X.

Fig. 47.

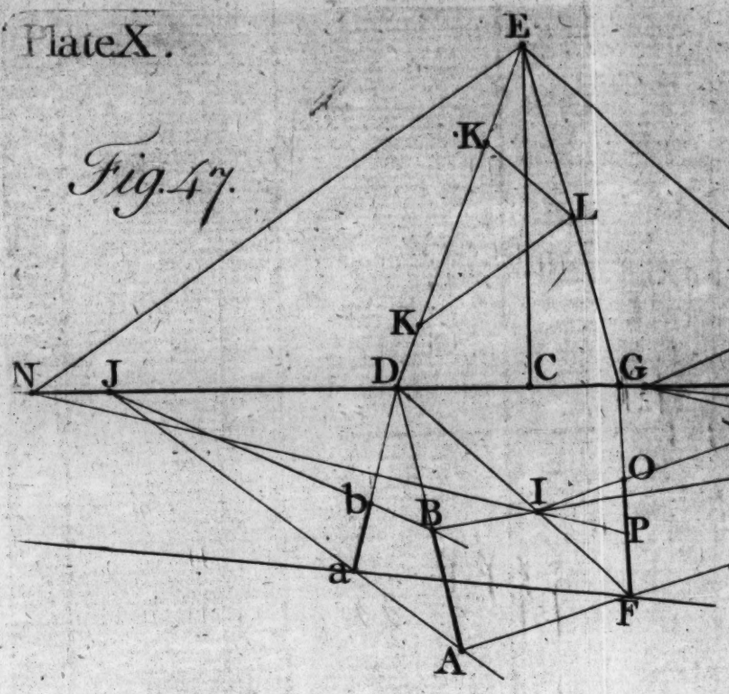


Fig. 48.

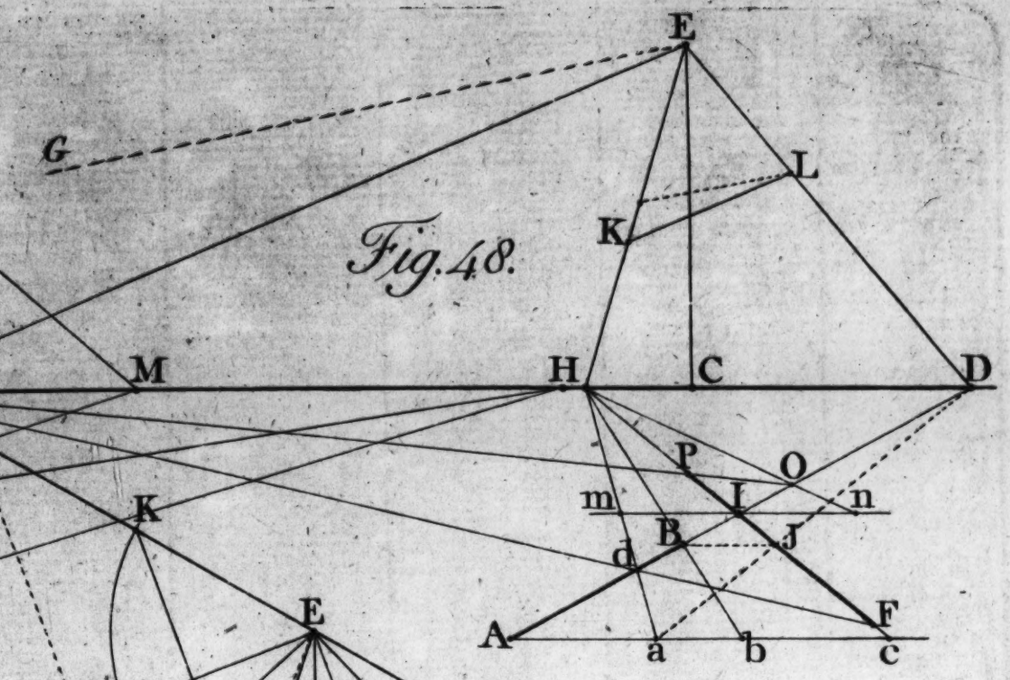


Fig. 49.

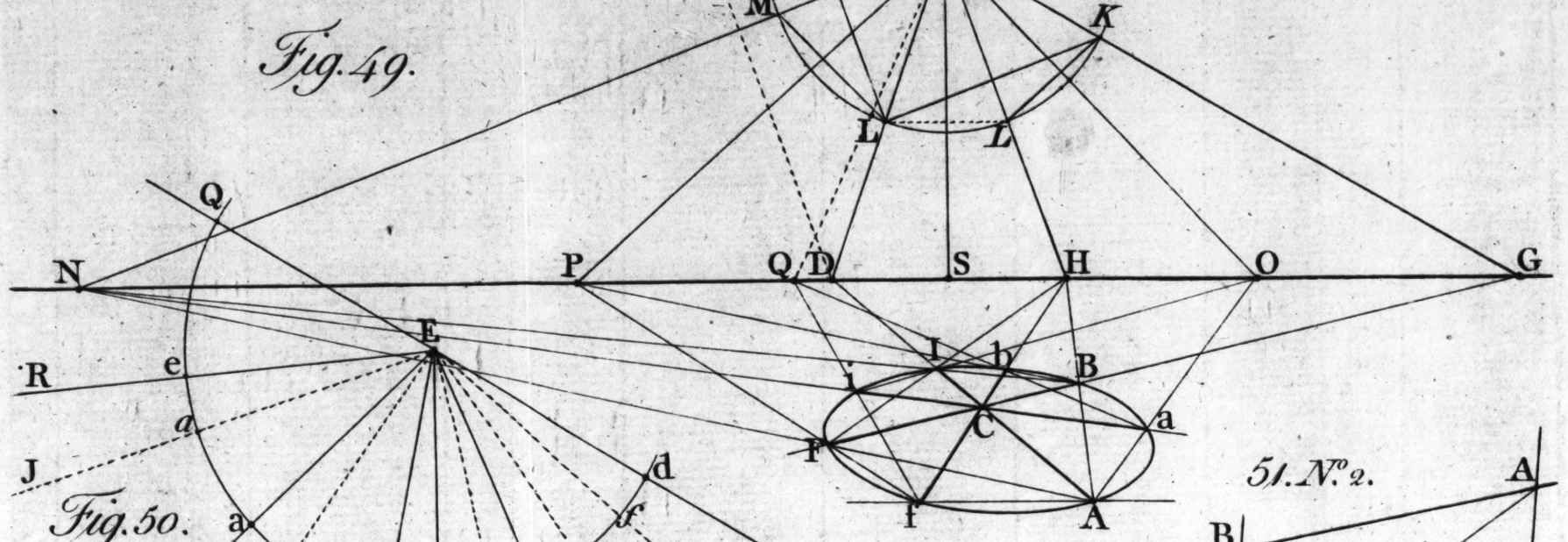
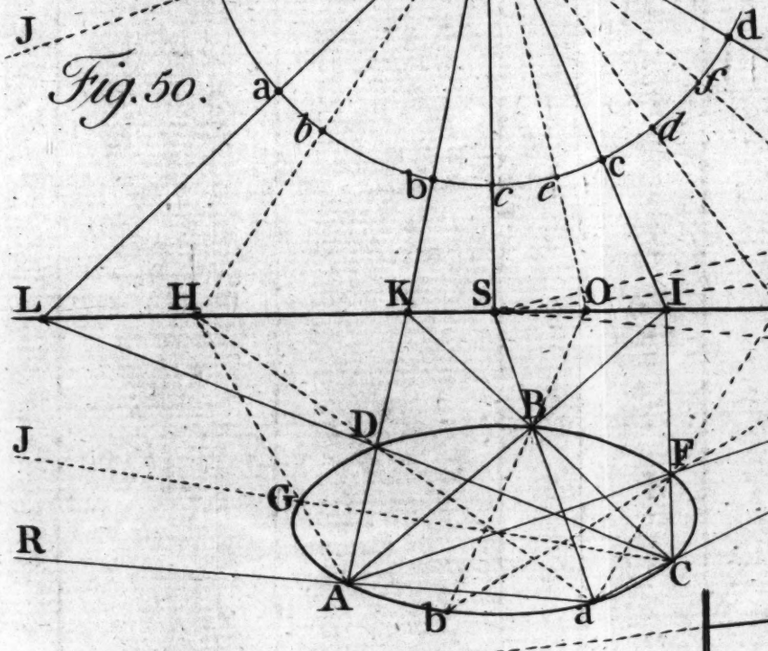
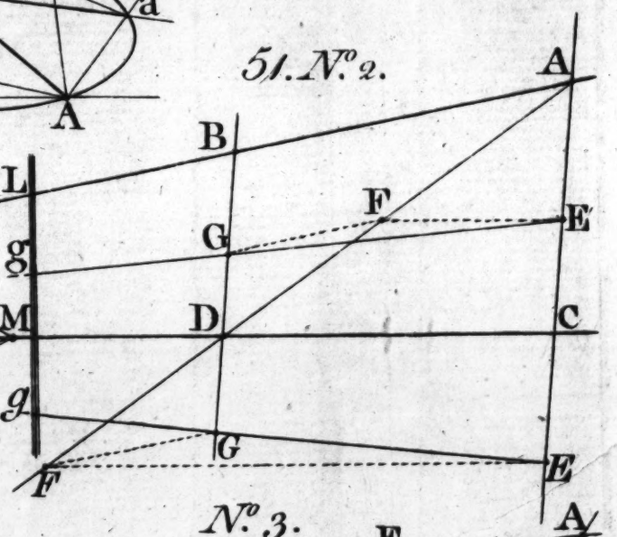


Fig. 50.

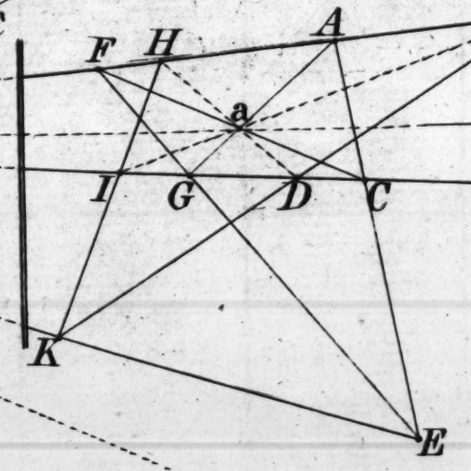


51. N.º 2.



51.

N.º 4.



N.º 3.

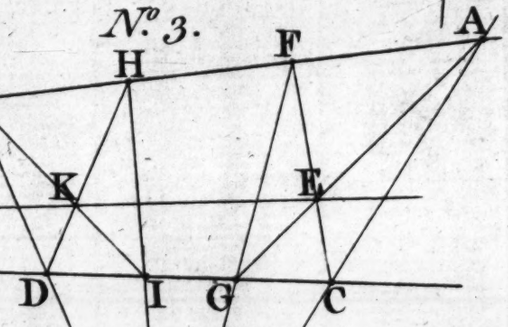
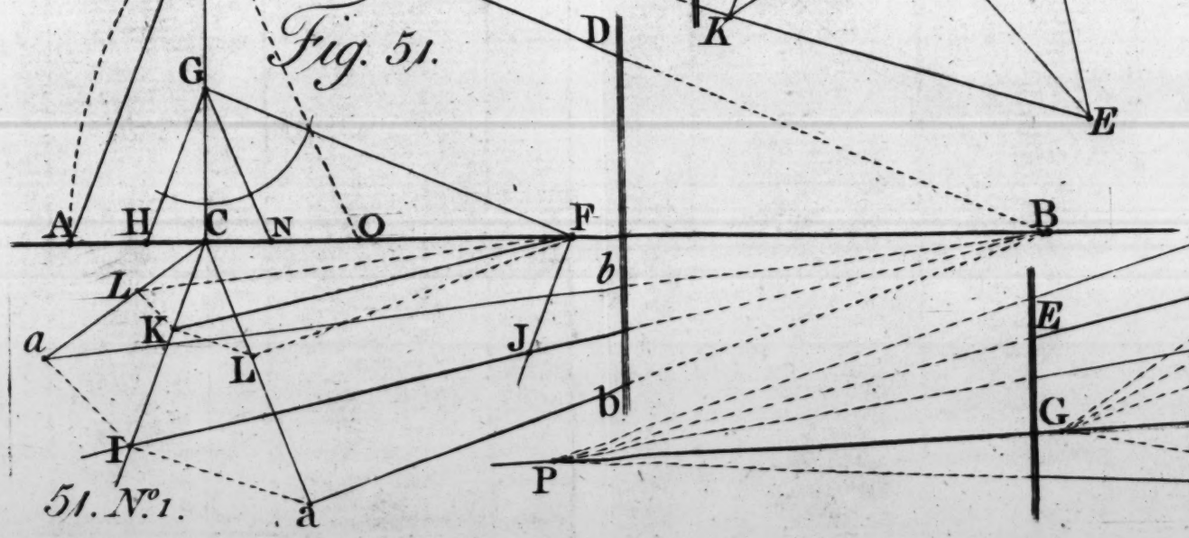


Fig. 51.



51. N.º 1.

N.º 5.

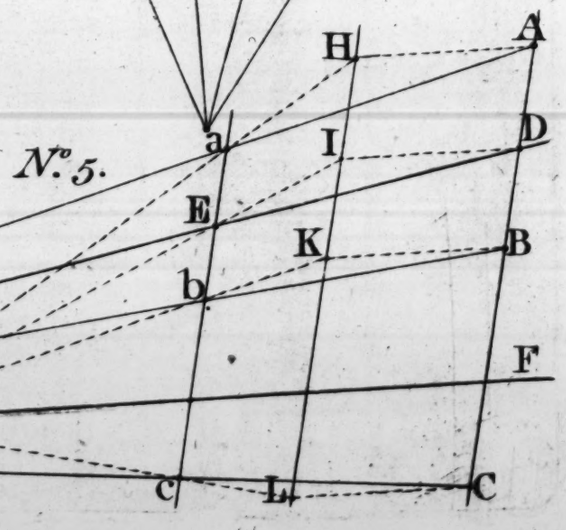


Plate XI.

Fig. 52.

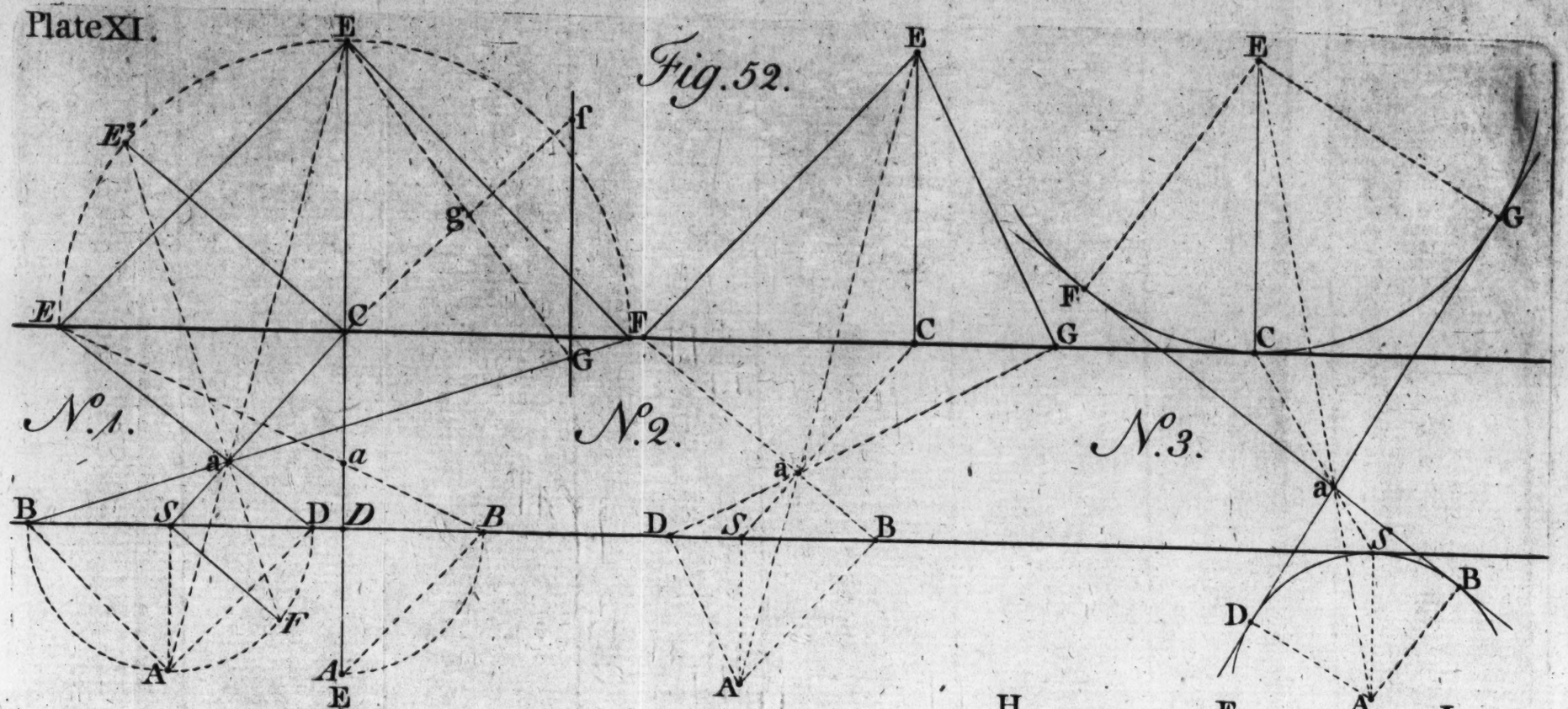


Fig. 53.

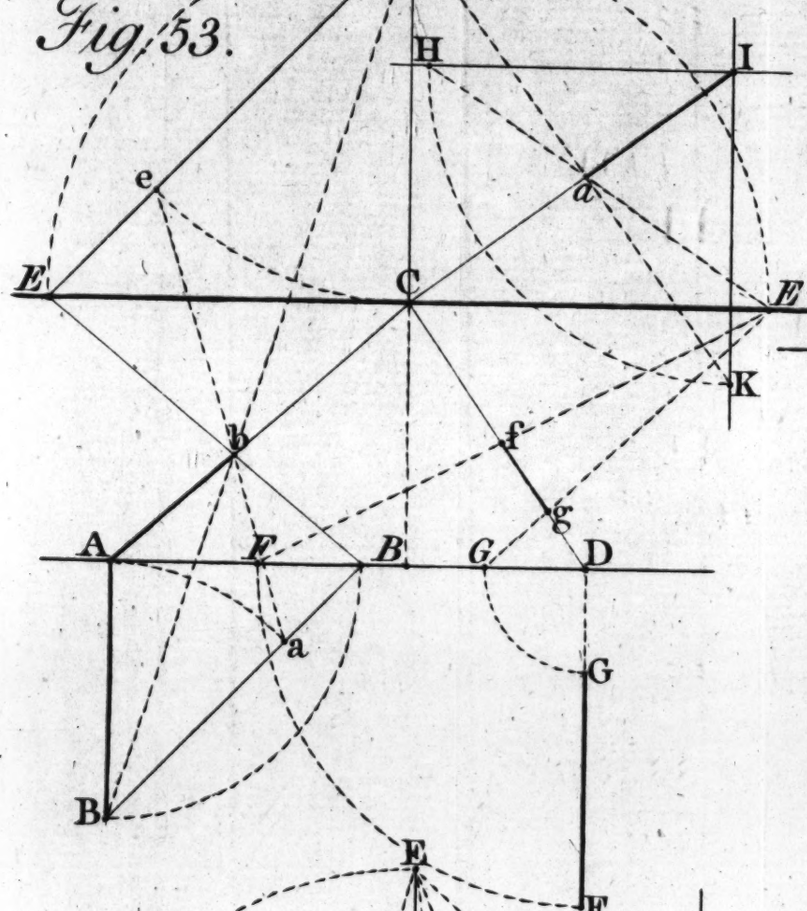


Fig. 54.

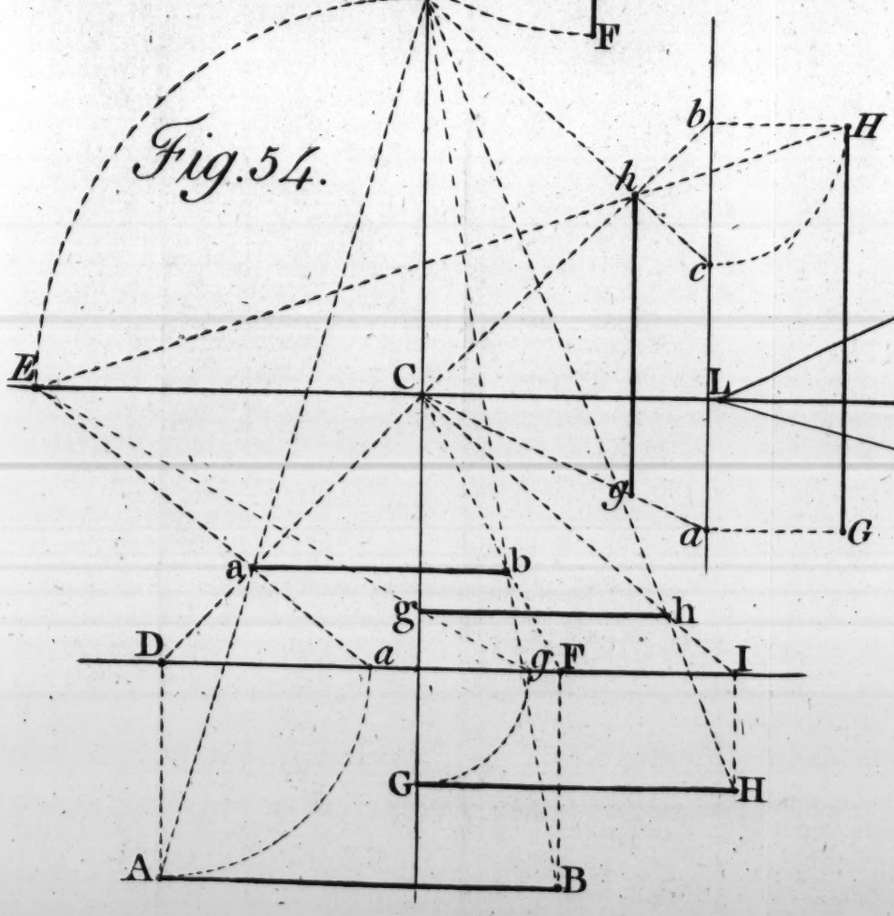


Fig. 55.

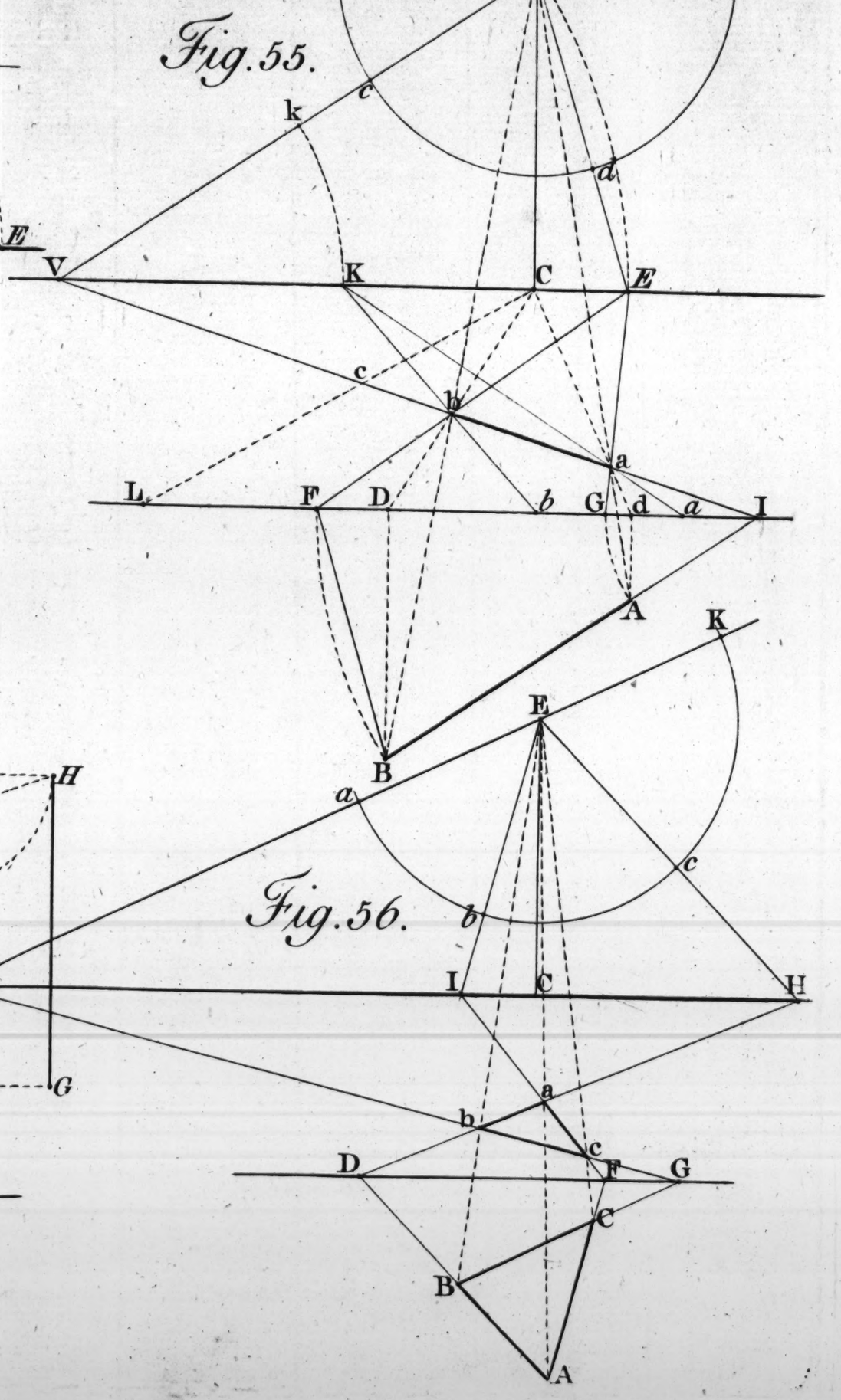


Fig. 56.

Plate XII.

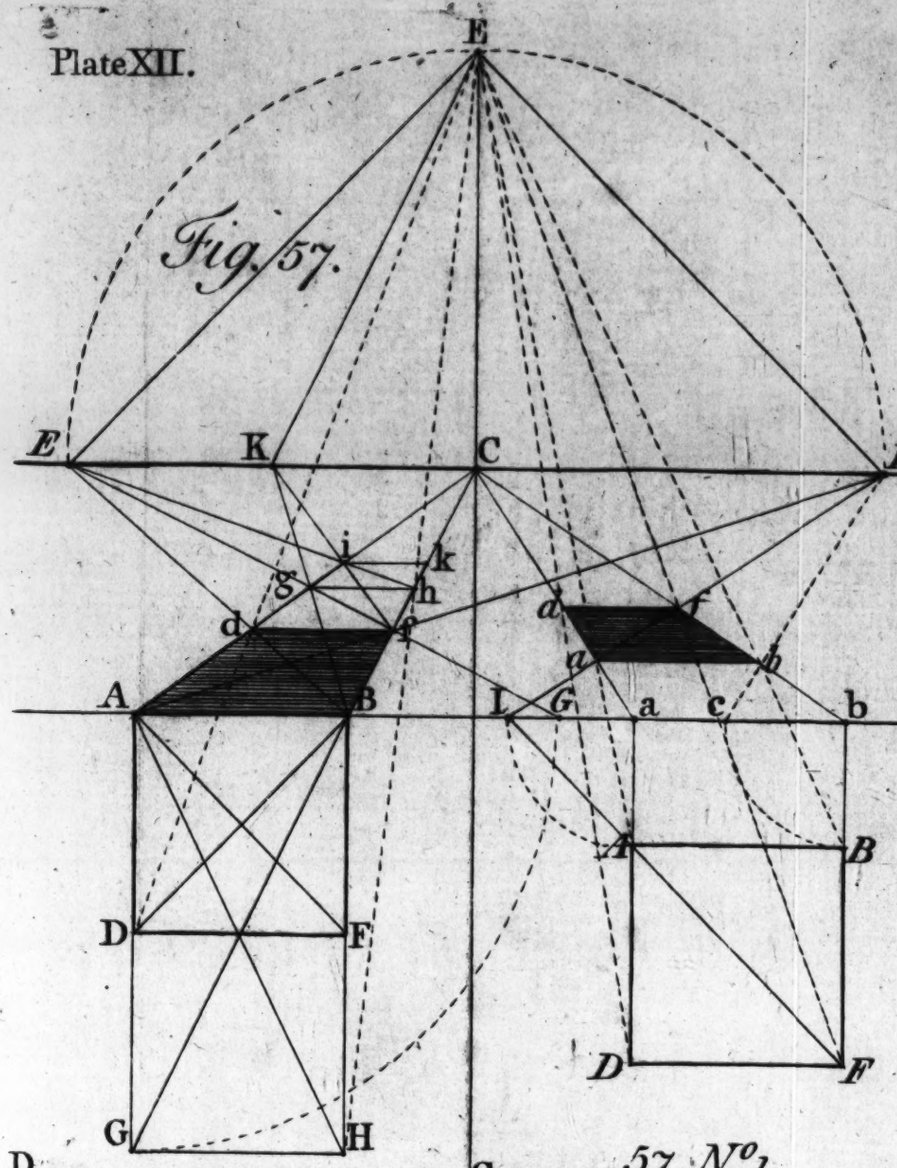


Fig. 57.

57. N.º 1.

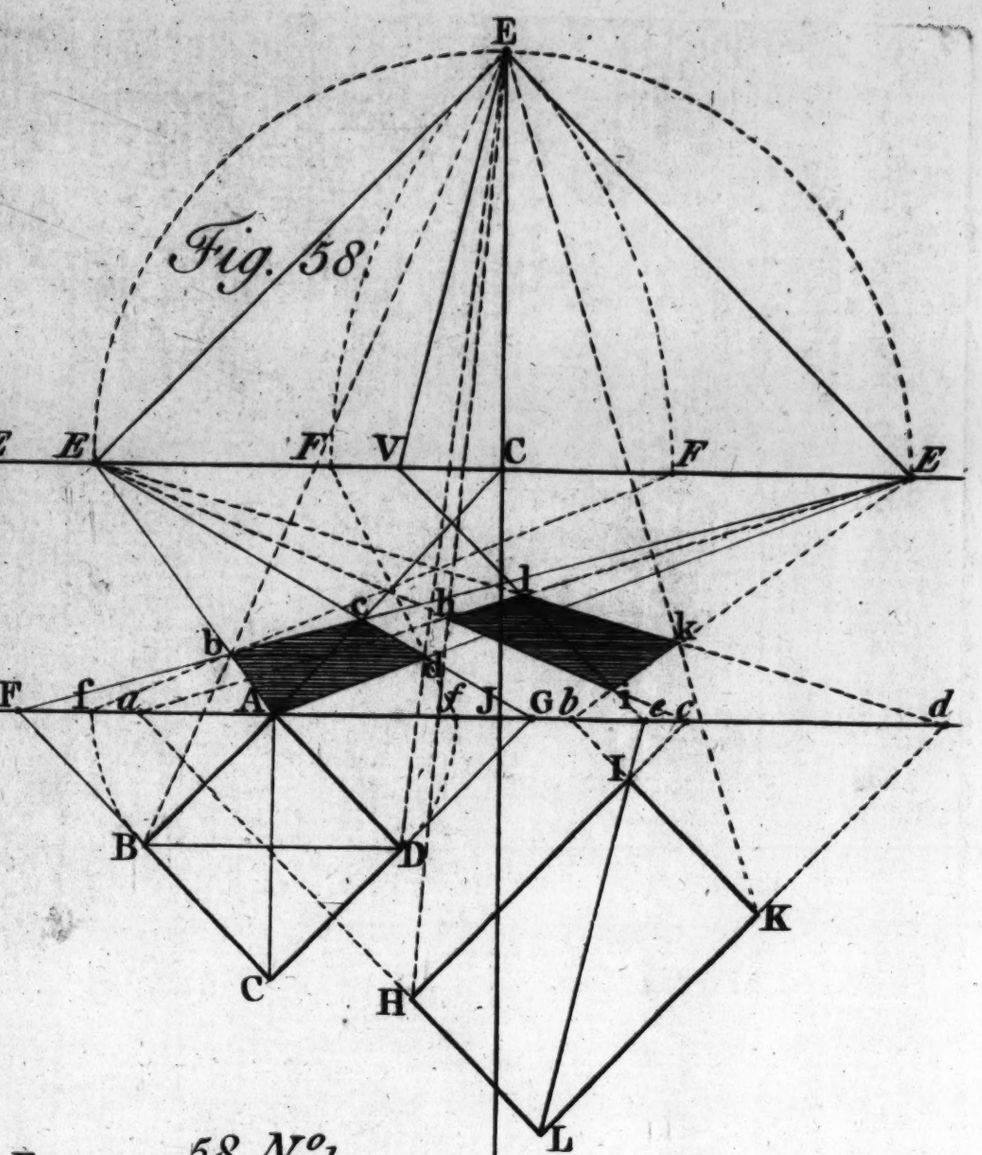


Fig. 58.

58. N.º 1.

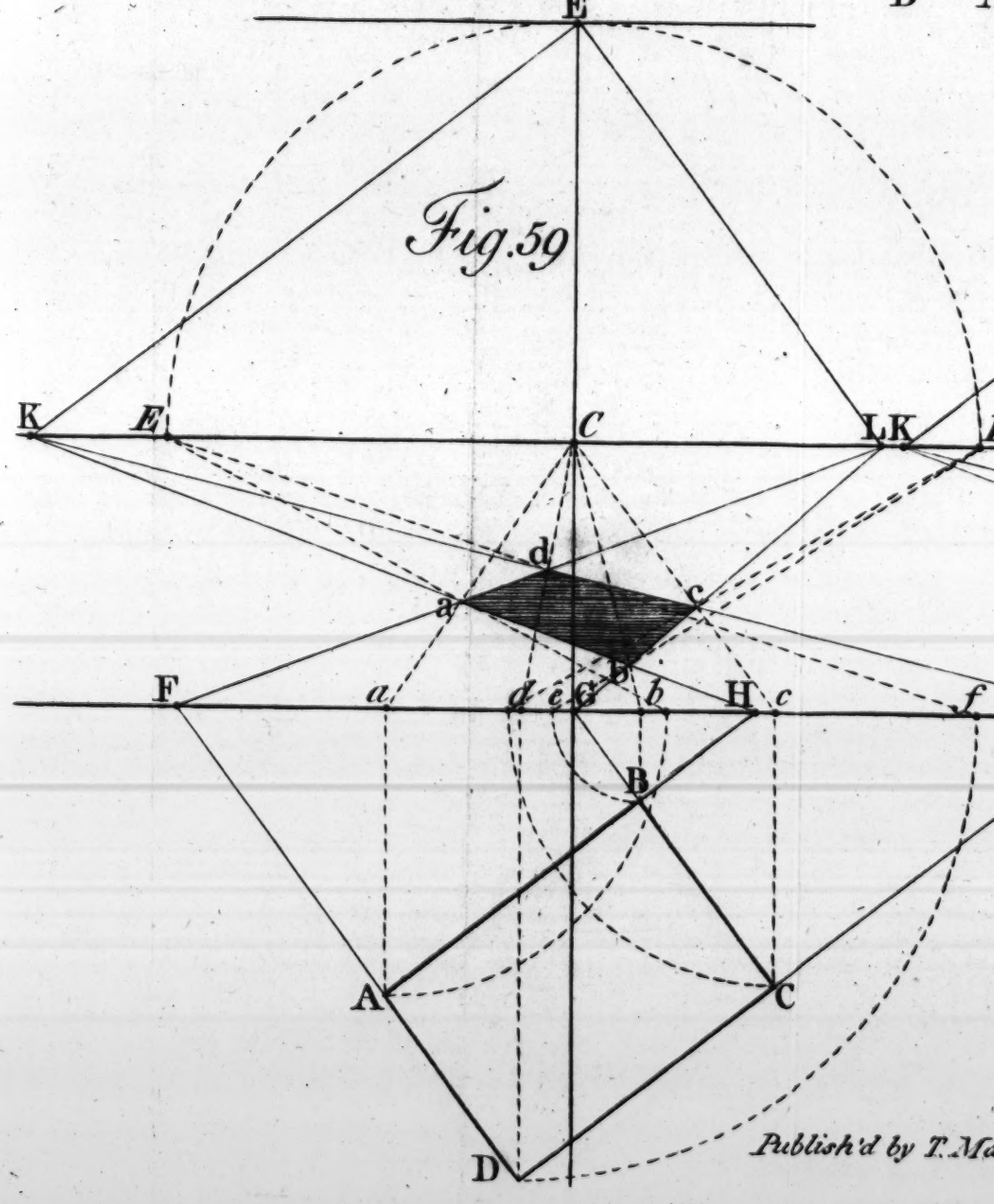


Fig. 59.

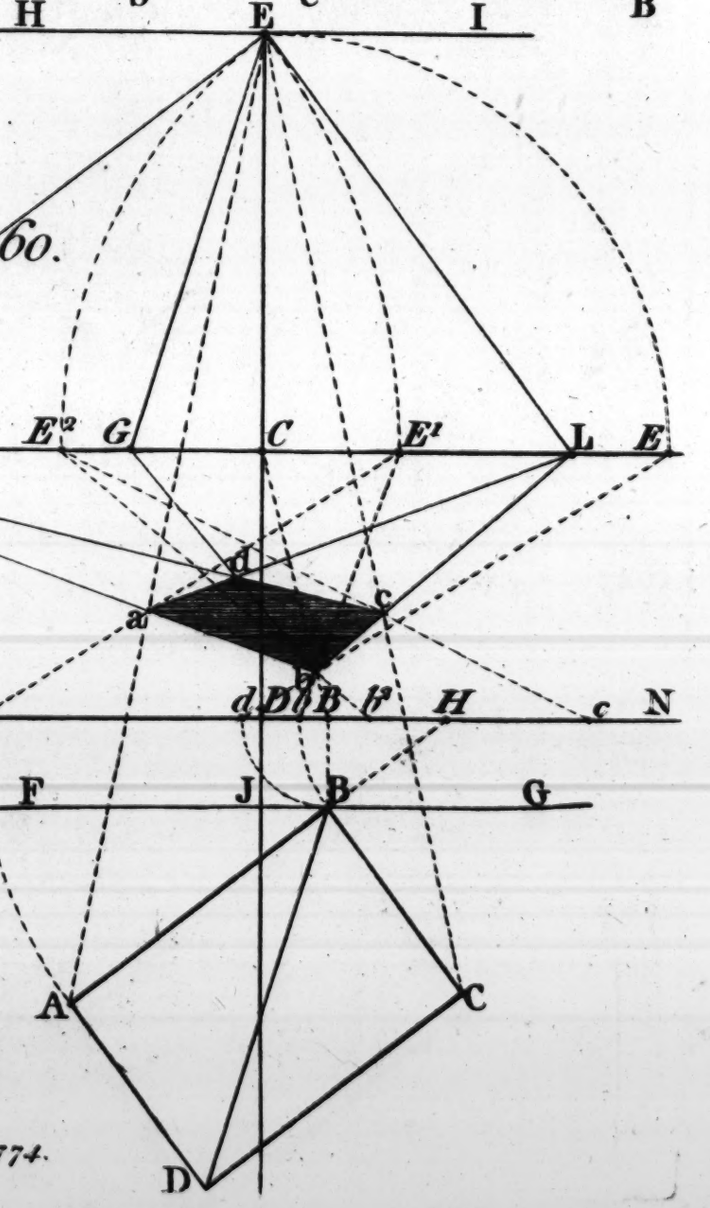


Fig. 60.

Publish'd by T. Malton May 2.d 1774.

Plate XIII.

Fig: 61.

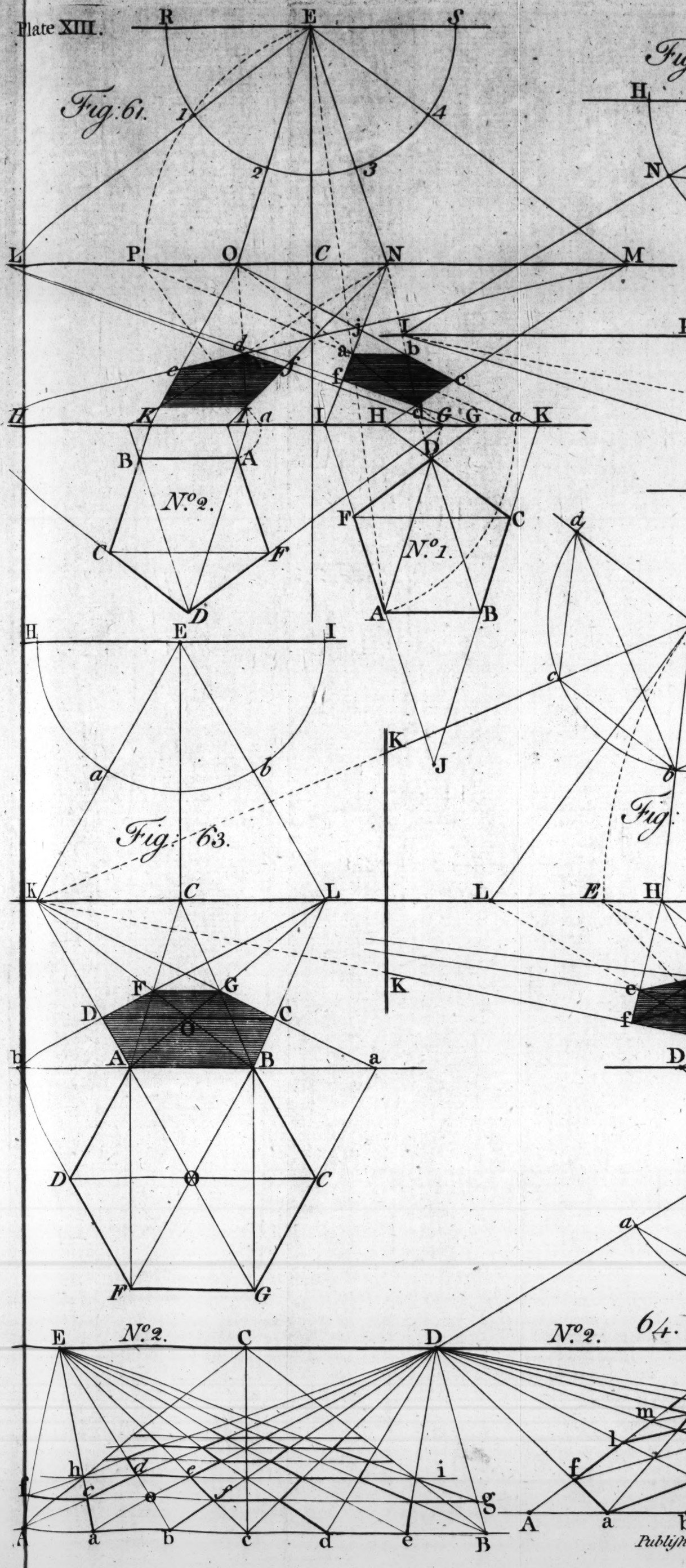


Fig: 62.

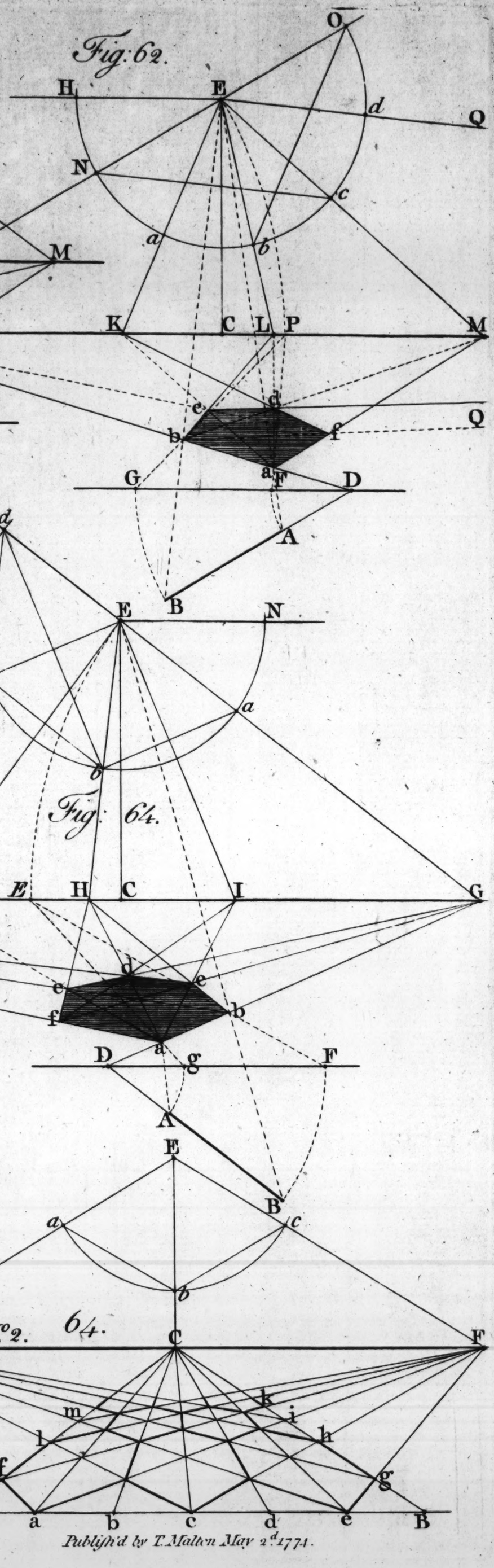


Fig: 63.

Fig: 64.





Publish'd by T. Malton May 2.d 1774.

Plate XIV.

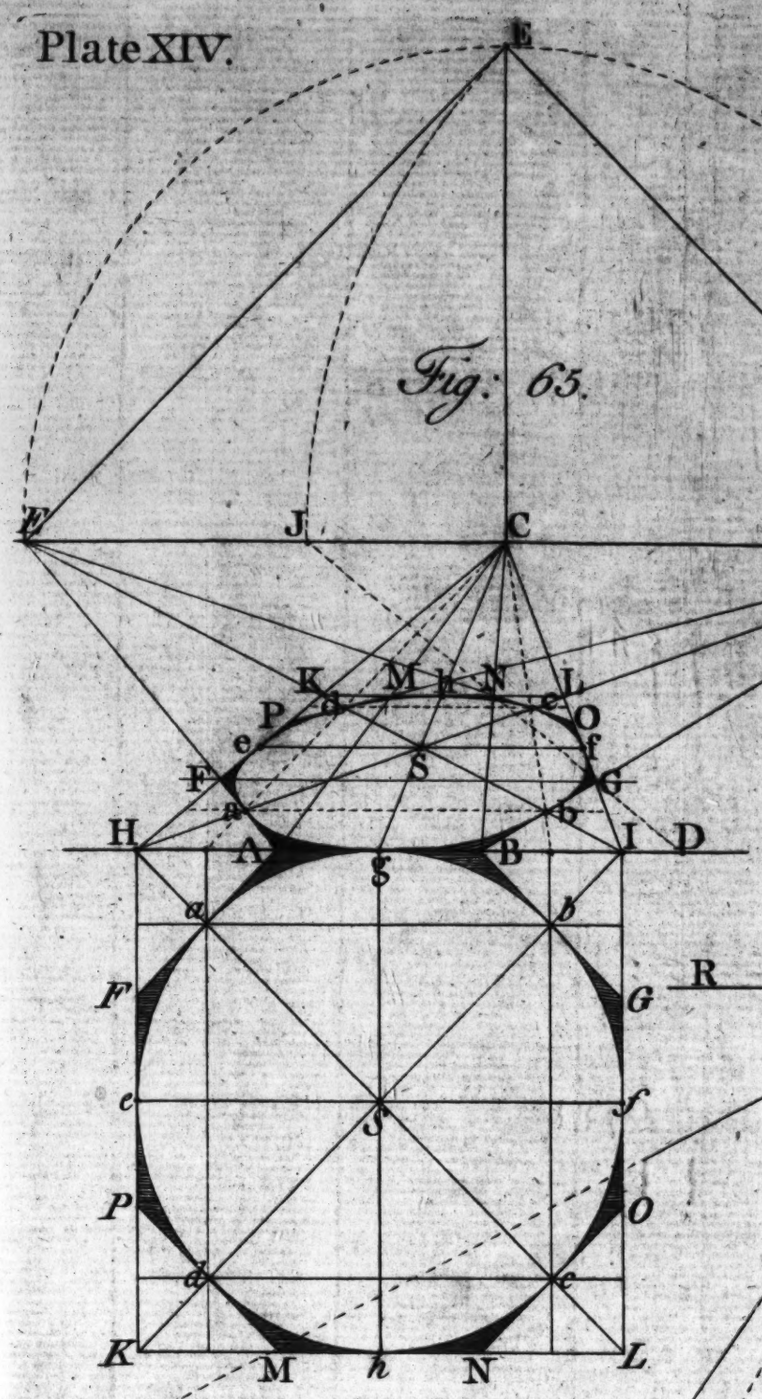


Fig. 65.

Fig. 66

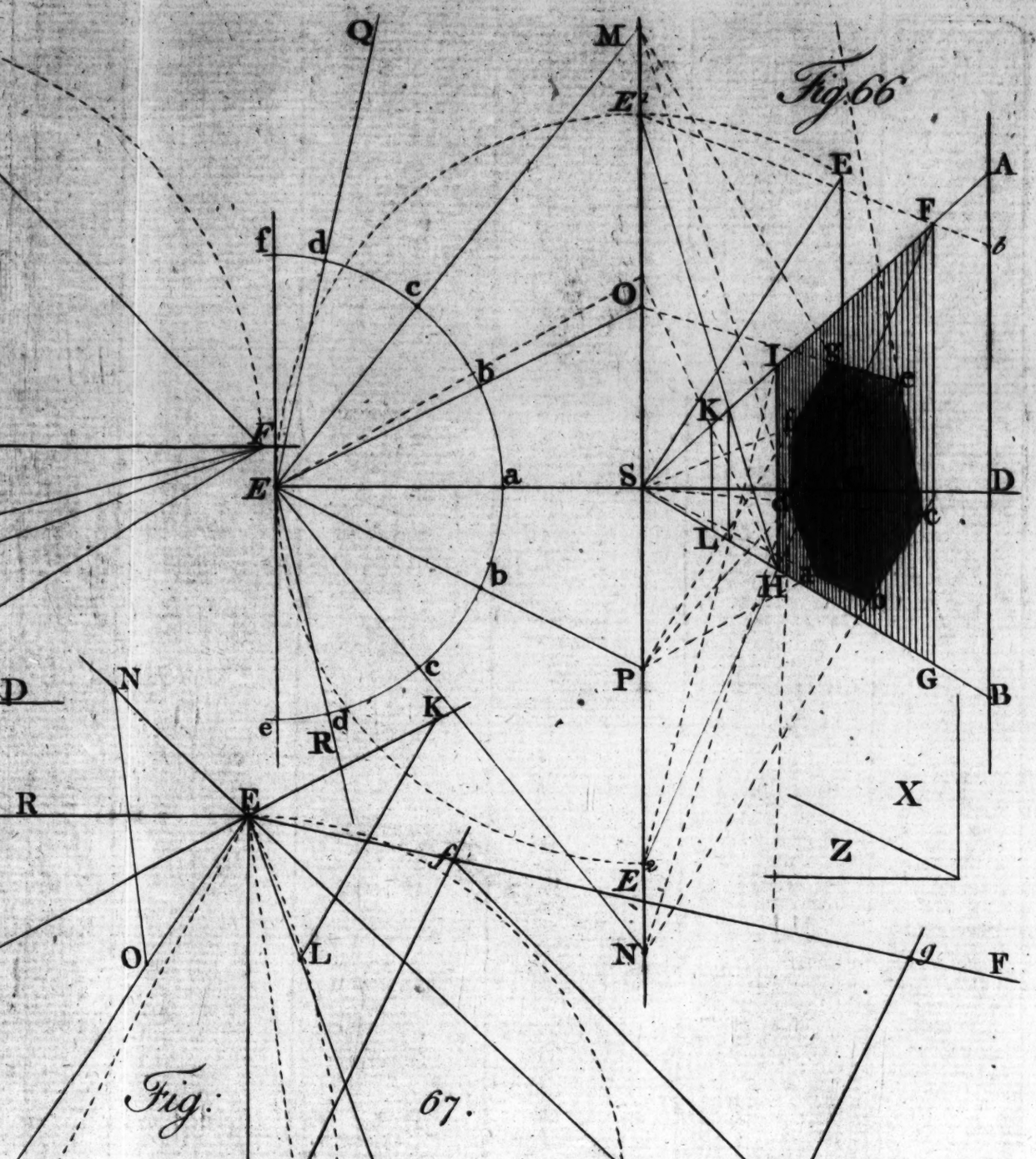


Fig. 67.

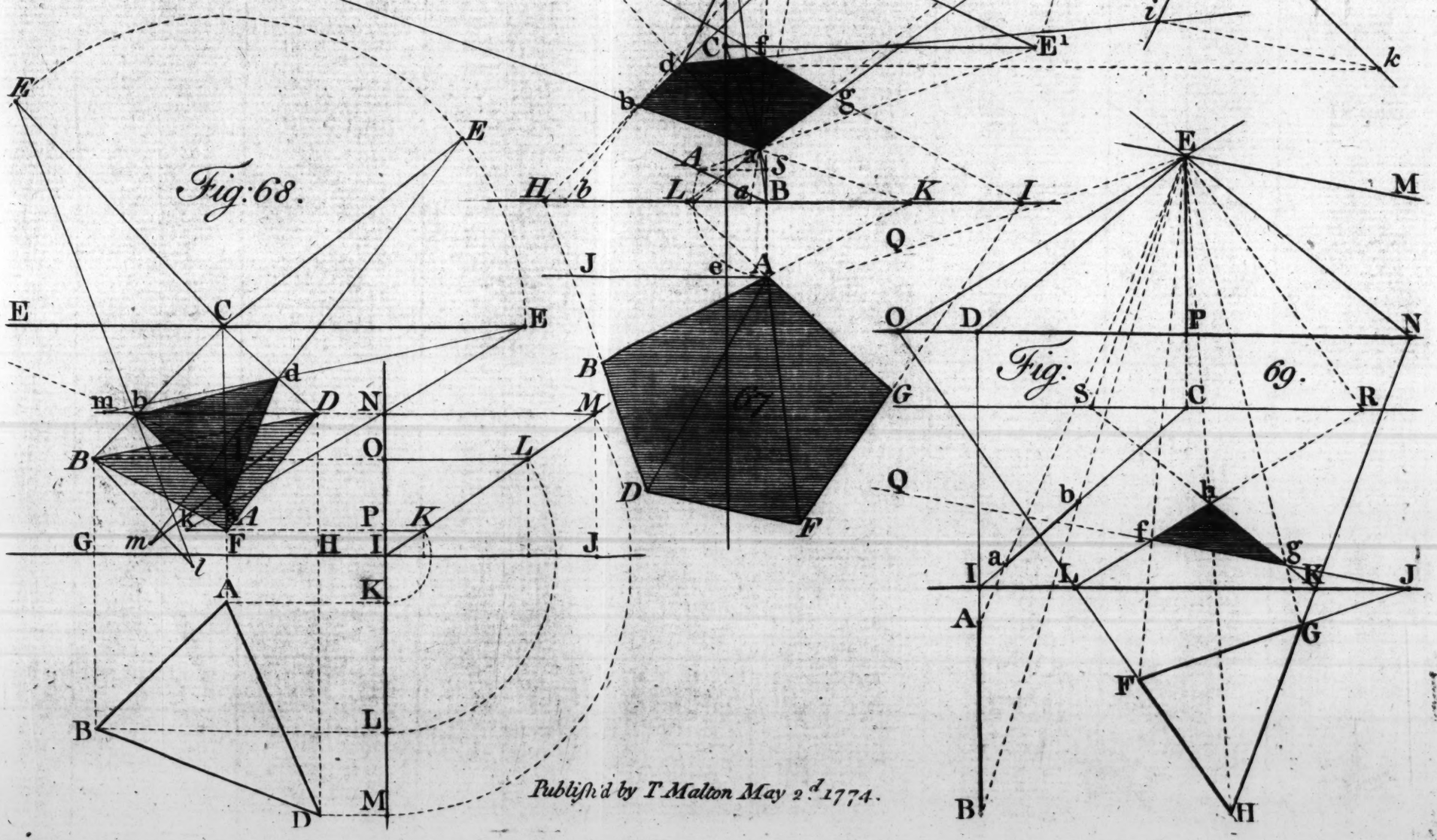
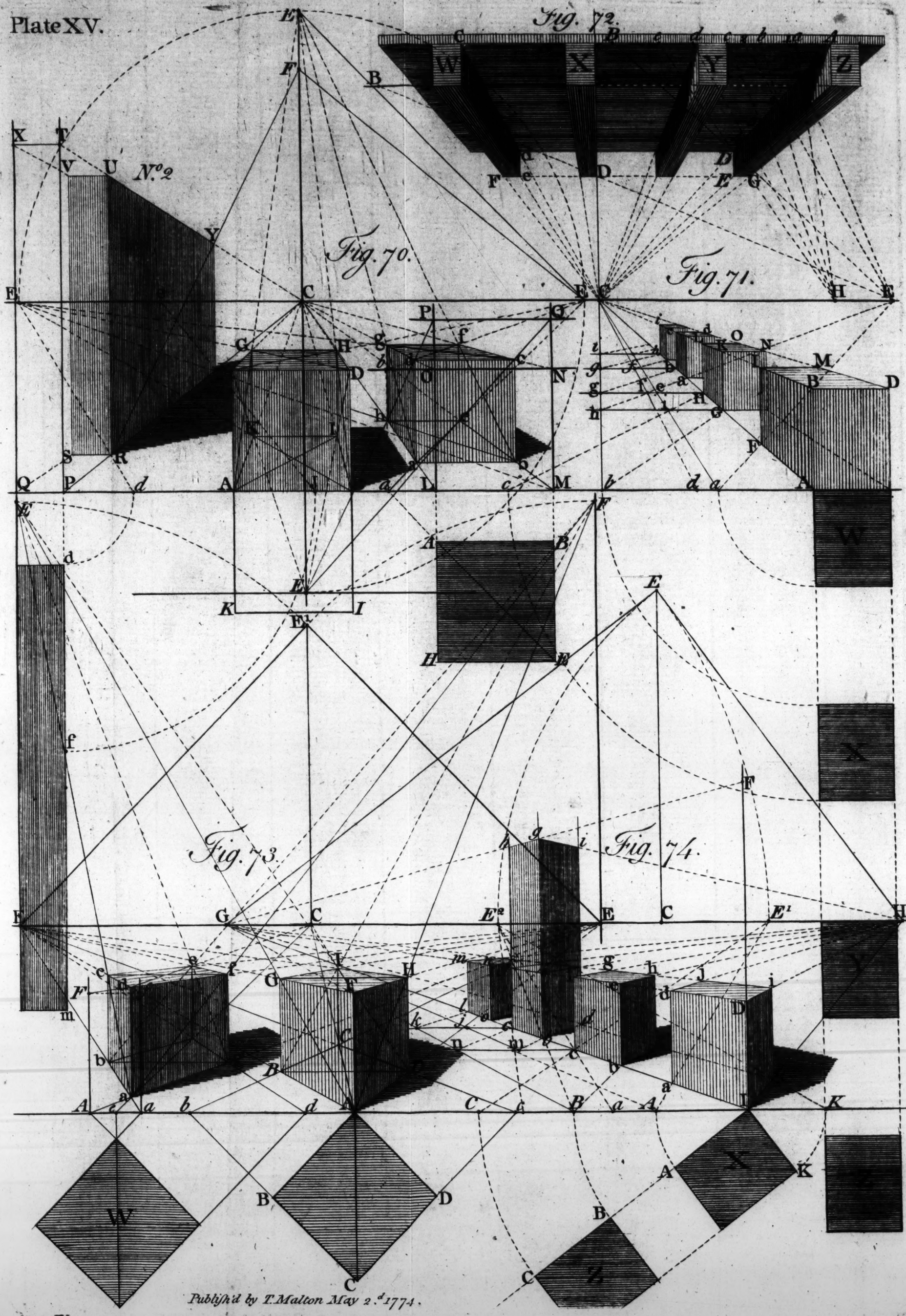


Fig: 68.

Published by T. Malton May 2.d 1774.

Plate XV.



Publish'd by T. Malton May 2.d 1774.

Plate XVI.

Fig. 75.

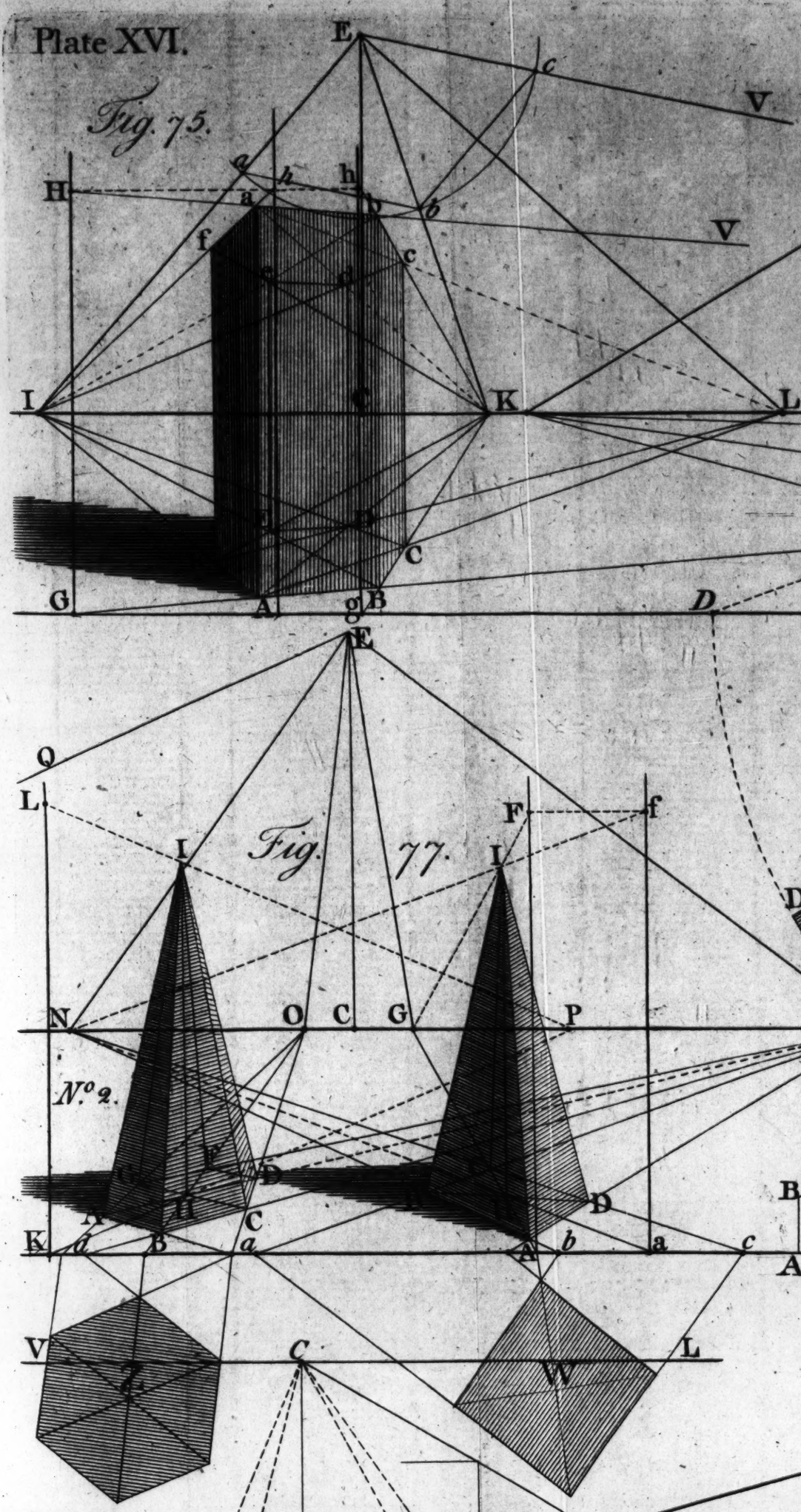


Fig. 76.

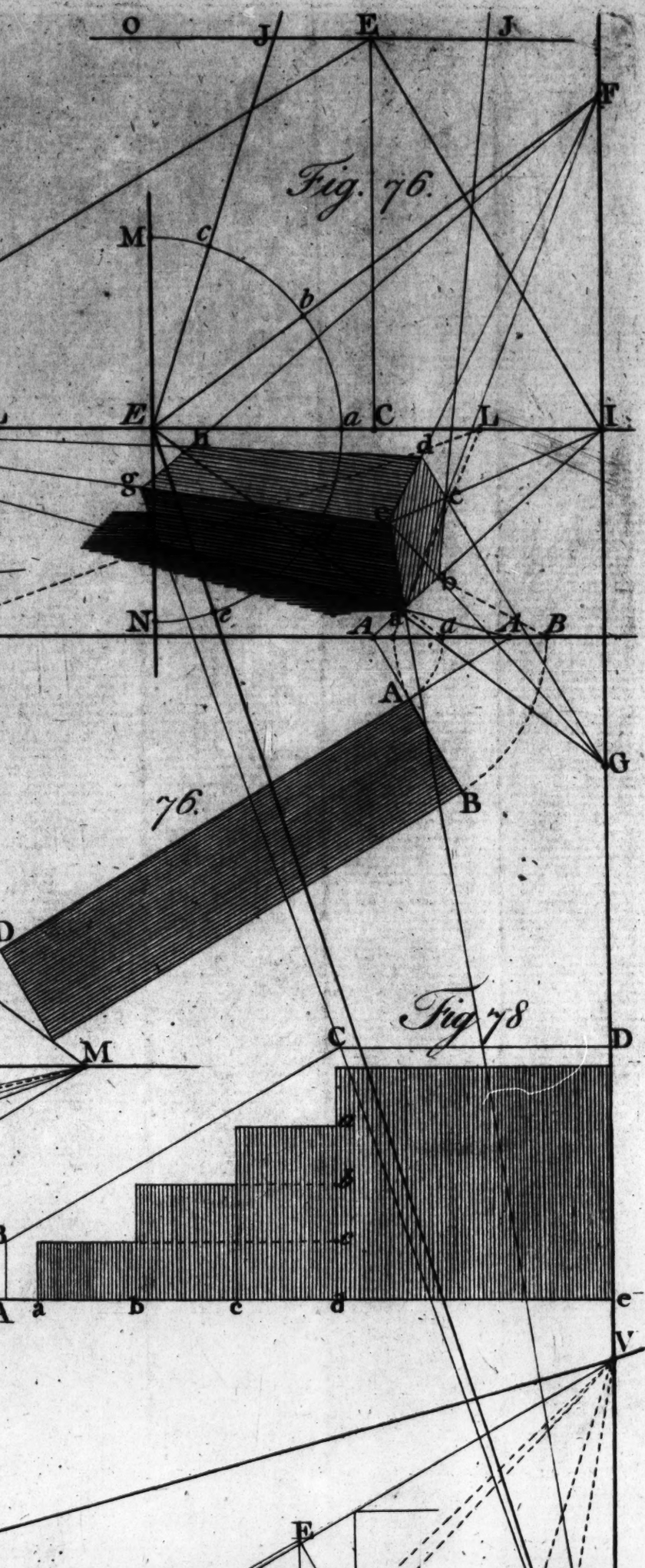


Fig. 77.

Fig. 78

Fig. 79.

Fig. 80.

Publish'd by T. Malton May 2d. 1771.

Plate XVII

Fig. 81.

Fig. 82.

81.

Fig. 83.

Fig. 84.

83.

Fig. 85.

N.1.

Publish'd by T. Malton May 2.d 1774.

Plate XVIII.

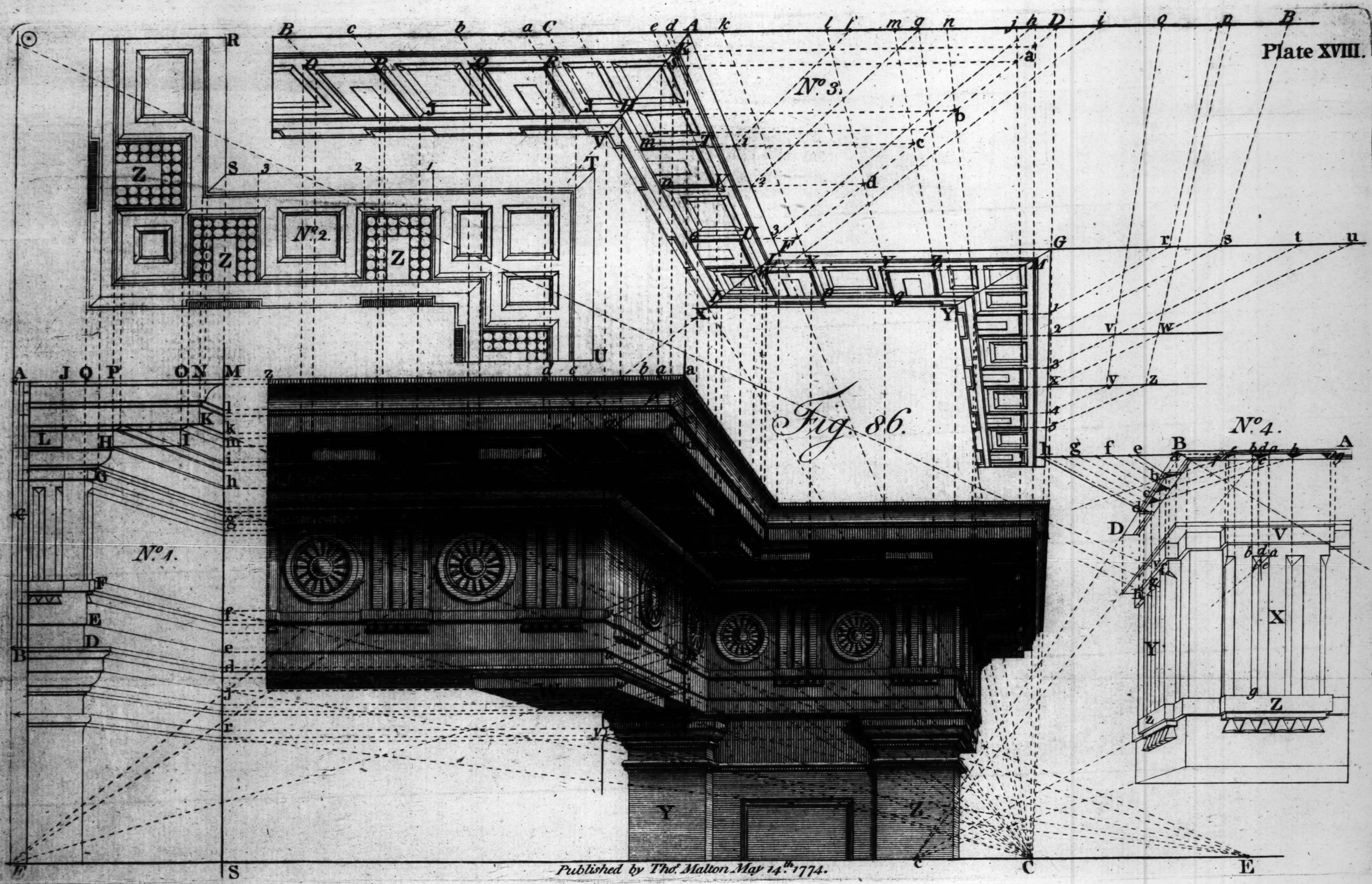


Fig. 86.

Published by Thos. Malton May 14th. 1774.

Plate XIX

Fig. 87.

Fig. 88.

88. N.º 2.

N.º 2.

Fig. 89.

N.º 1.

Fig. 90.

N.º 1

Published by Tho.s Malton May 28. 1774.

Plate XX.

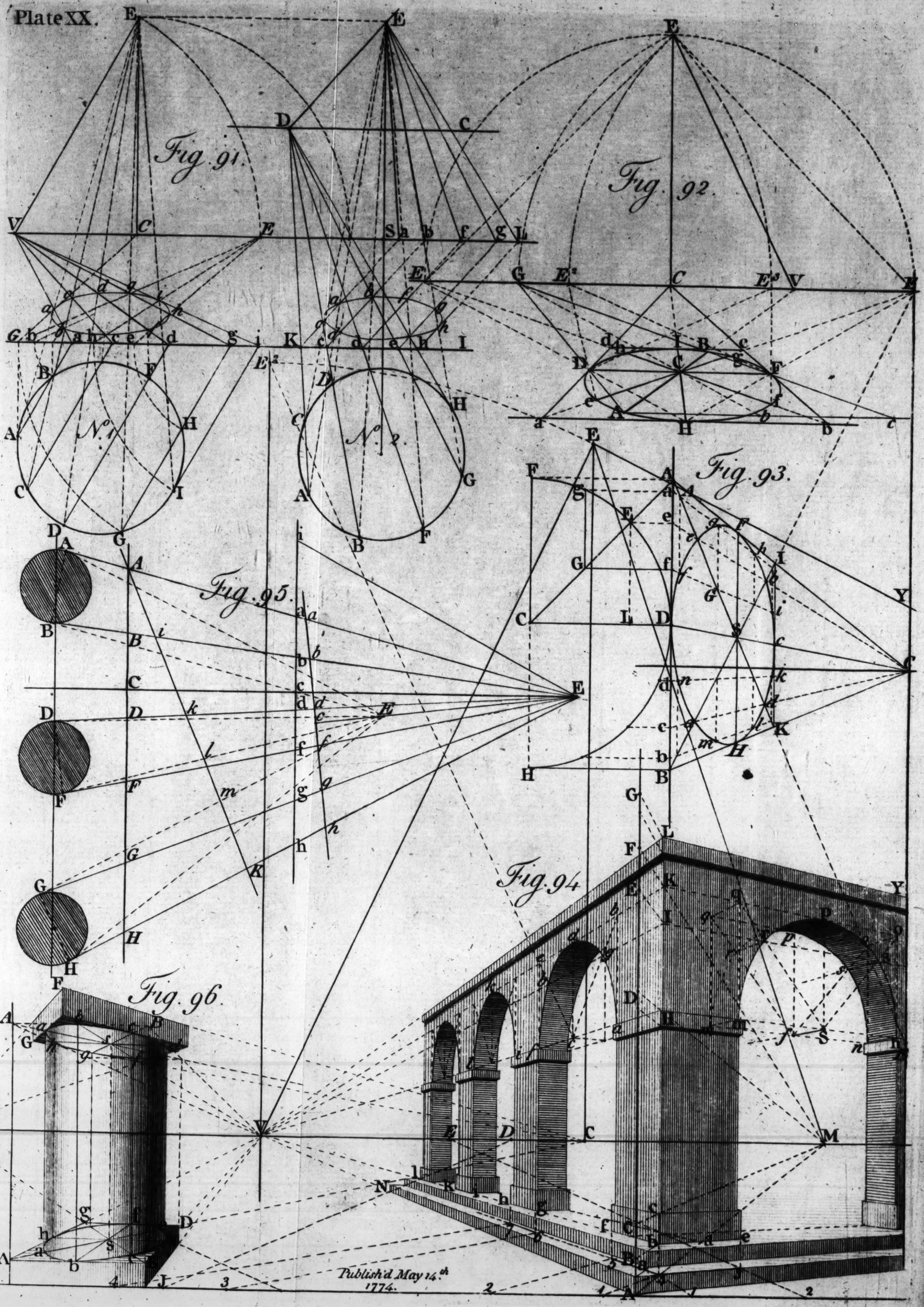

Fig. 91.
Fig. 92.
Fig. 93.
Fig. 95.
Fig. 94
Fig. 96.
N°. 1.
N°. 2.
Publish'd May 14th 1774.

Plate XXI.

Fig. 97.

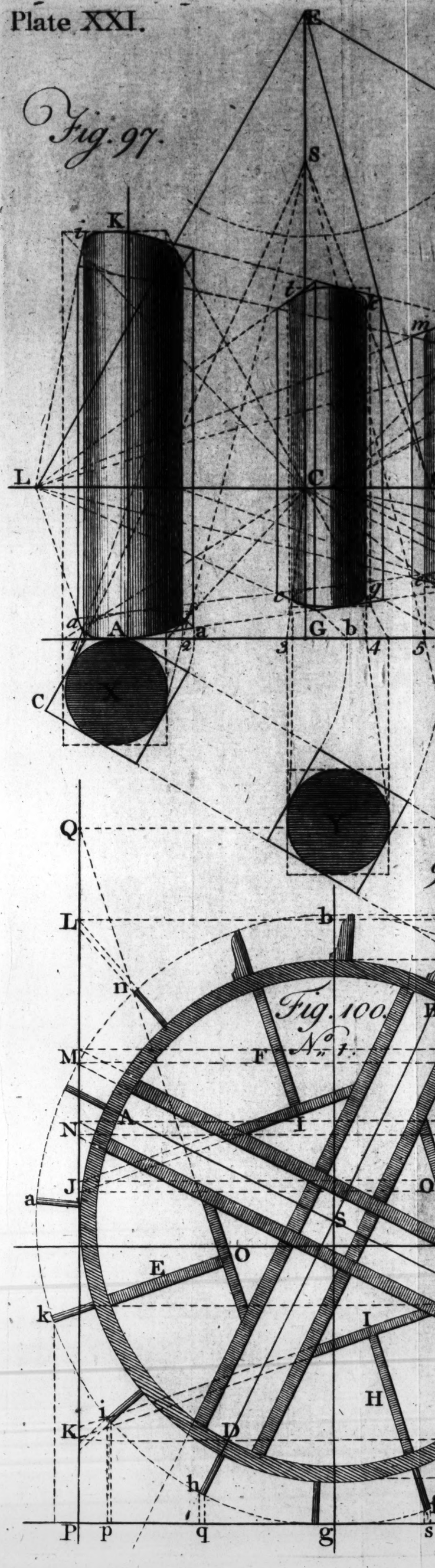


Fig. 99.

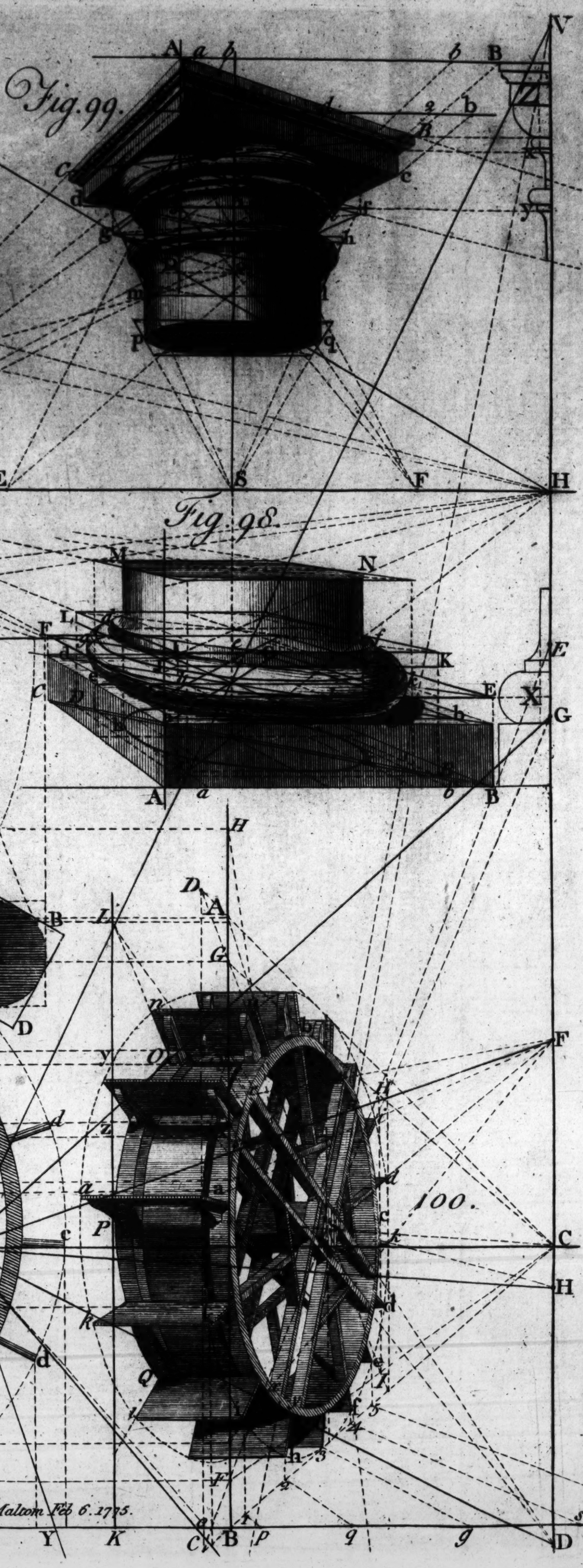


Fig. 98.

Fig. 100. No. 1.

Fig. 100.

Publish'd by T. Malton Feb 6. 1795.

Plate XXII.

Fig. 101.

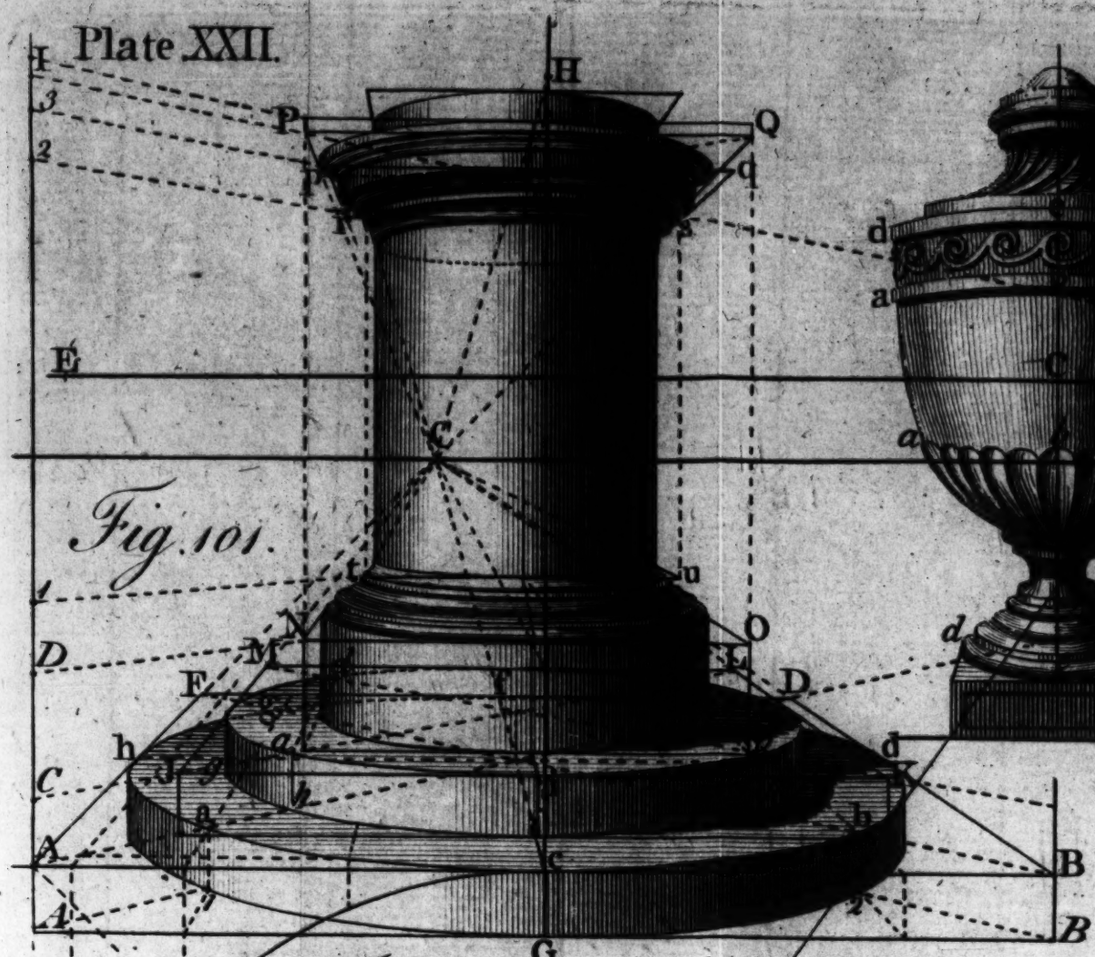


Fig. 102.

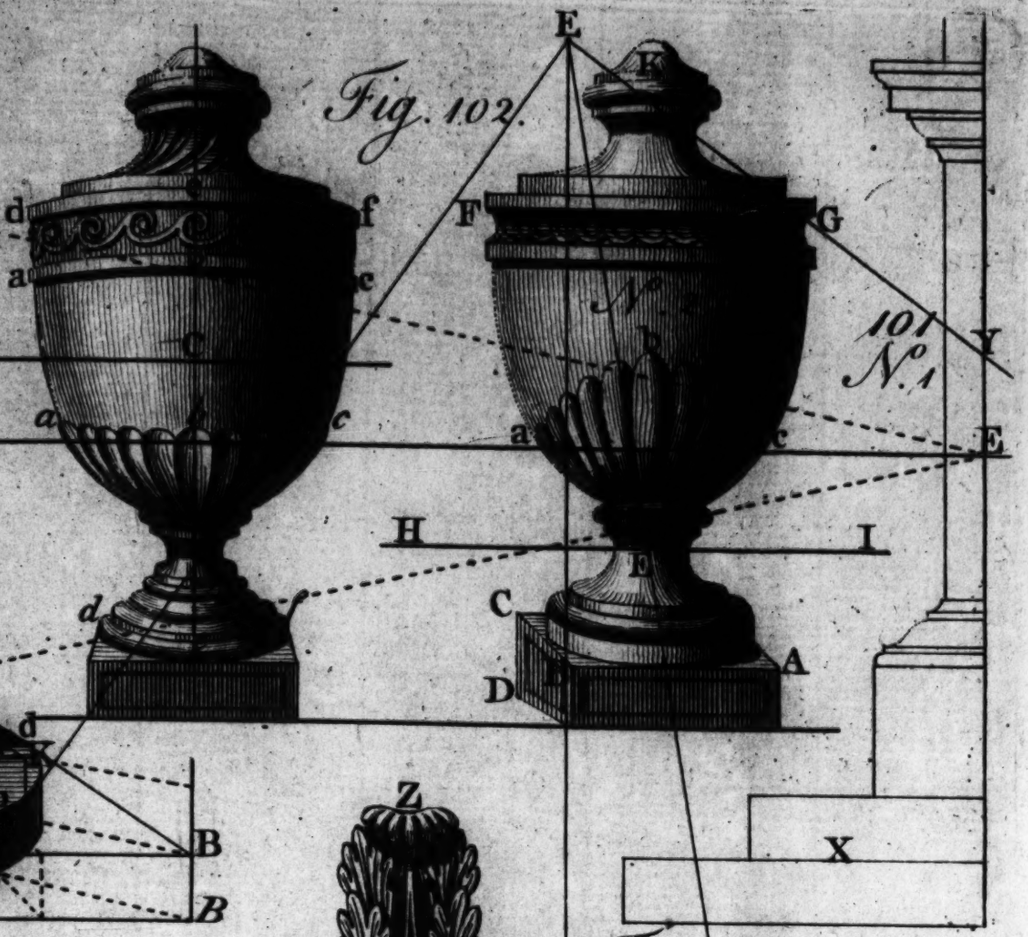
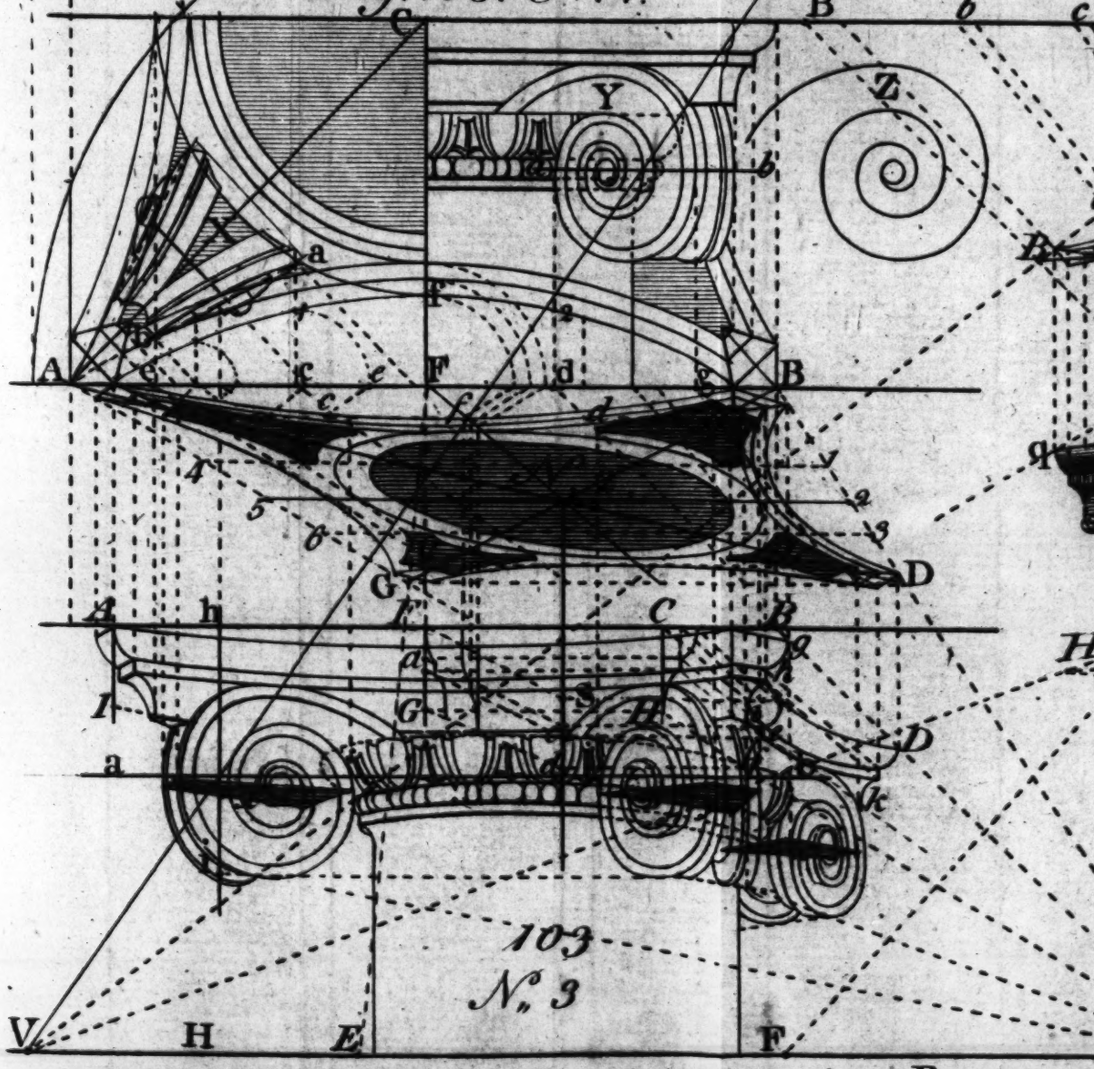


Fig. 103. N.o 1.



103
N.o 3

Fig. 105.

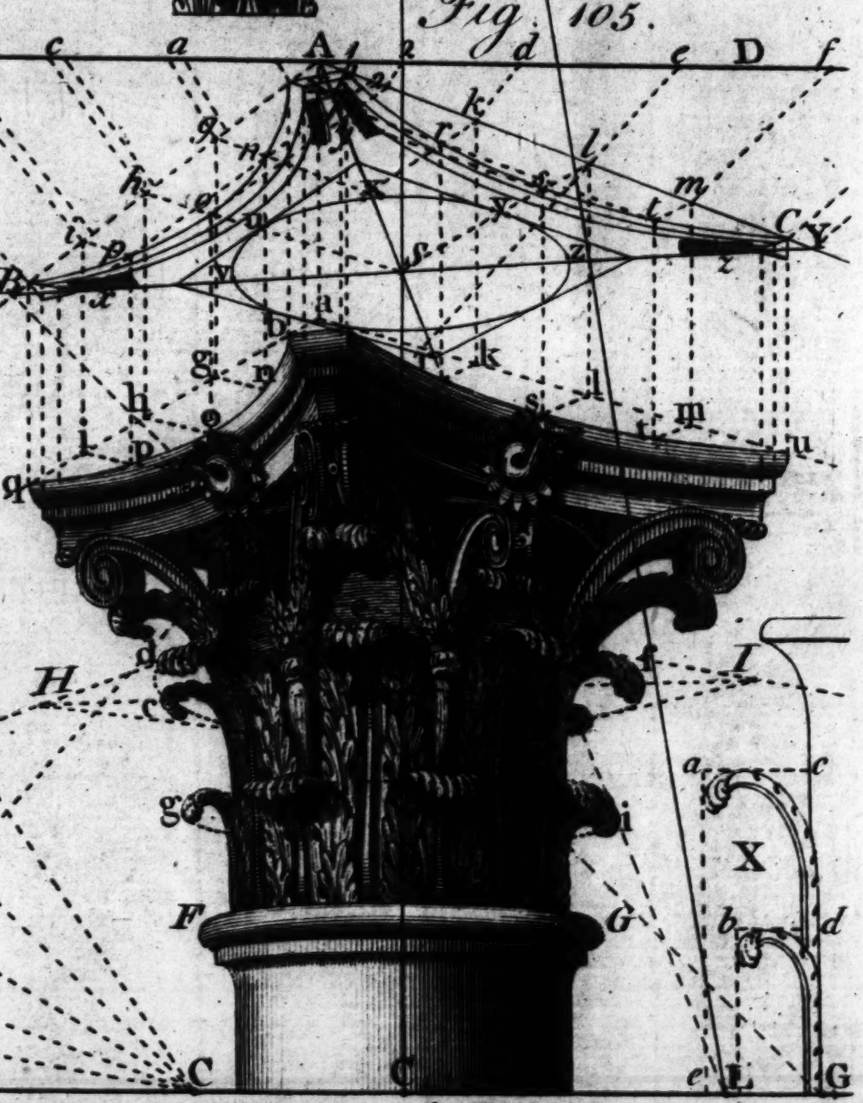
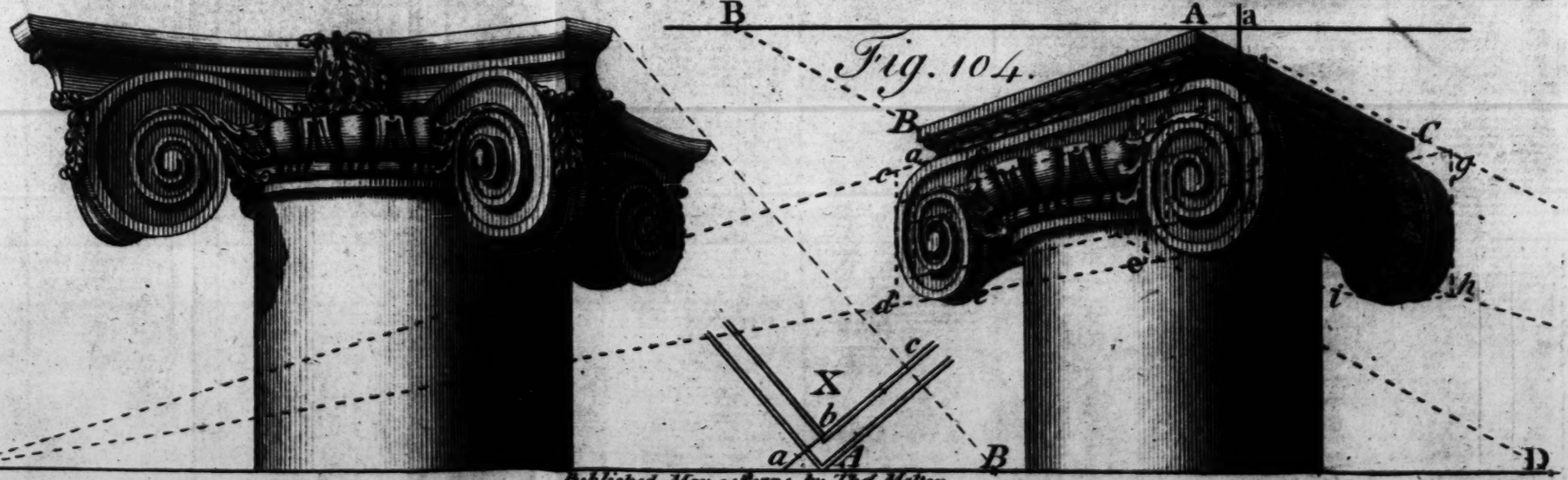


Fig. 104.



Published May 14.th 1774 by Thos. Malton.

Fig. 106.

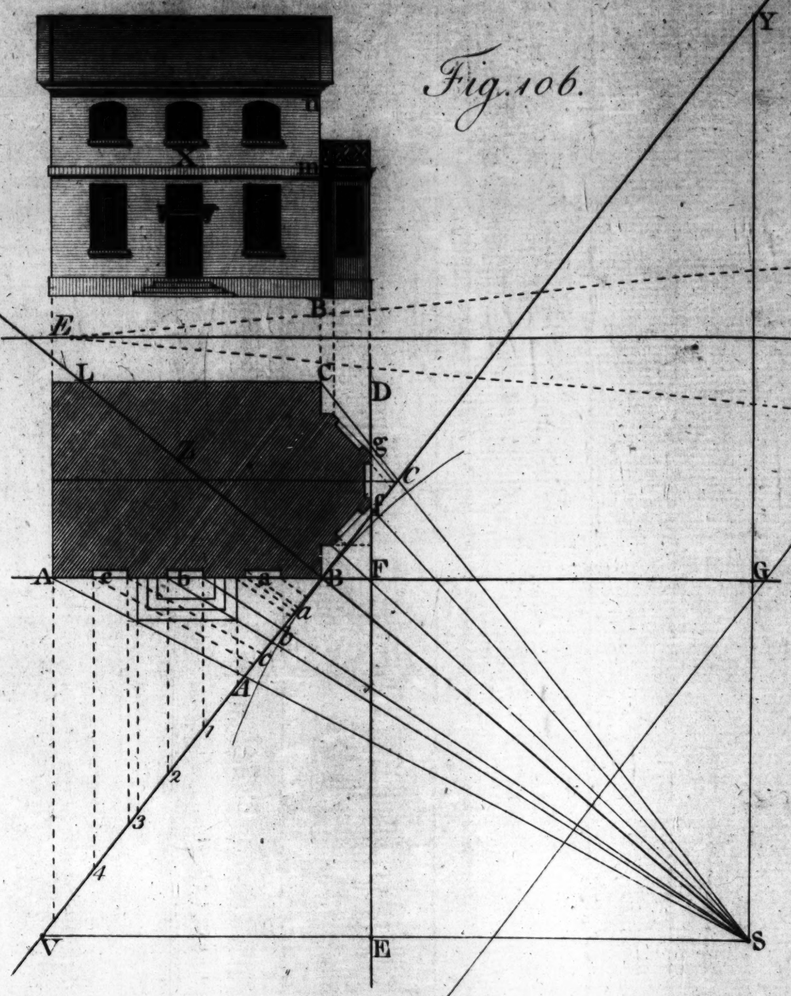


Fig. 107.

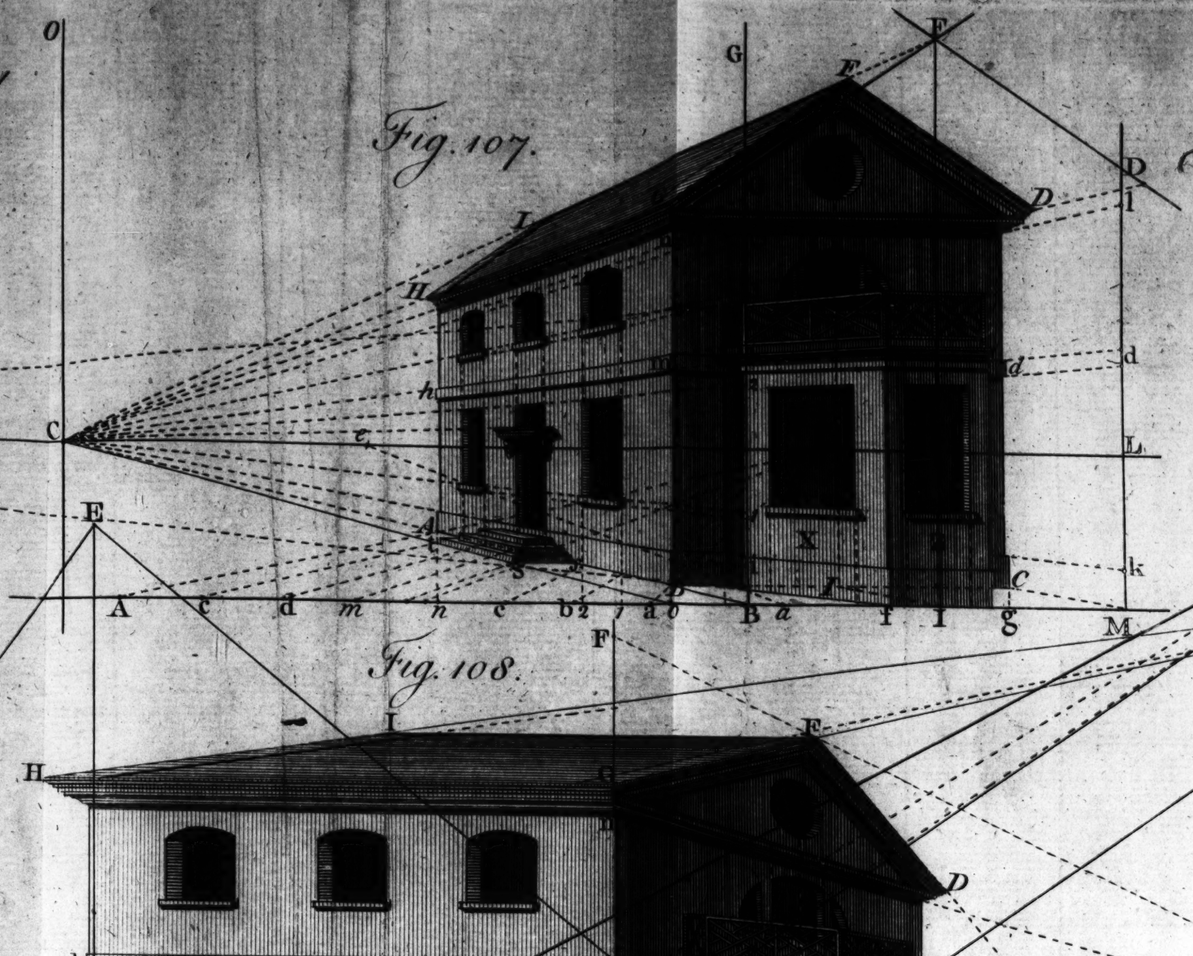


Fig. 108.

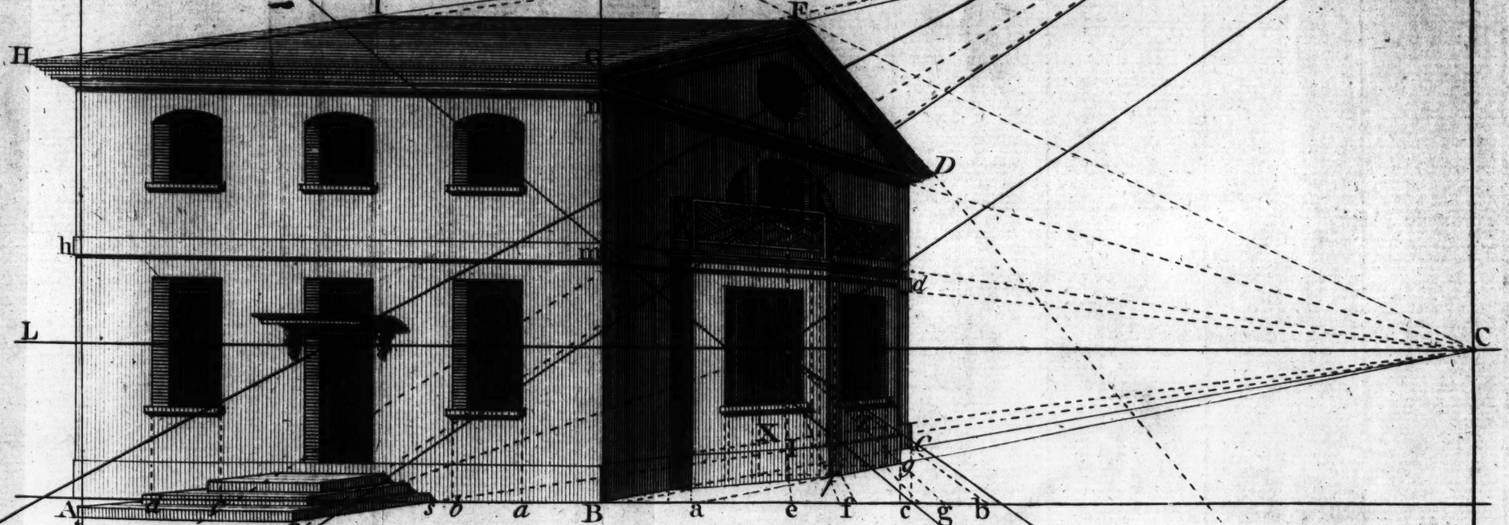
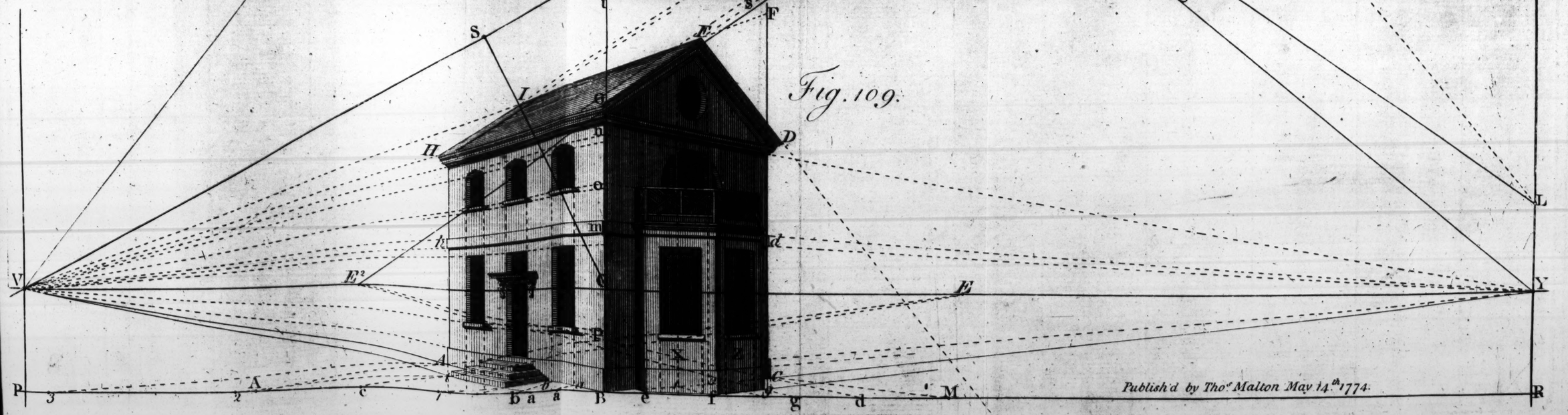


Fig. 109.



Publish'd by Thos Malton May 14th 1774.

Plate XXIV.

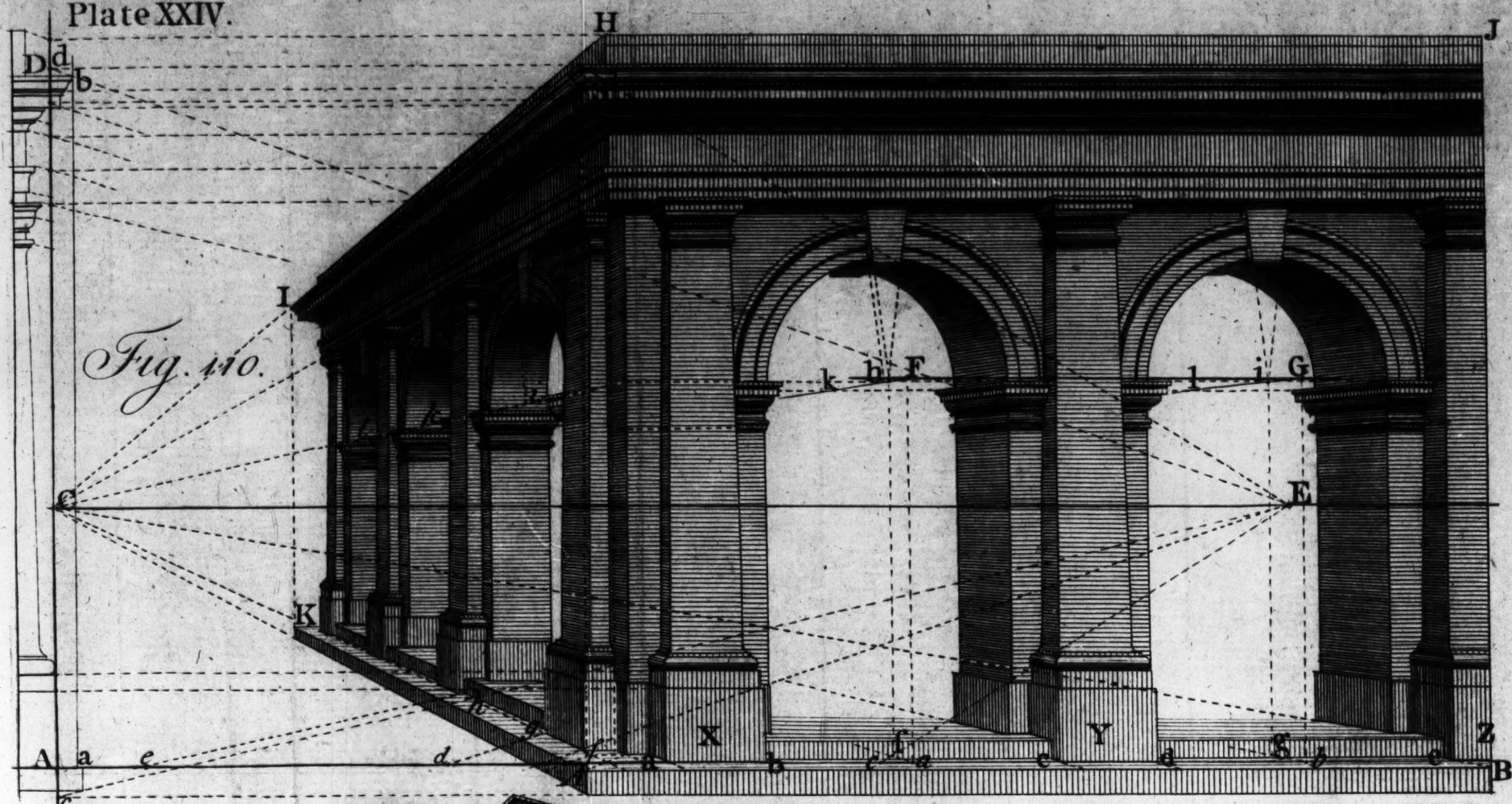


Fig. 110.

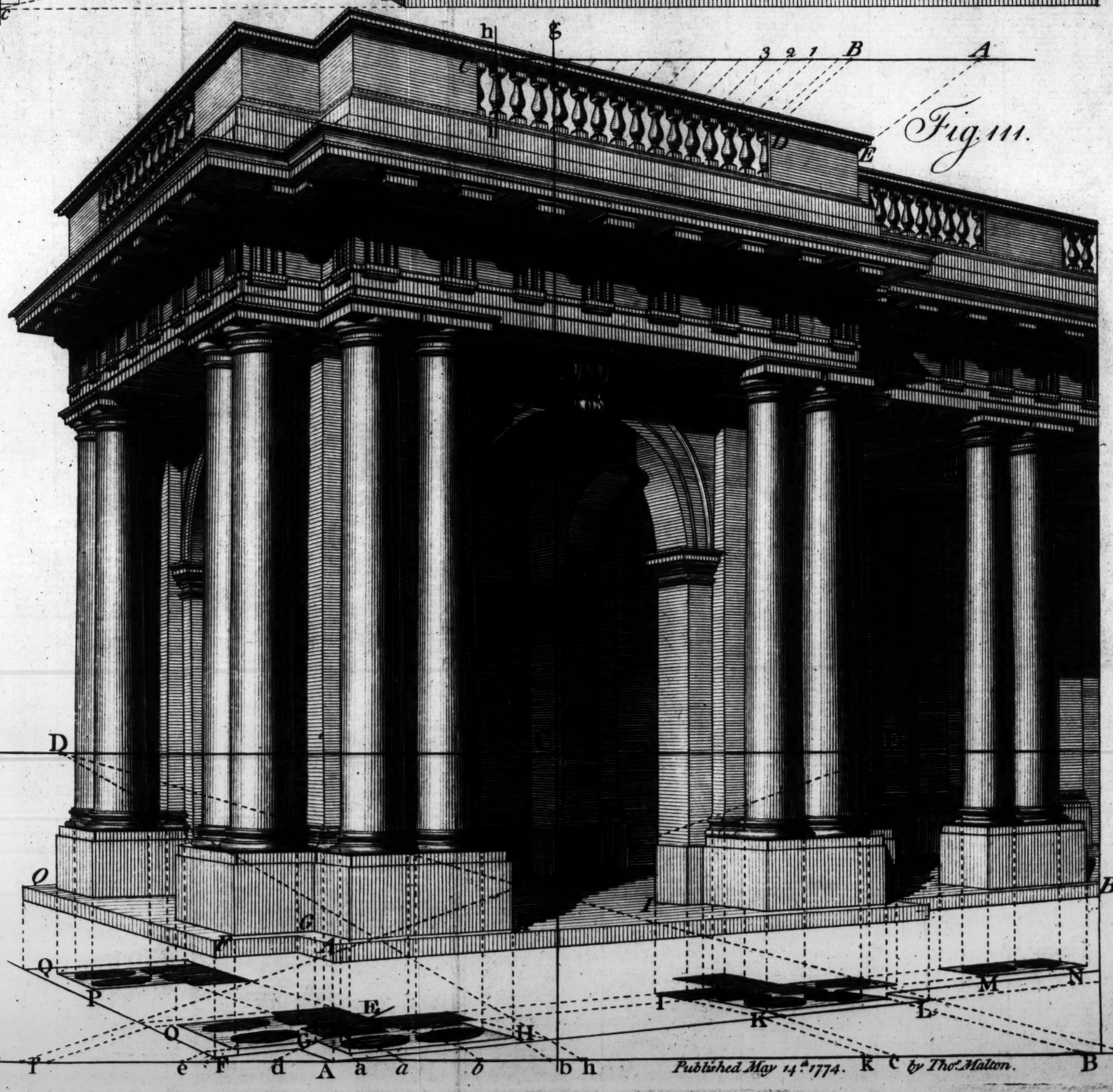
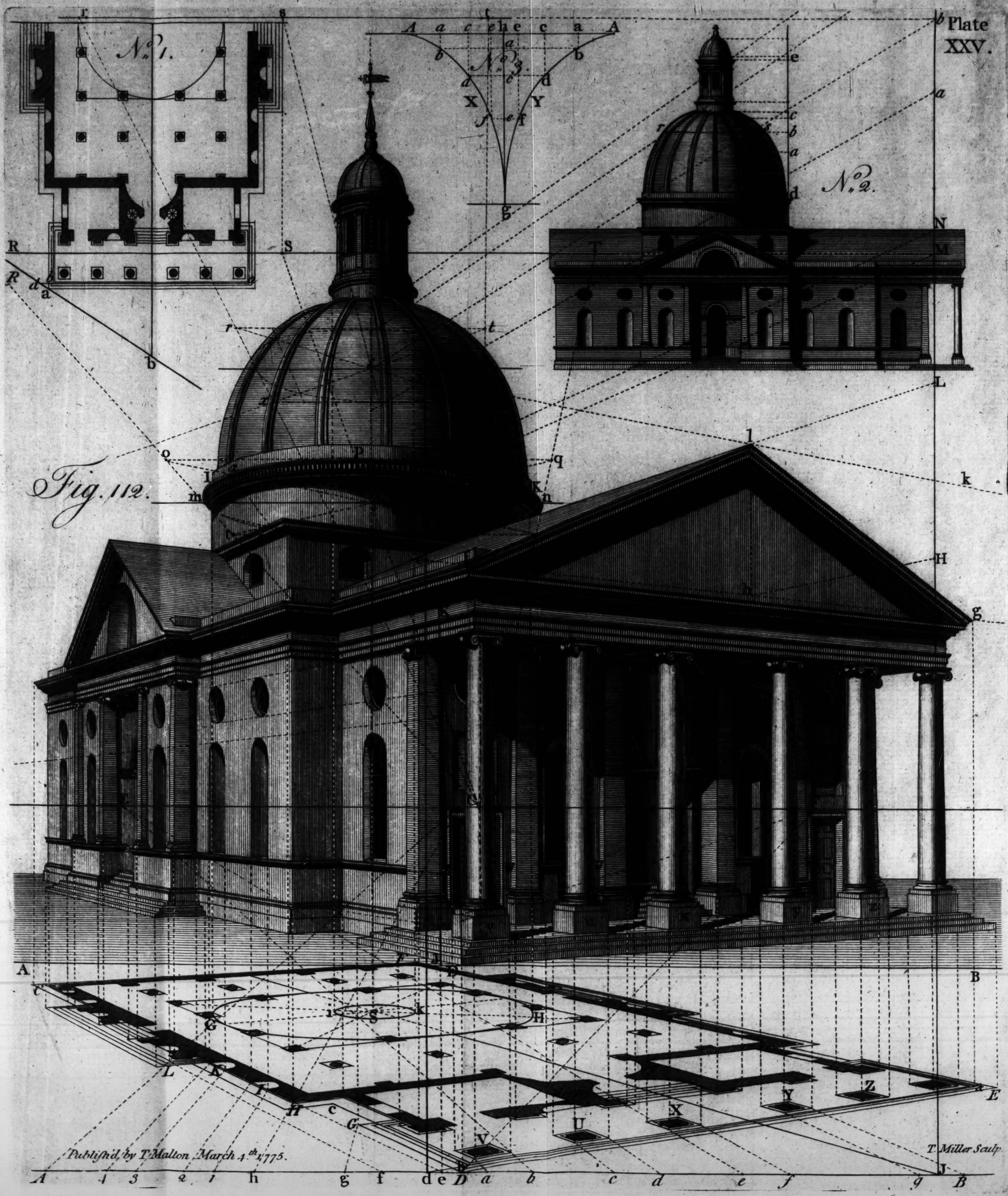


Fig. 111.

Published May 14th 1774. by Thos Malton.

Plate XXV.

Fig. 112.



Published by T. Malton March 4th 1775.

T. Miller Sculp

Plate XXVI.



Plate XXVII.



Plate XXVIII.



Plate XXIX.

Fig. 113.

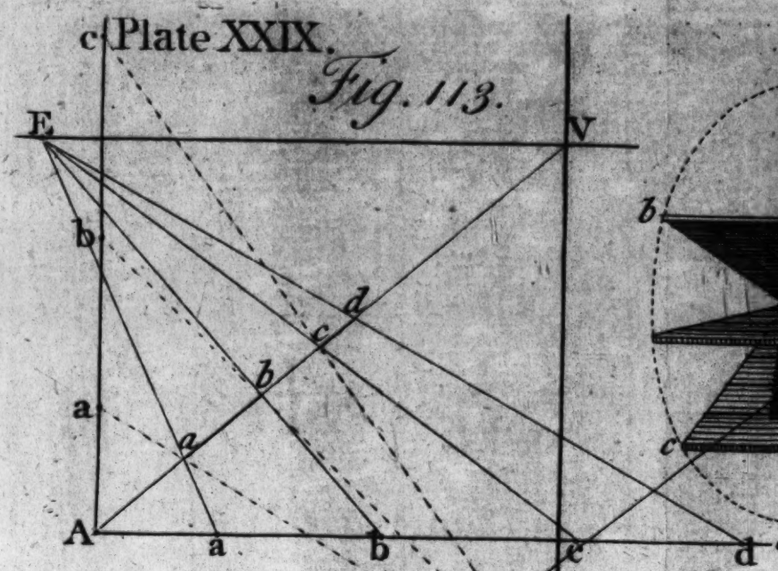


Fig. 116.

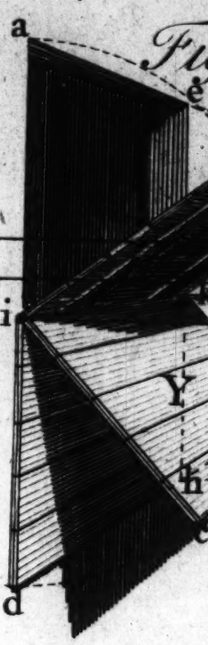


Fig. 116. N.2.

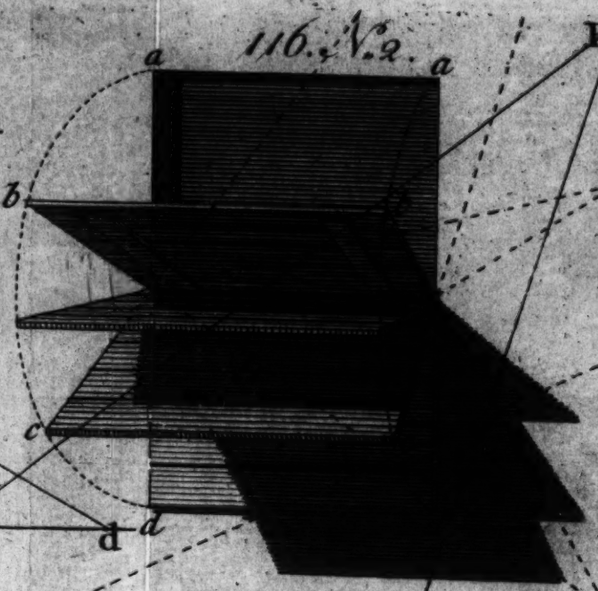


Fig. 115.

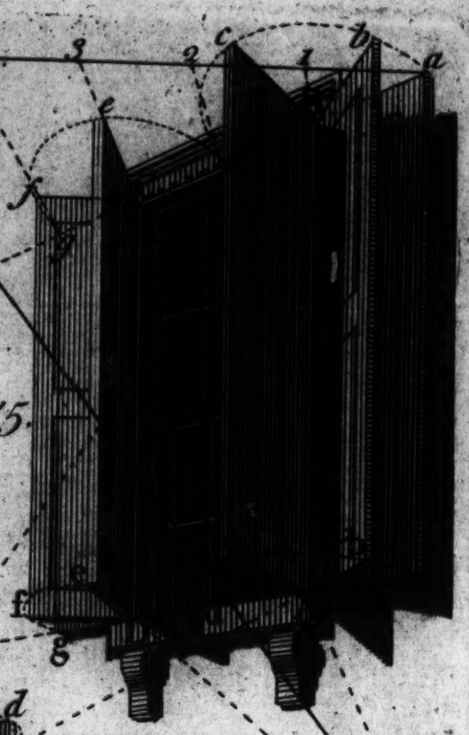


Fig. 114.

Fig. 118.

Fig. 120.

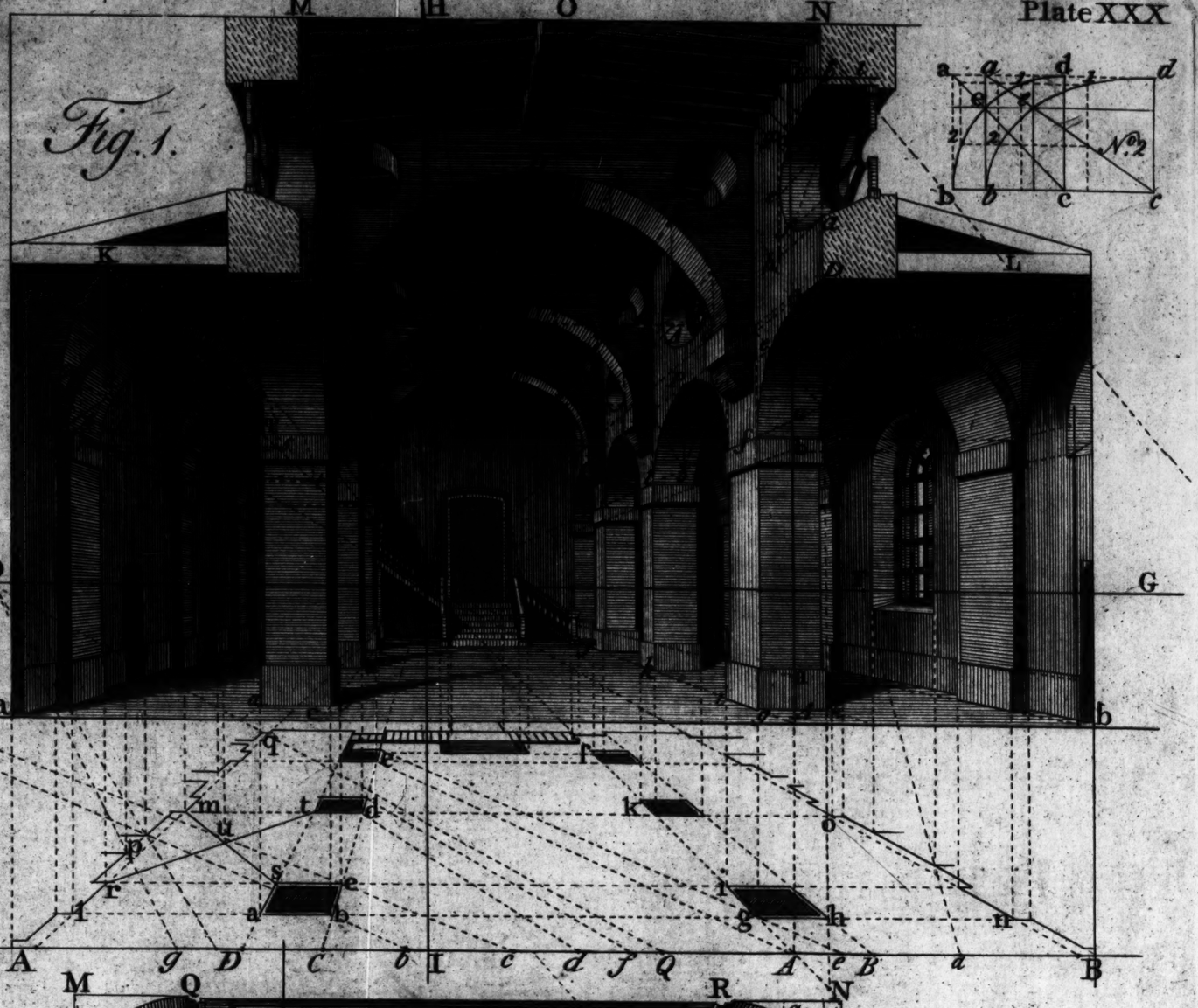
Fig. 117.

Fig. 119.

Publish'd by T. Malton Feb.y 6.th 1773.

Plate XXX

Fig. 1.



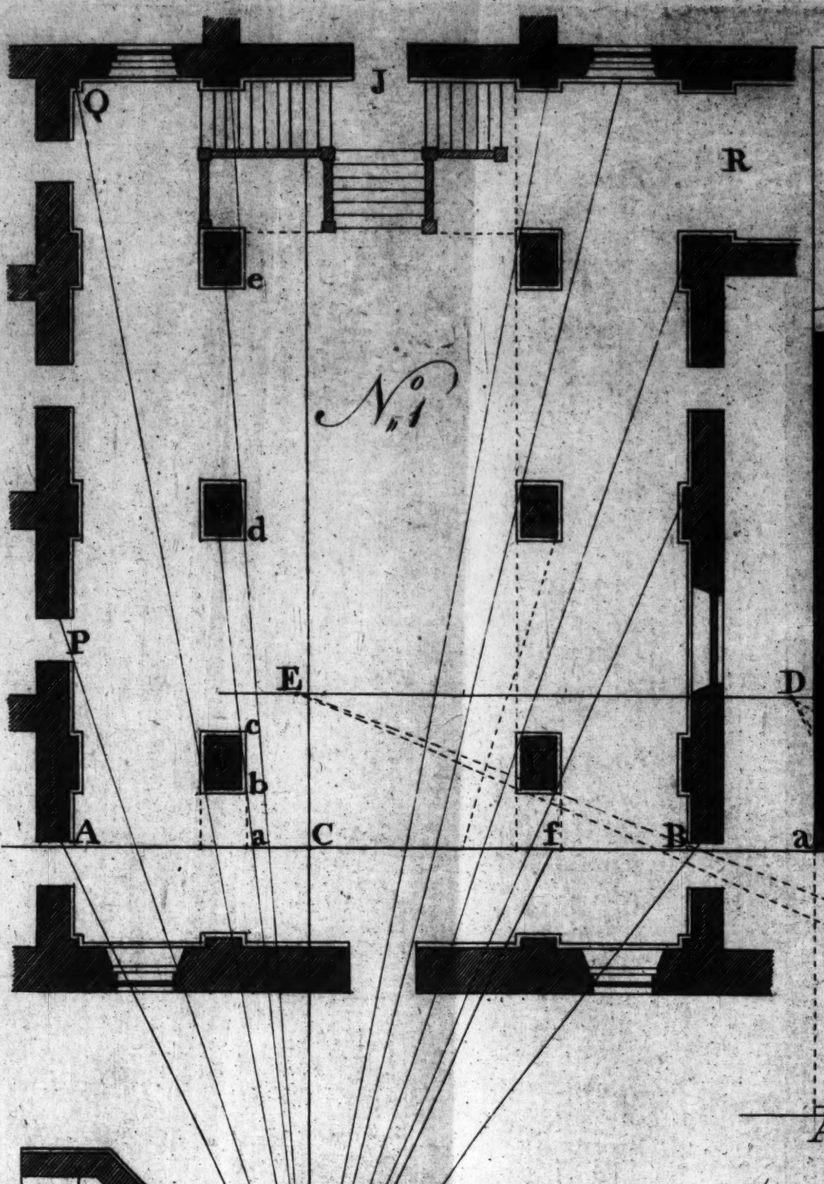


Fig. 2.

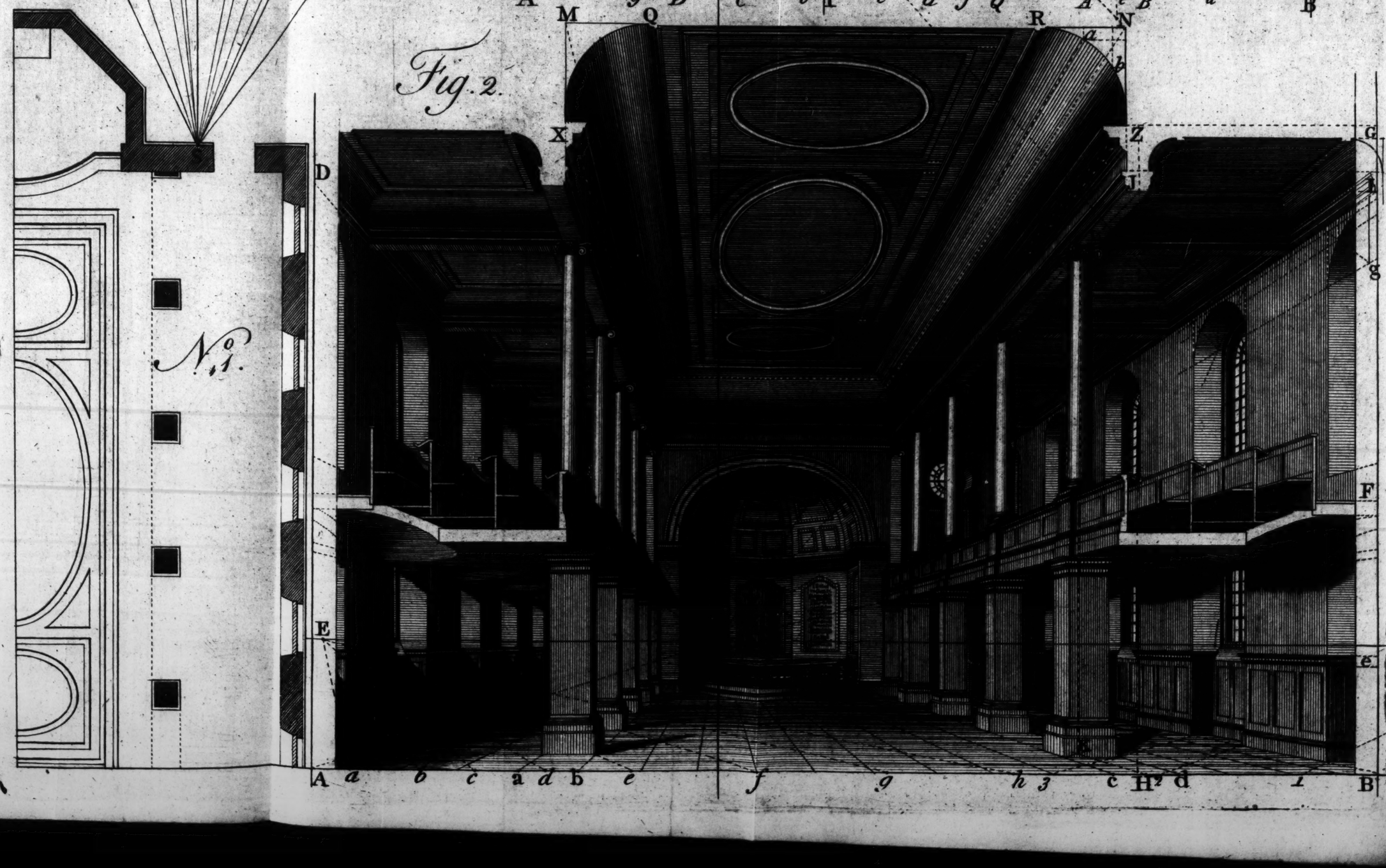


Plate XXI

Fig. 1.



Fig. 2.



Publish'd by T. Malton March 16th. 1778.

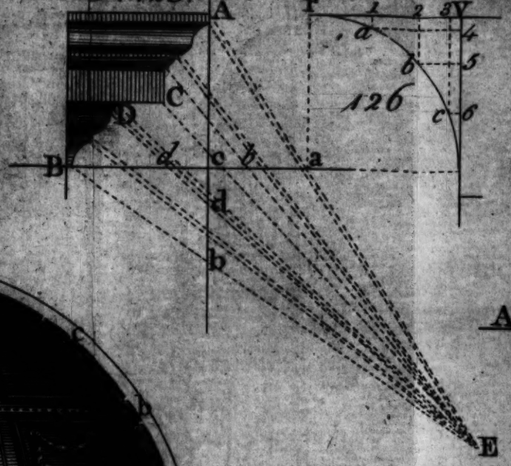
Plate XXXII.



Published by T. Malton March 1st 1775.

Pl: XXXIII

Fig. 125.



121. E No. 2.

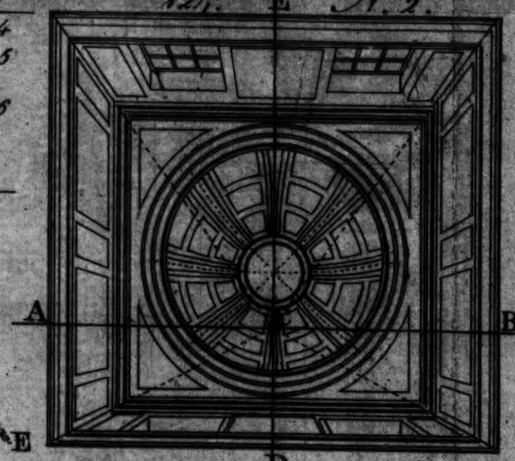


Fig. E 124.

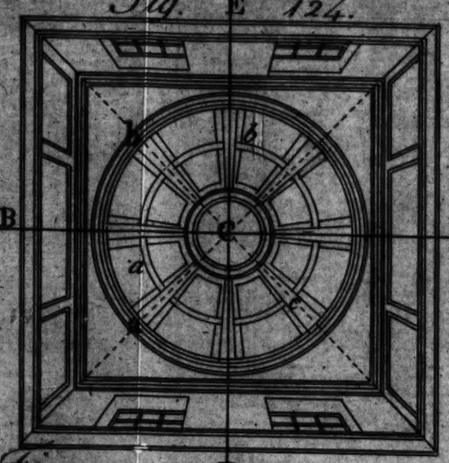
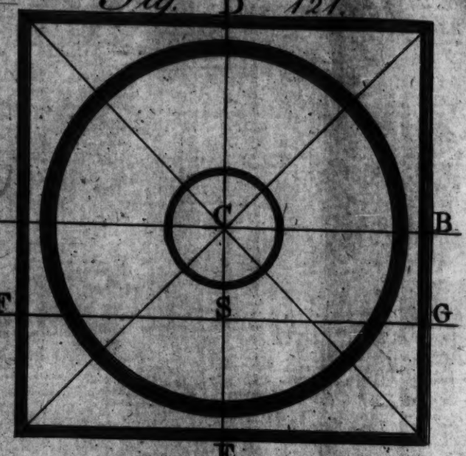


Fig. 123 D

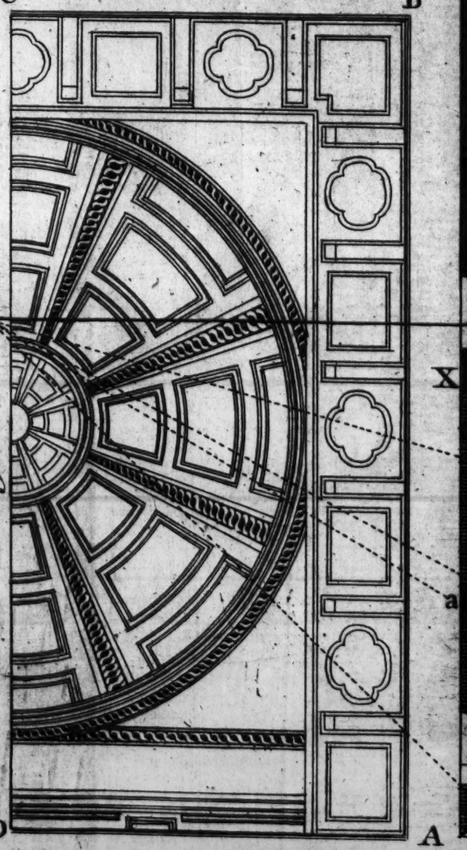
Fig. D 121.



122. No. 2.



Fig. 122.



Publish'd by T. Malton March 1st 1776.

Miller Sculp.

Plate XXXIV.

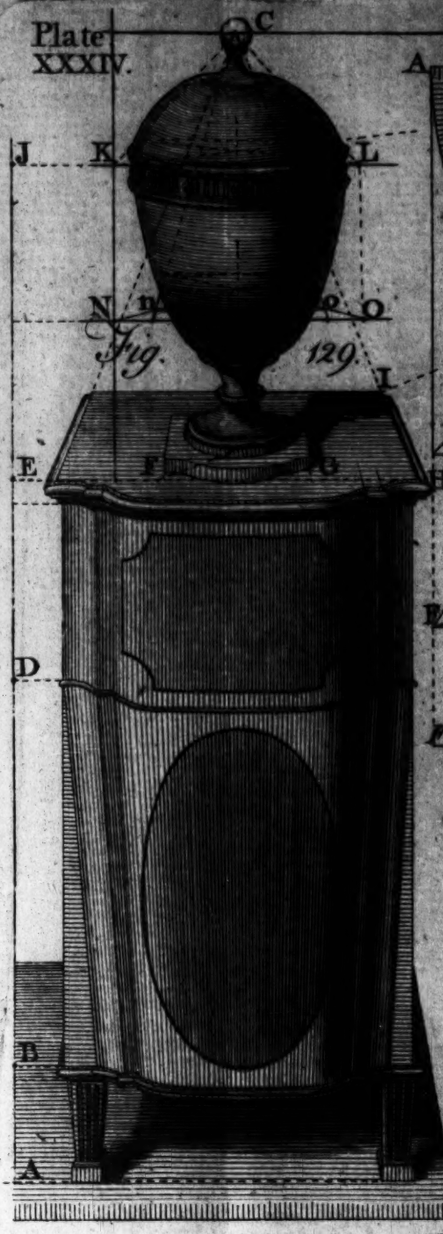


Fig. 129.

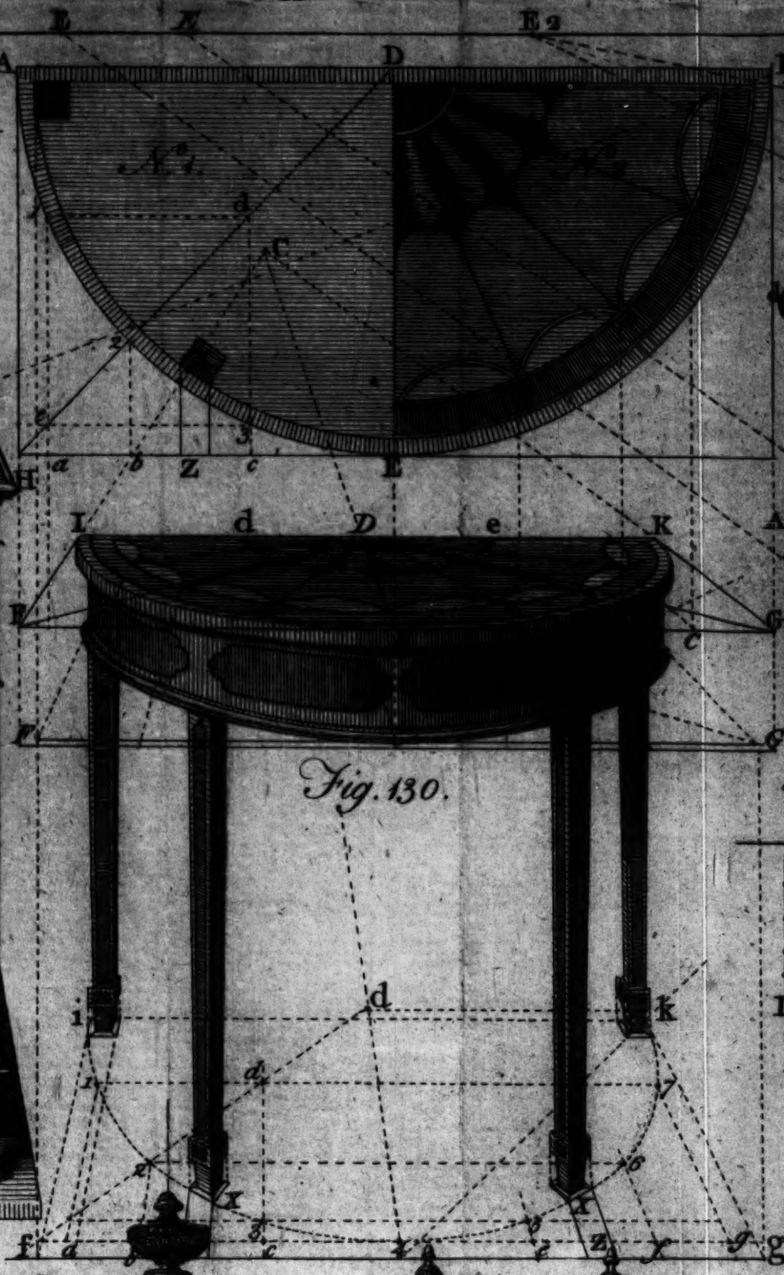


Fig. 130.

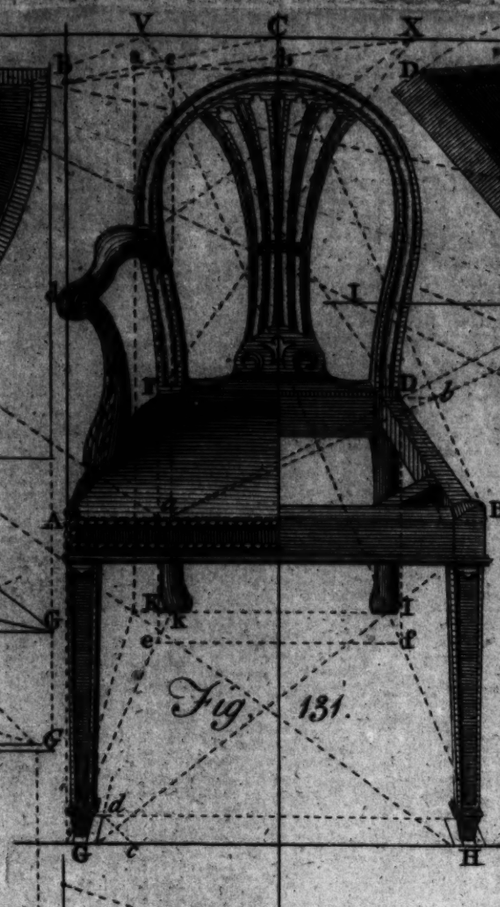


Fig. 131.

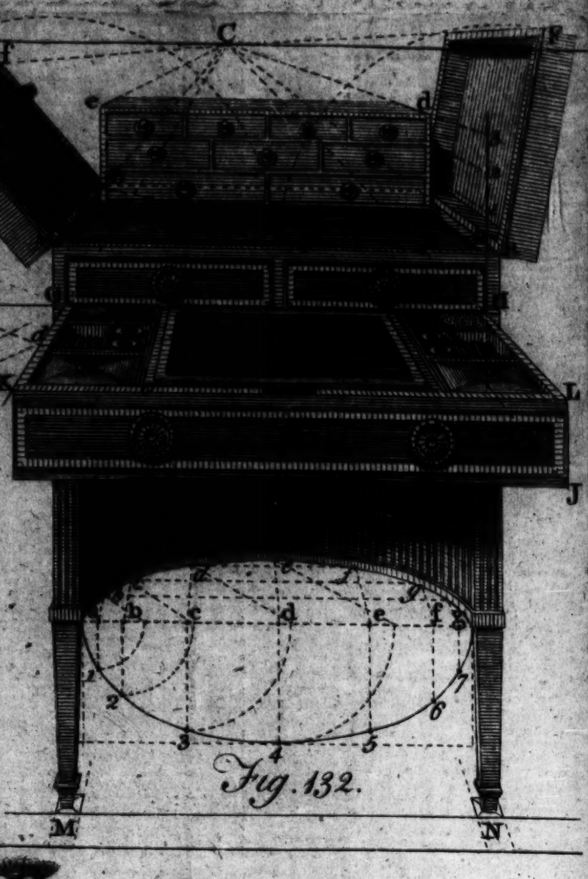


Fig. 132.

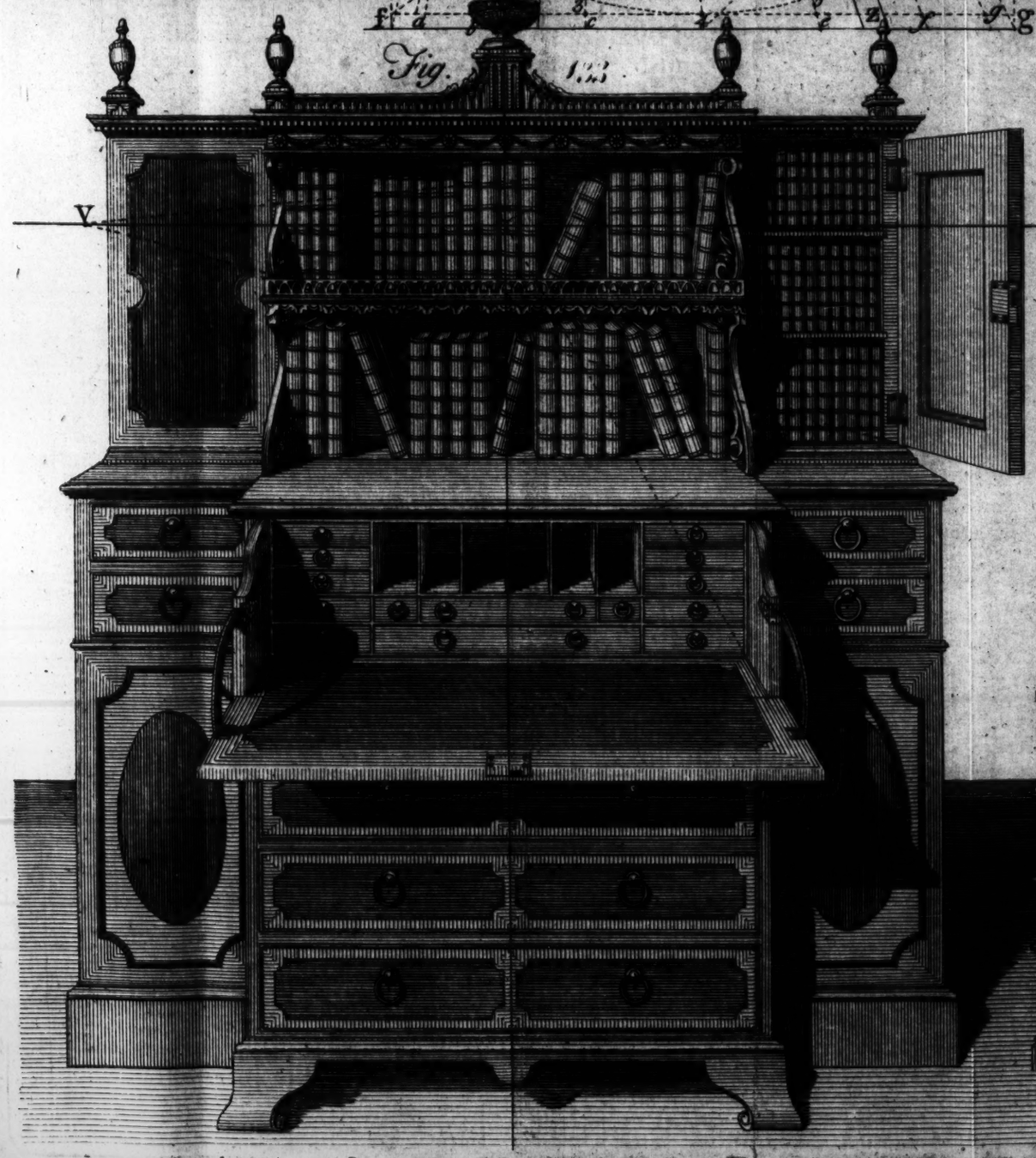


Fig. 133.

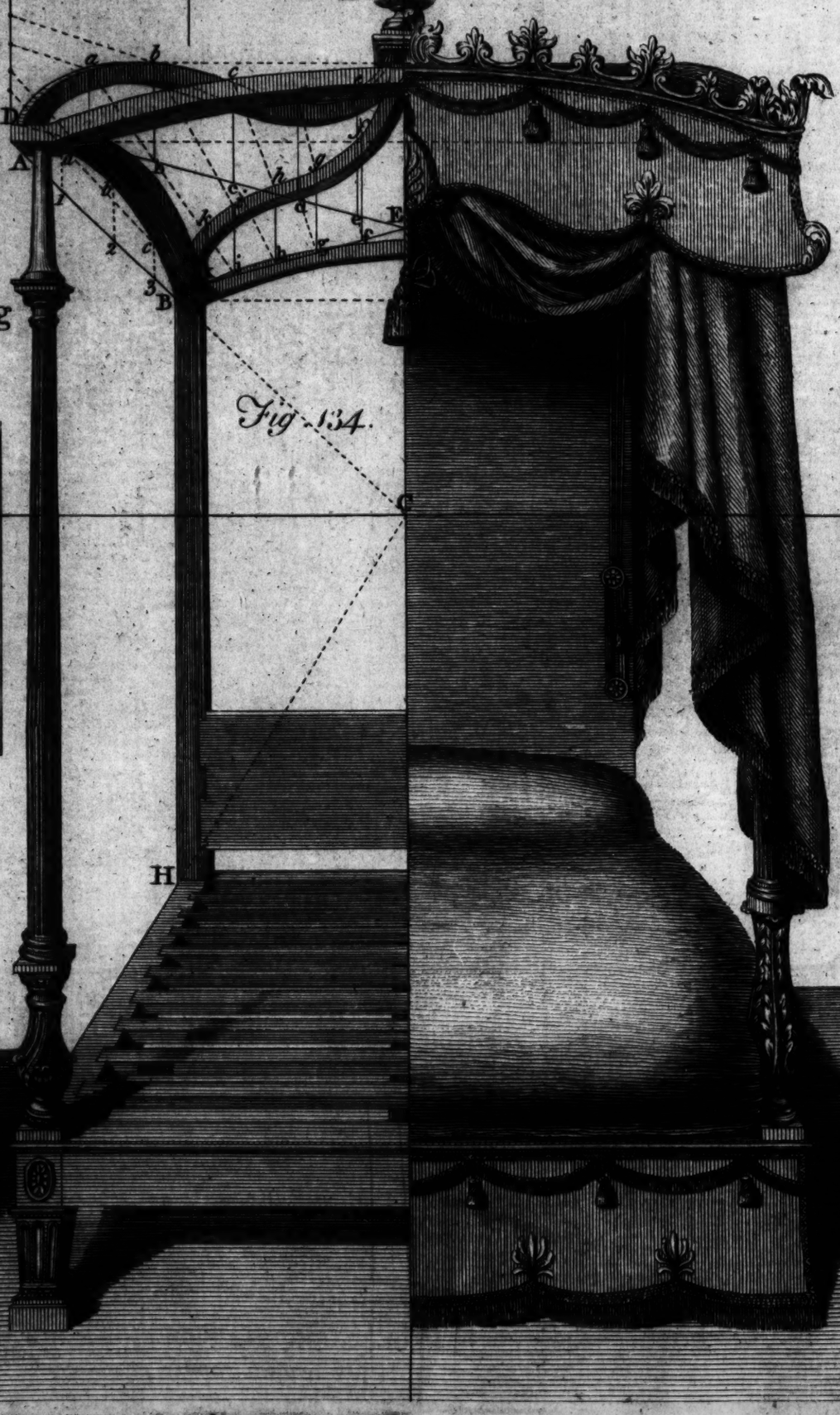


Fig. 134.

Plate XXXV.

Fig. 135.

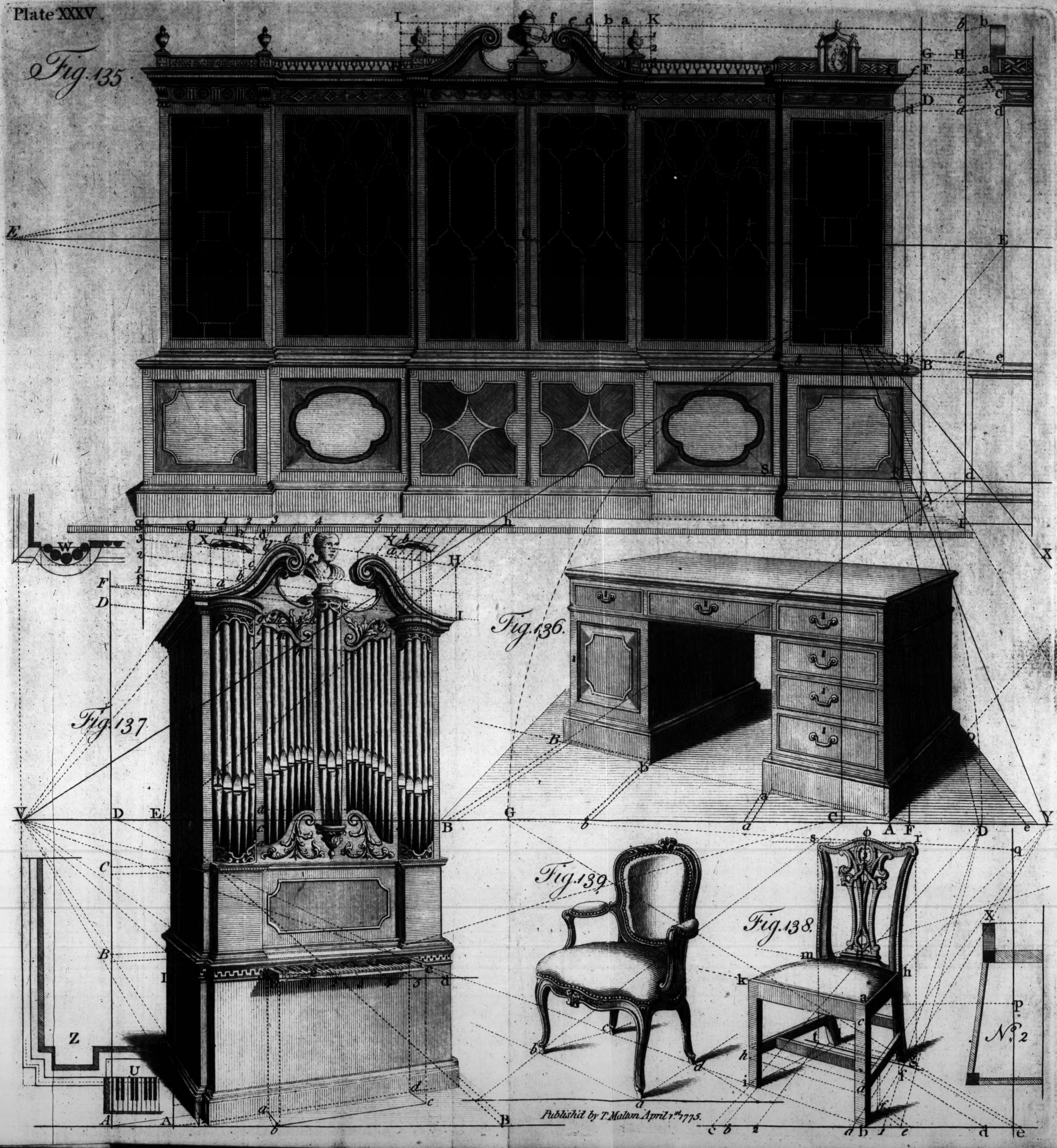


Fig. 137.

Fig. 136.

Fig. 139.

Fig. 138.

Published by T. Malton April 1st 1775.

Plate XXXVI

Fig. 142.

Fig. 143.

Fig 144.

Fig. 145.

Published by T. Mallon, April 1st 1775.

Miller Sc.

Plate XXXVII.

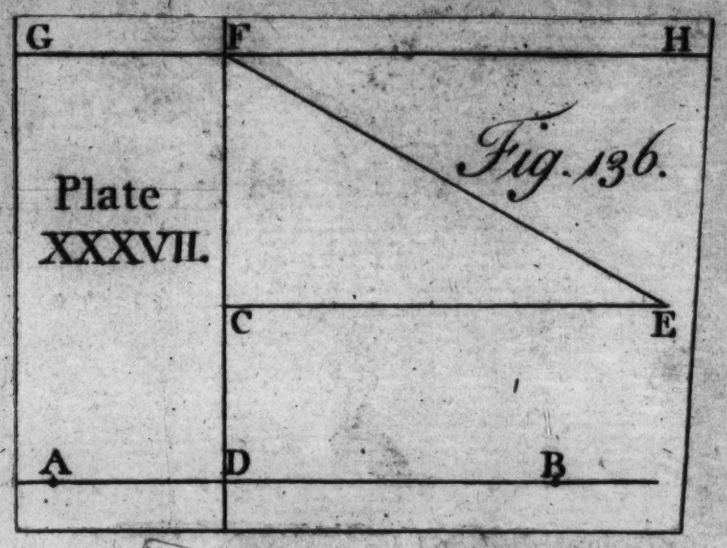


Fig. 136.

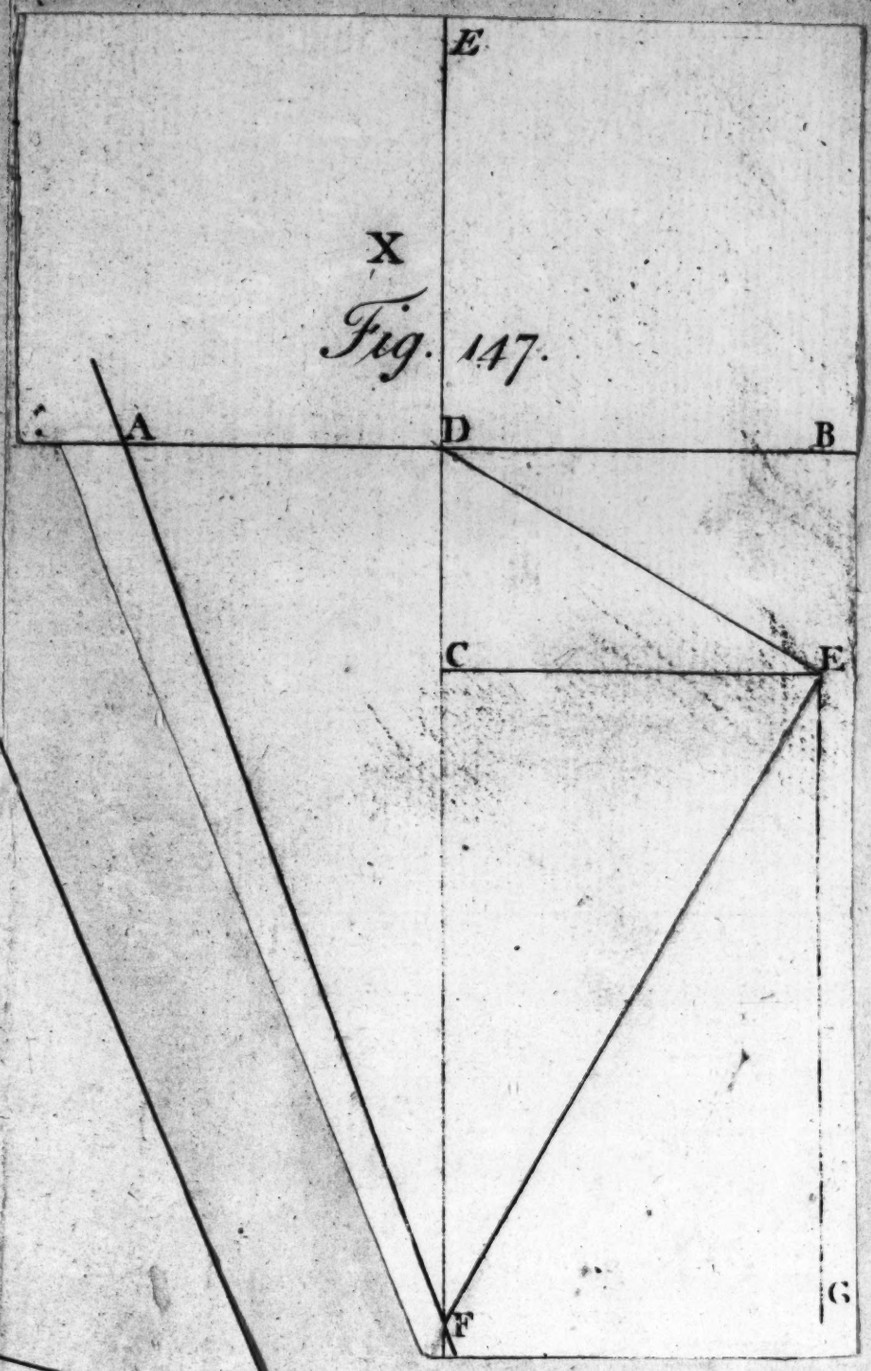


Fig. 147.

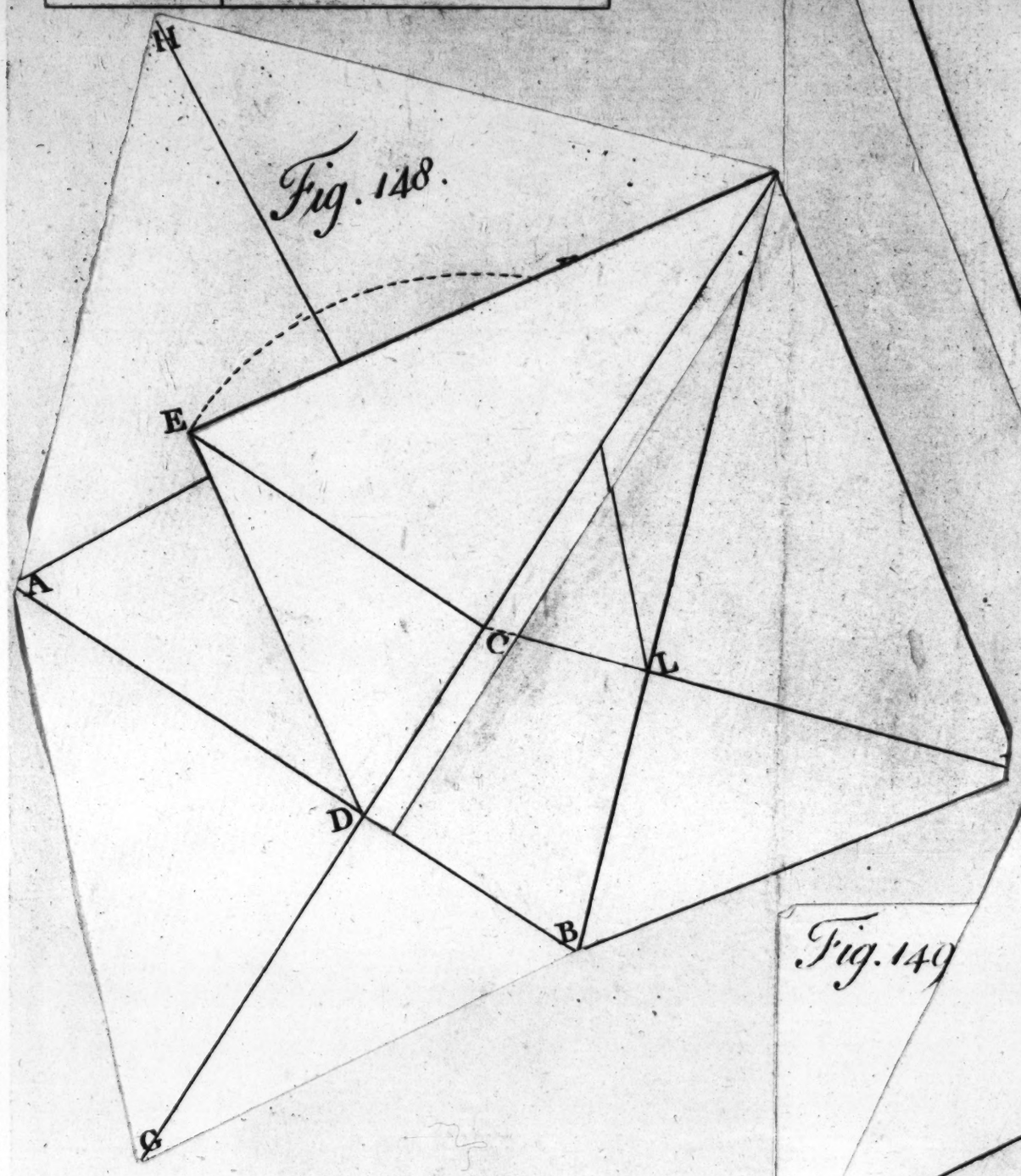


Fig. 148.

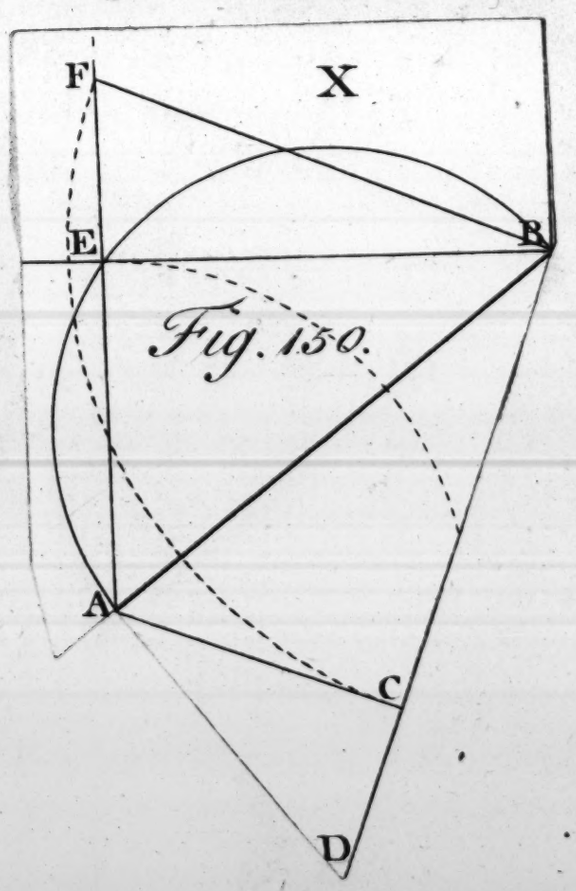


Fig. 150.

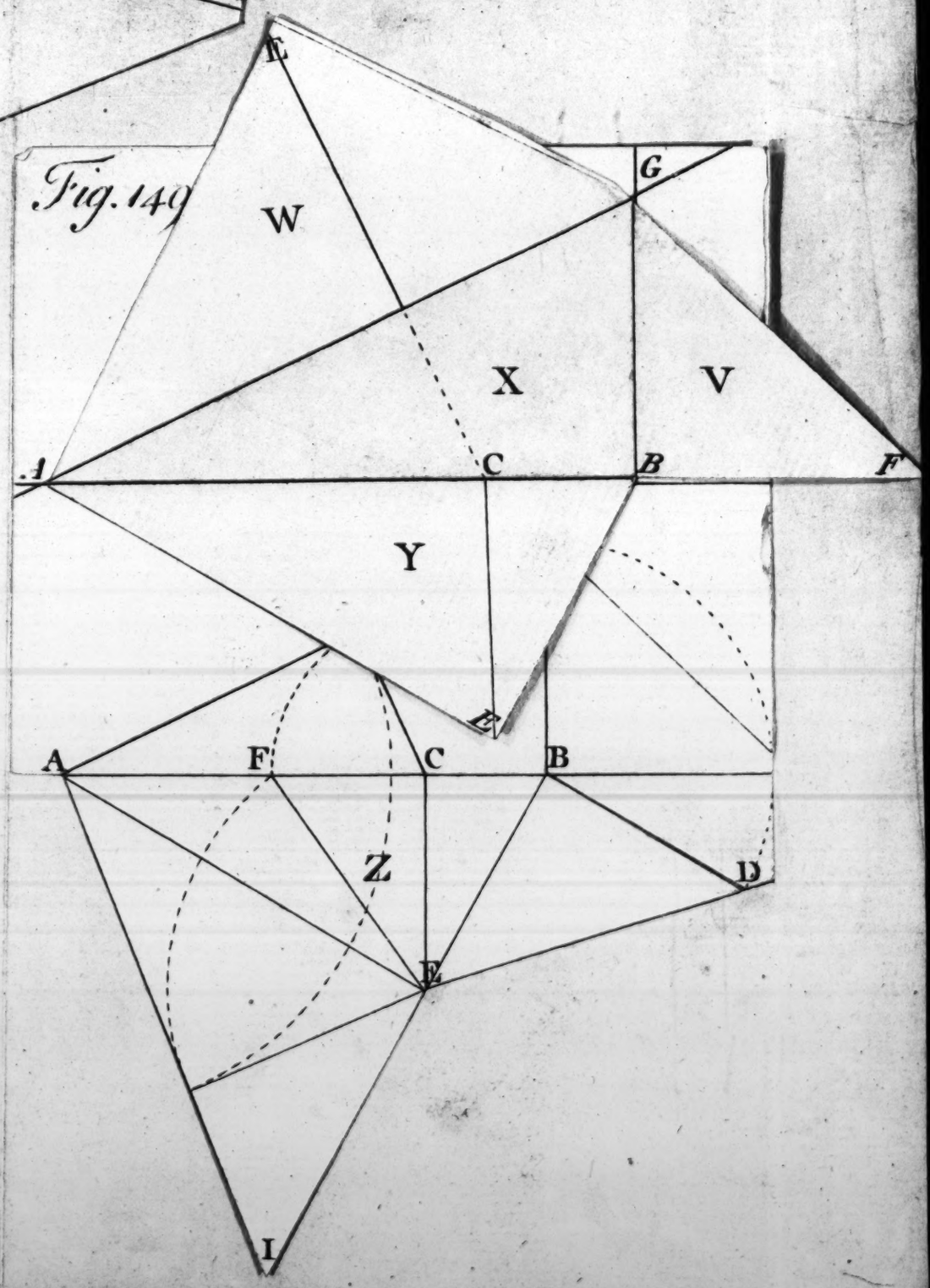
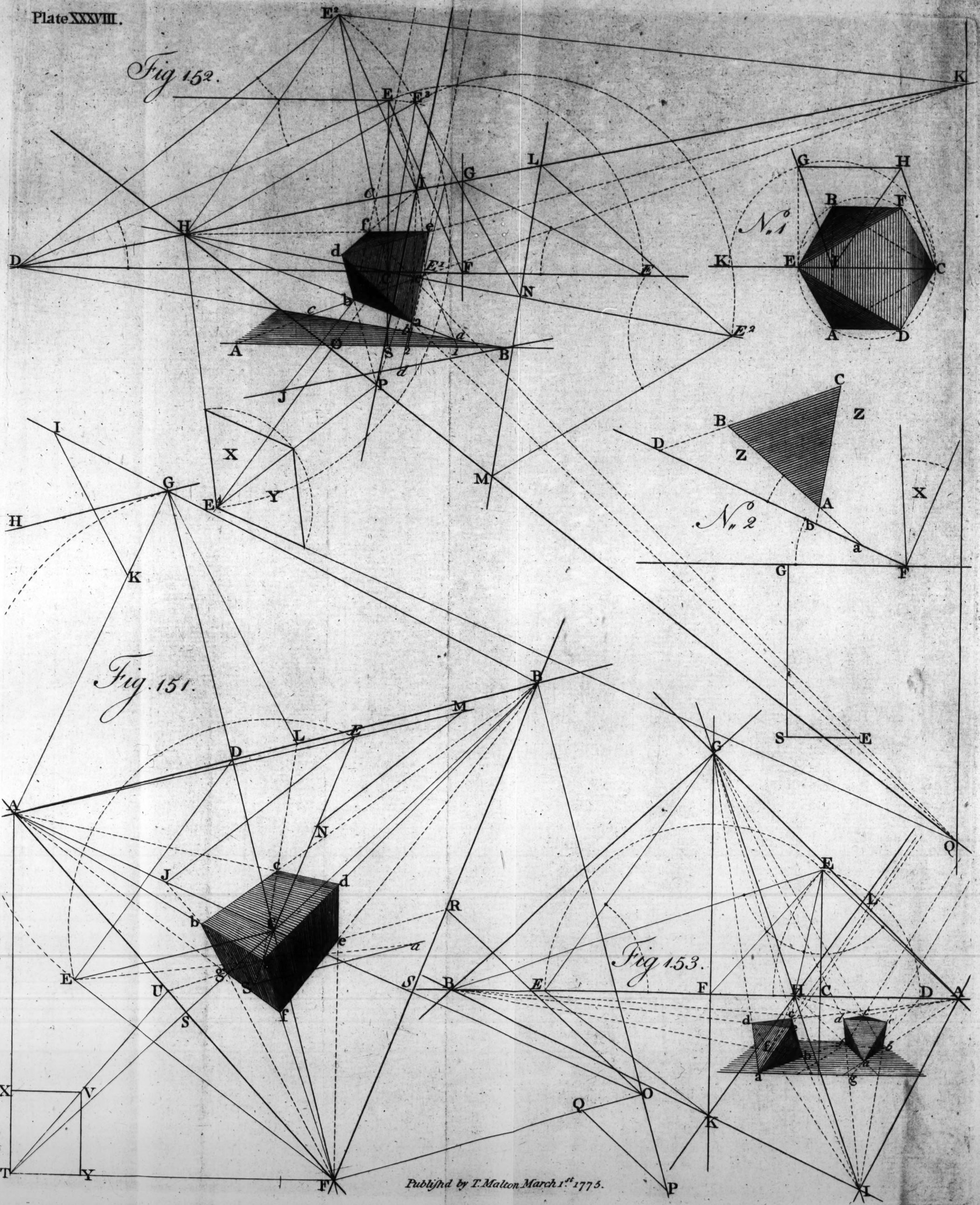


Fig. 149.

Plate XXXVIII.

Fig 152.



Publish'd by T. Malton March 1st 1775.

Plate XXXIX

Fig. 155.

N°. 1.

N°. 2.

N°. 3.

N°. 4.

N°. 5.

Fig. 154

N°. 1.

Published by T. Malton March 10th 1775.

Plate XL.

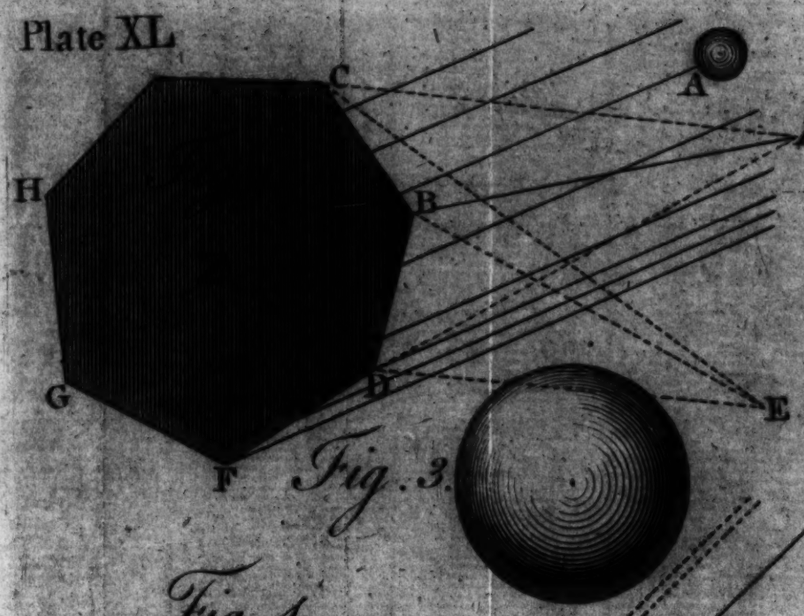


Fig. 3.

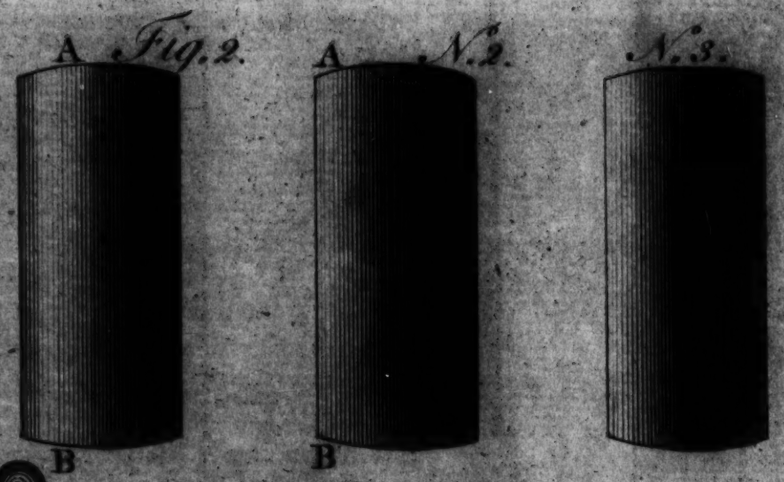
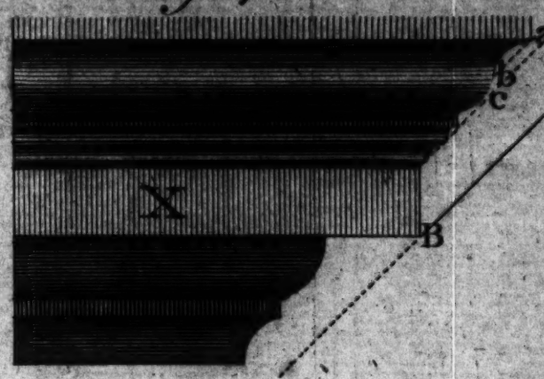


Fig. 2. N.° 2. N.° 3.

Fig. 4.



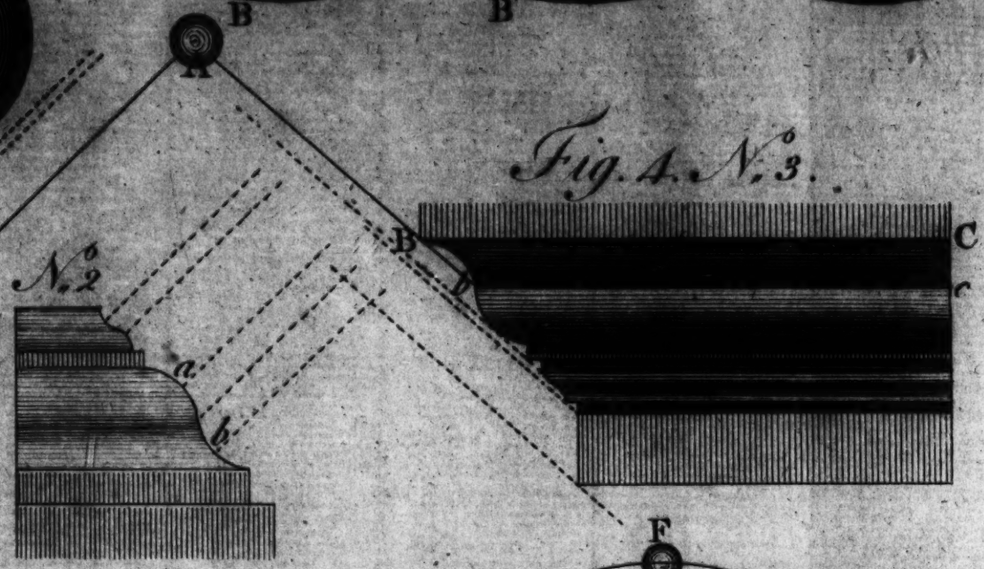


Fig. 4. N.° 3.

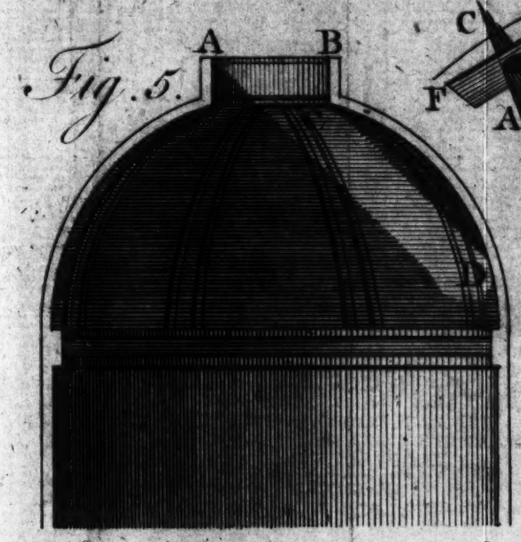


Fig. 5.

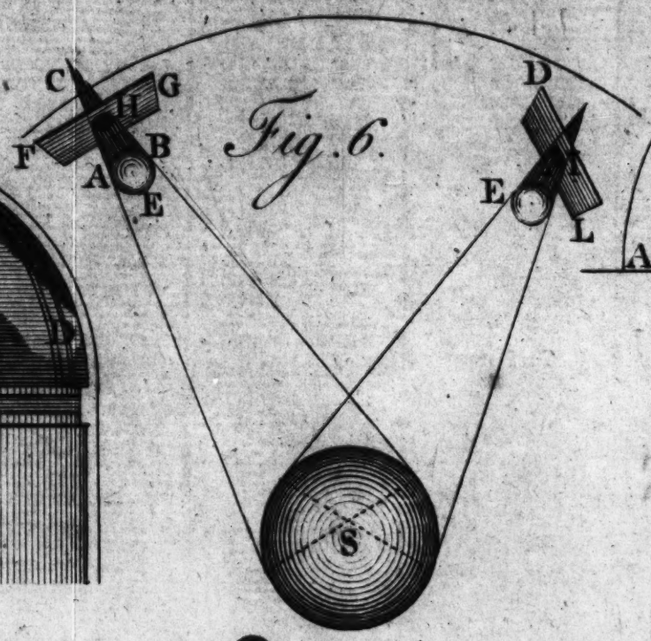


Fig. 6.

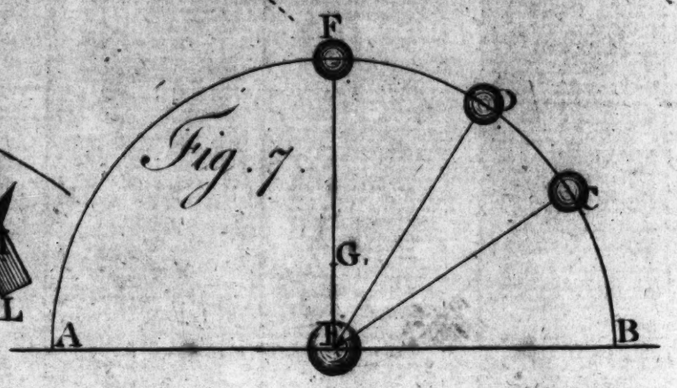


Fig. 7.

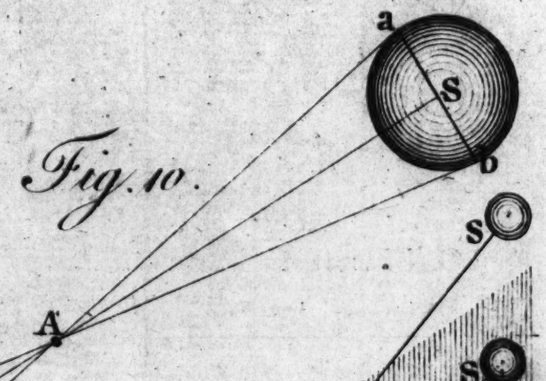


Fig. 10.

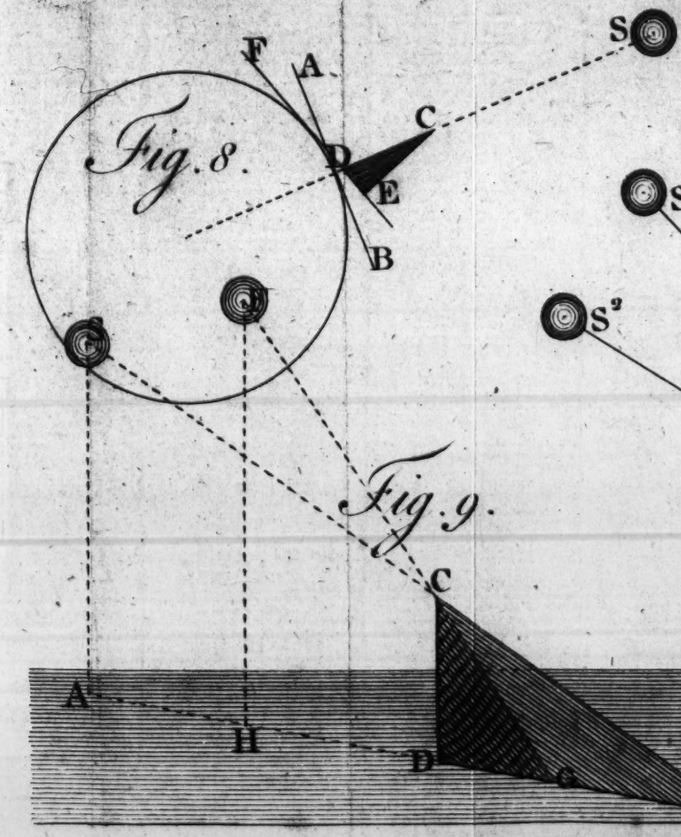


Fig. 8.

Fig. 9.

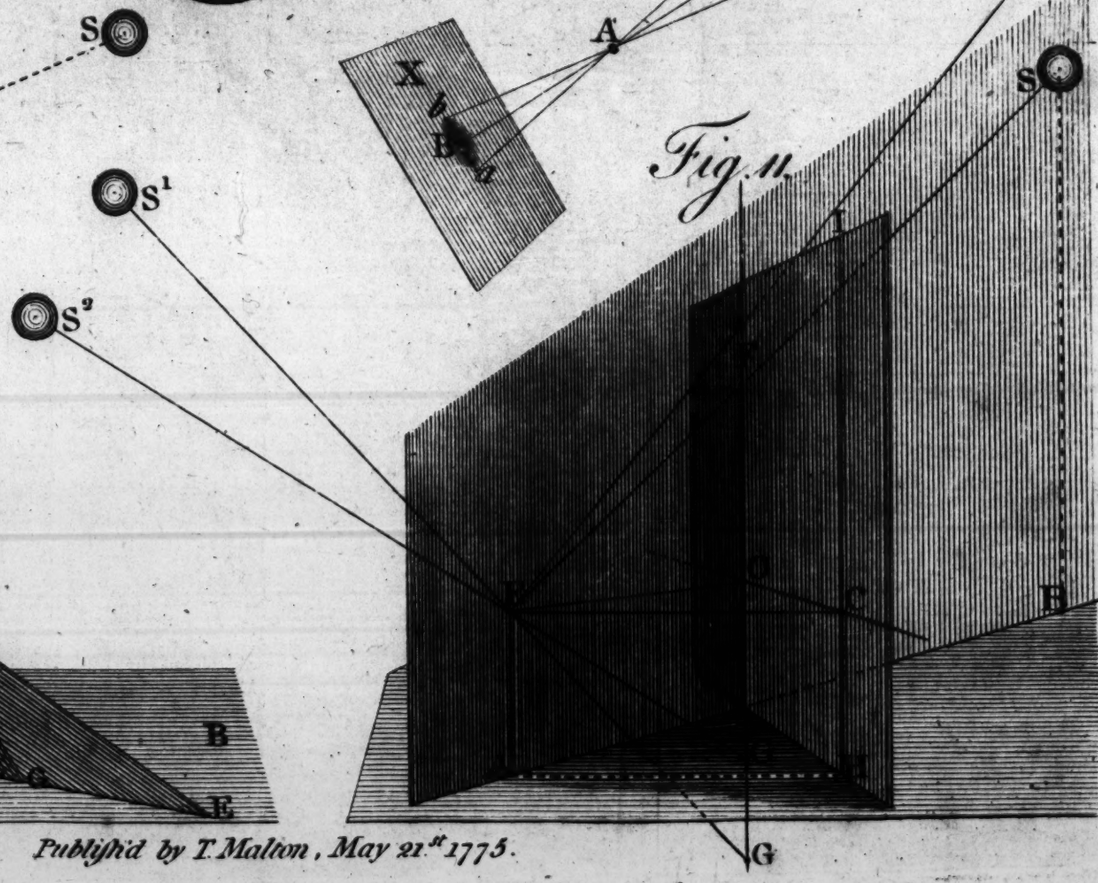


Fig. 11.

Publish'd by T. Malton, May 21.st 1775.

Plate XLI

Fig. 12.

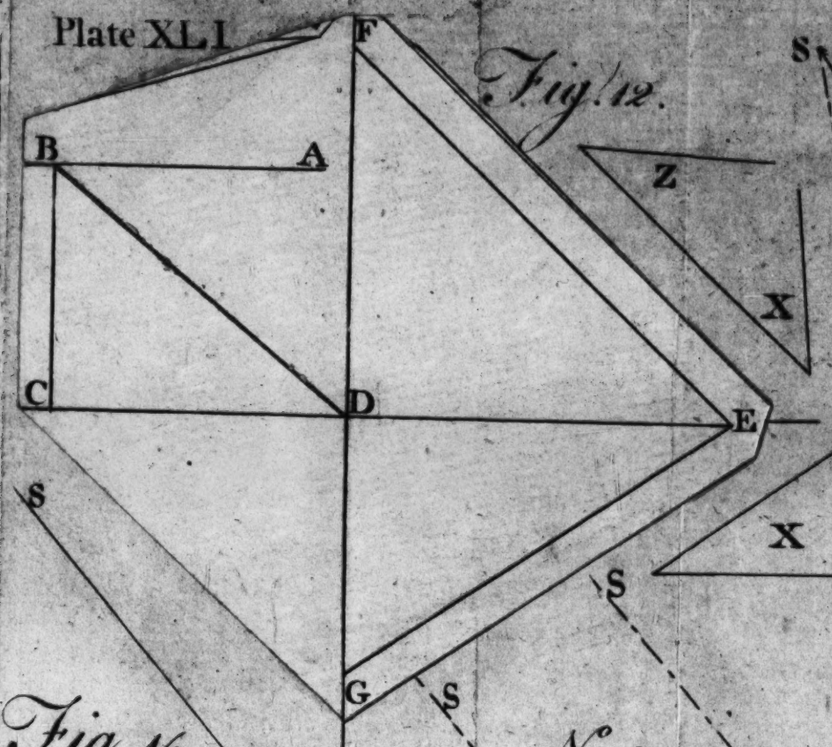


Fig. 13.

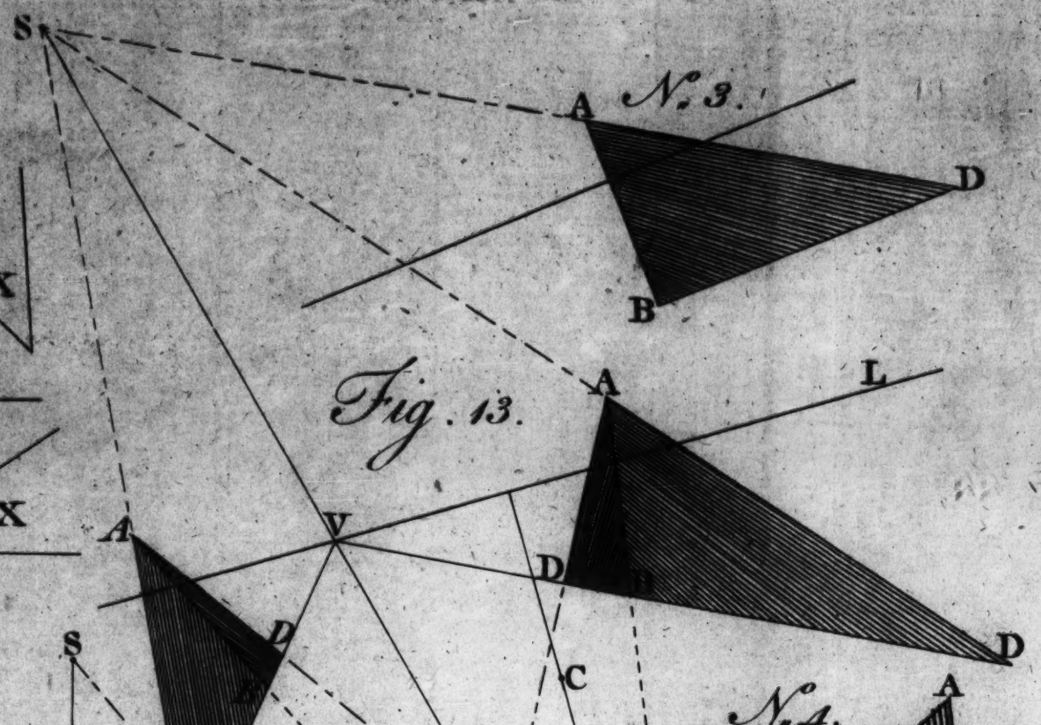
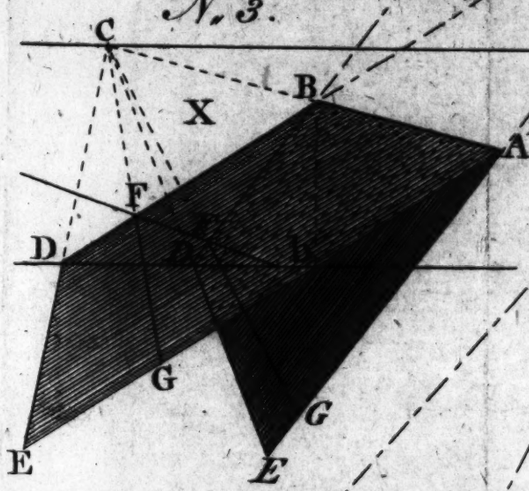


Fig. 14.





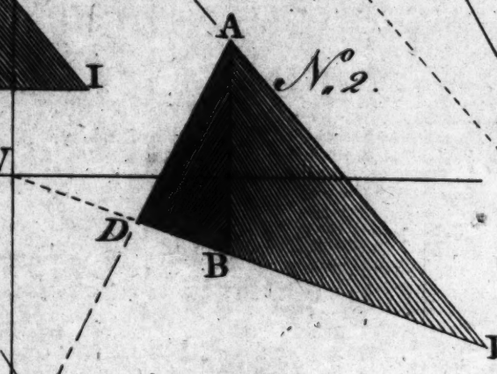
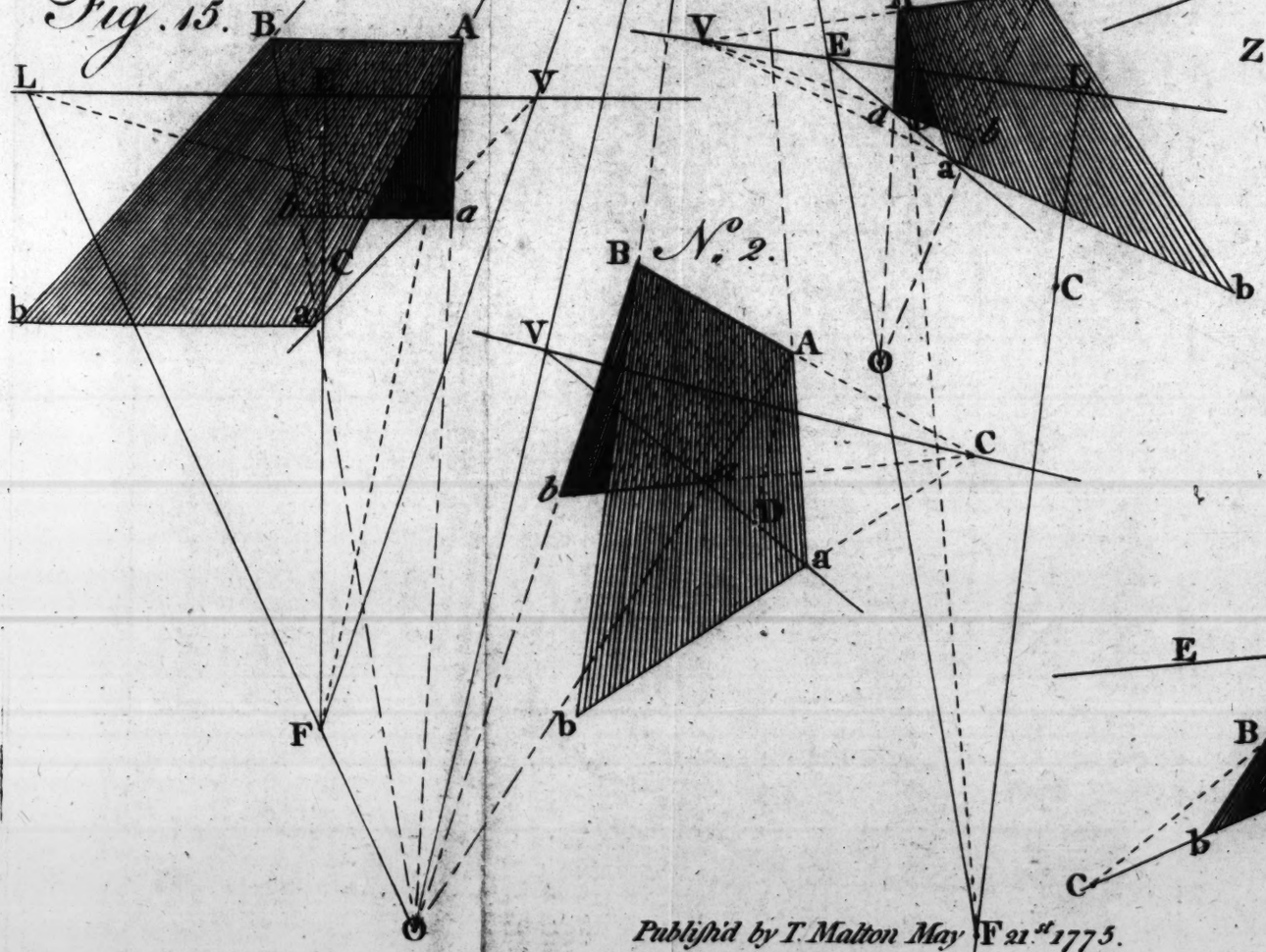
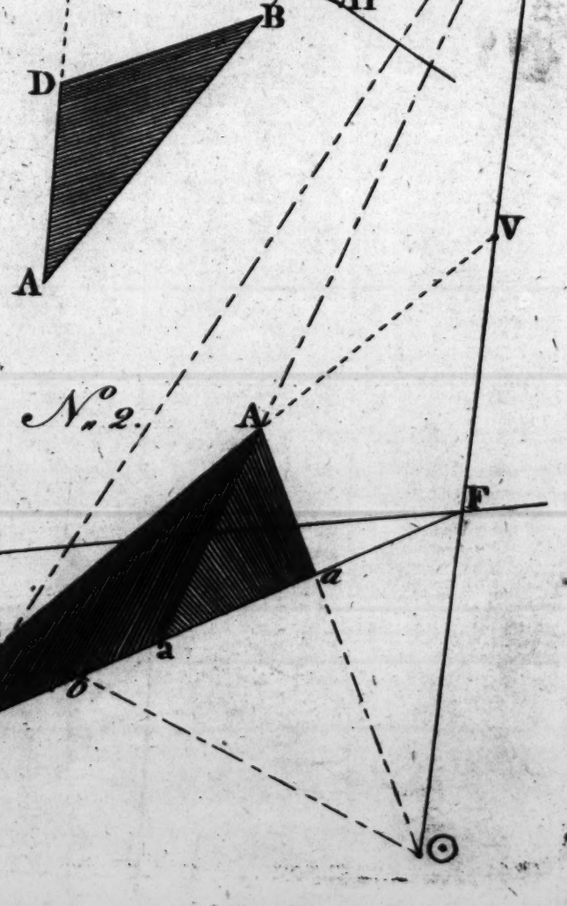


Fig. 16.



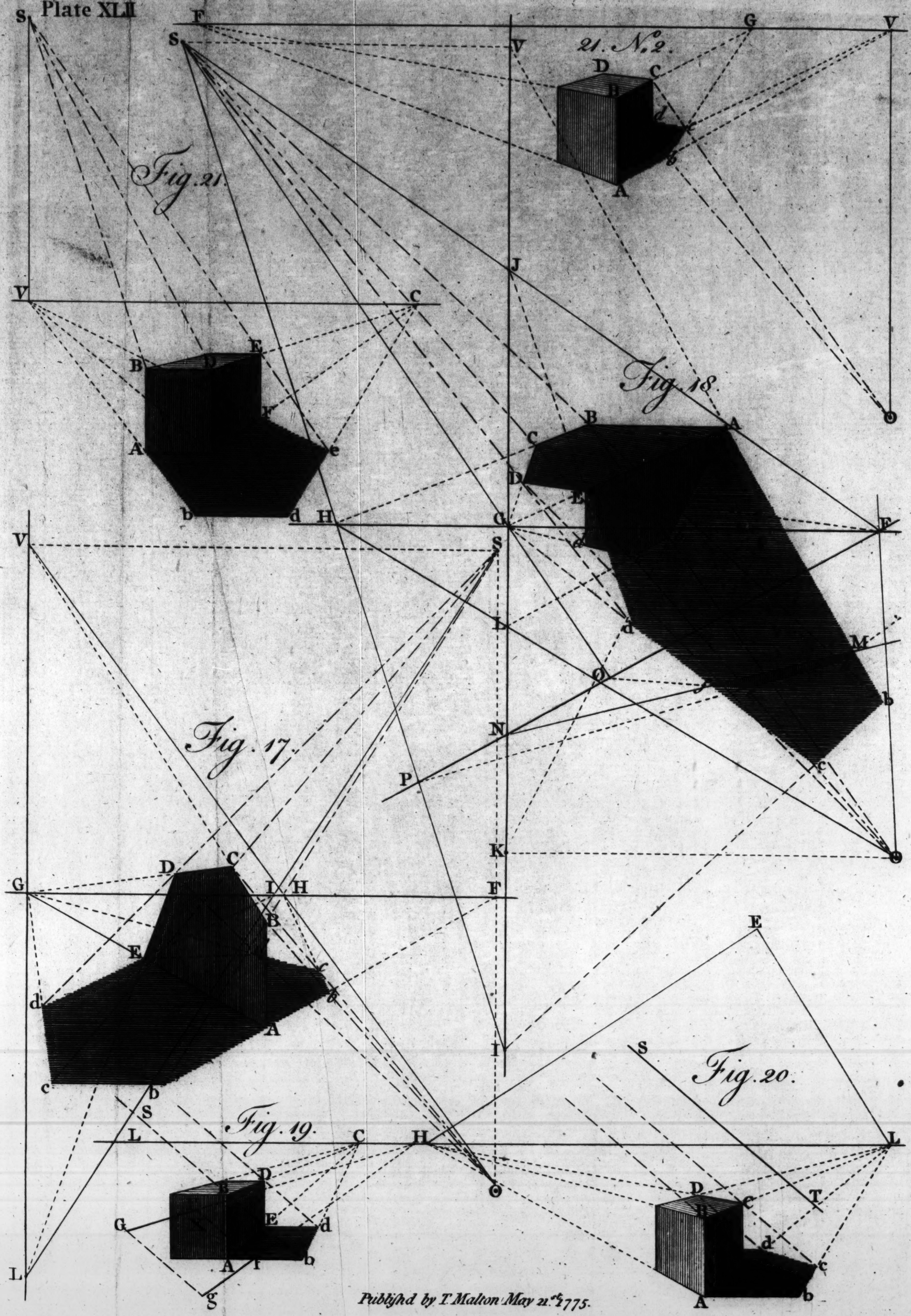
Fig. 15.





Published by T. Malton May 21st 1775.

Plate XLII



Publish'd by T. Malton May 21.st 1775.

Plate XLIII.

Fig. 22.

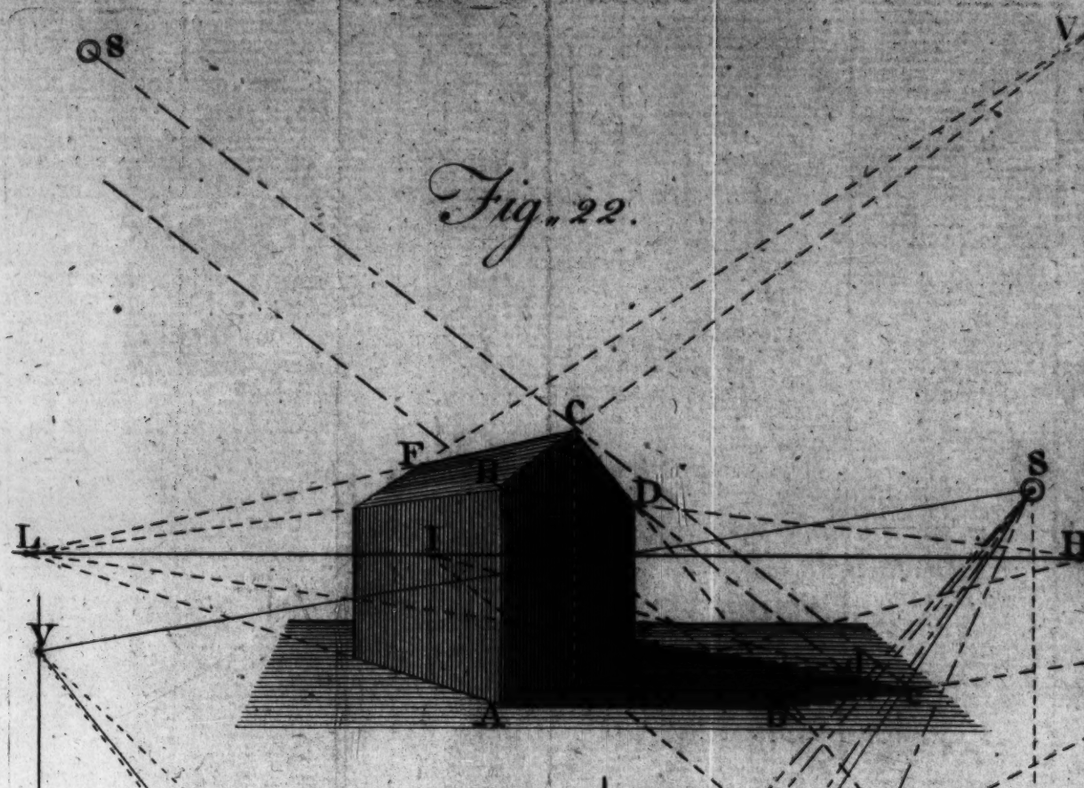


Fig. 24.

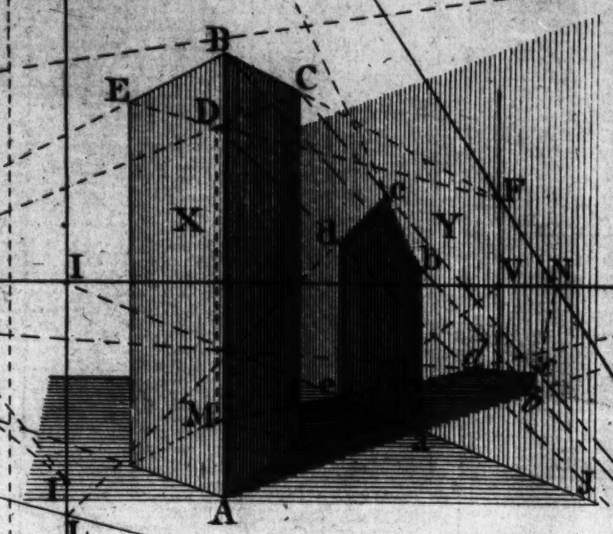


Fig. 23.

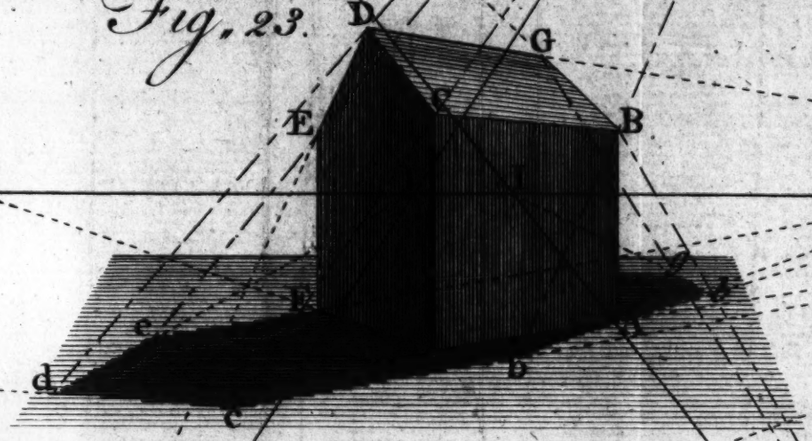


Fig. 26.

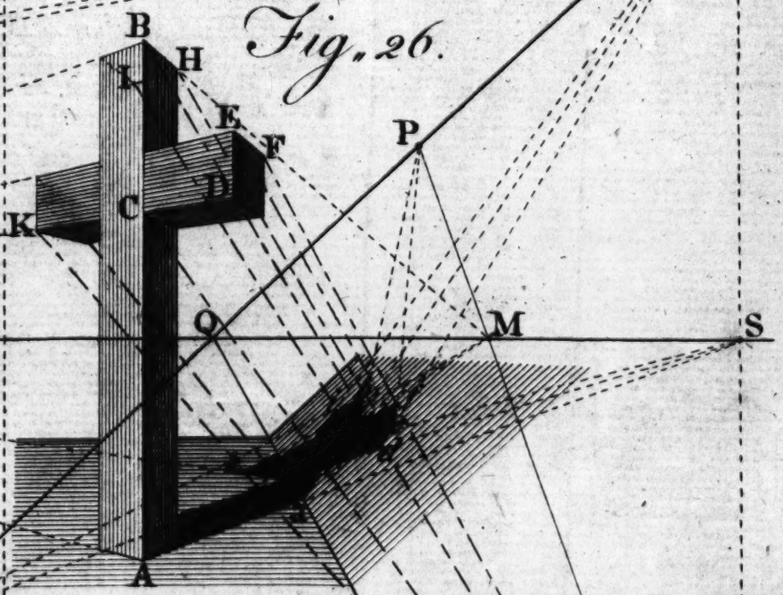
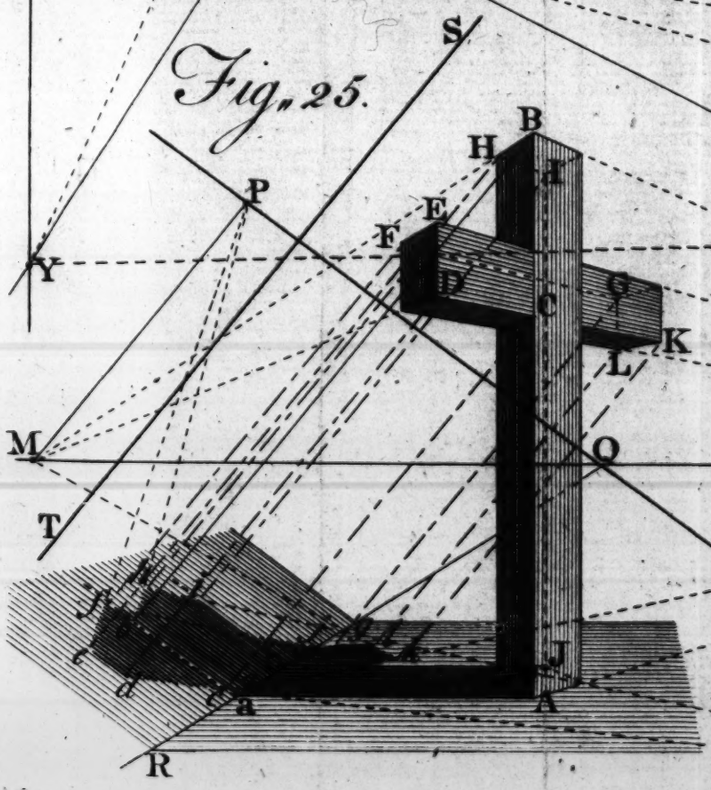


Fig. 25.



Publish'd by T. Malton May 21.st 1775.

Plate XLIV.

Fig. 29.



N°. 2.

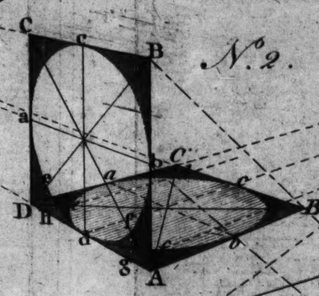
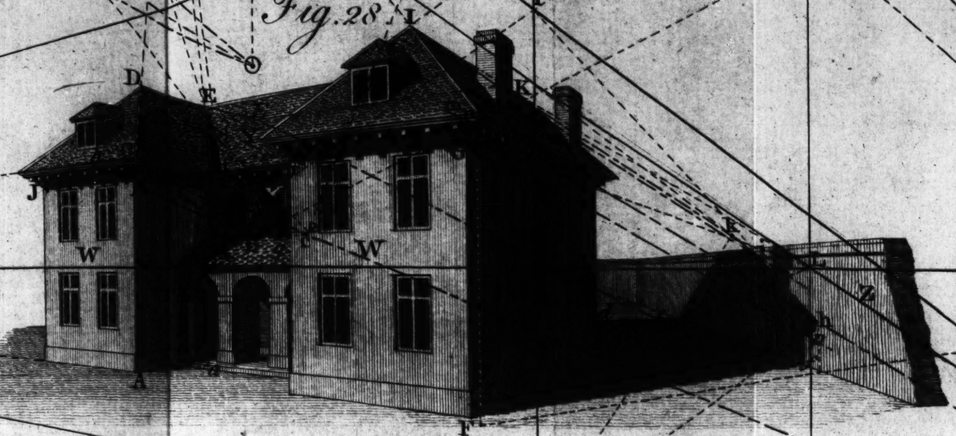


Fig. 28.



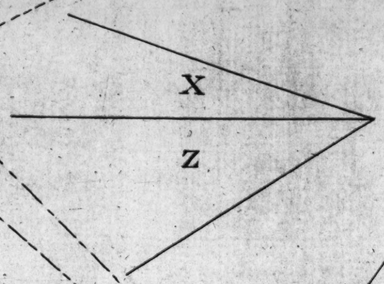


Fig. 27.

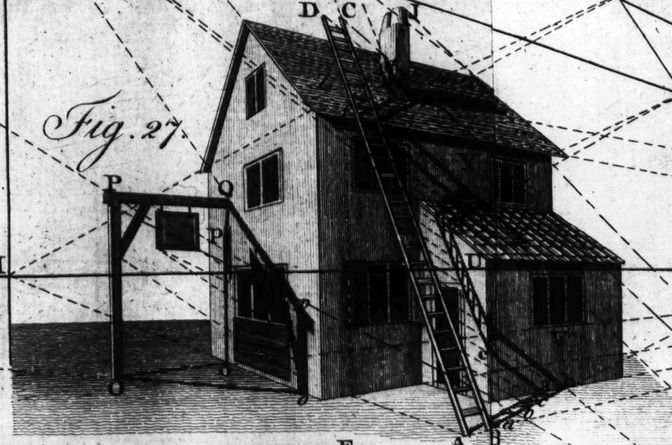


Fig. 30.

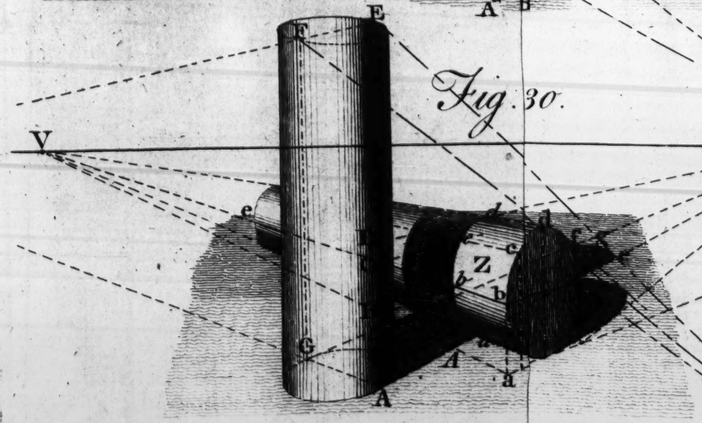
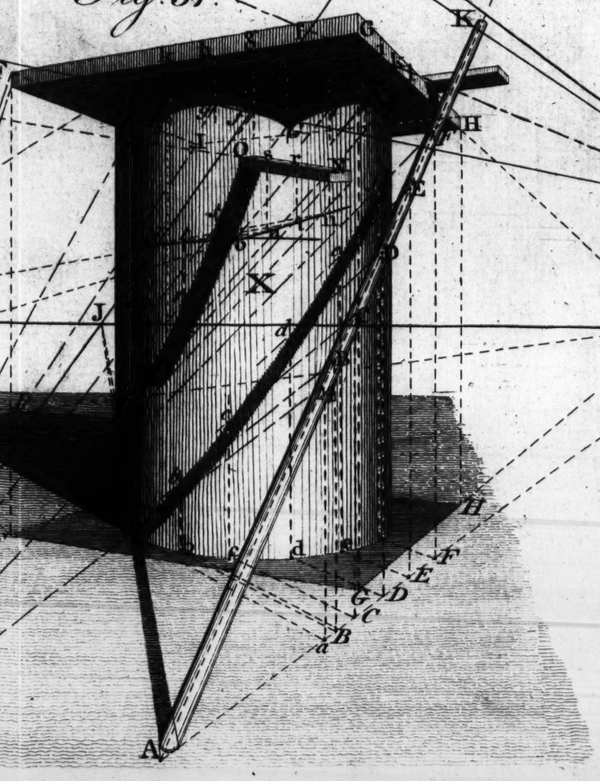


Fig. 31.



Publish'd by Thos. Malton June 20th 1775.

Plate XLV

Fig. 32.

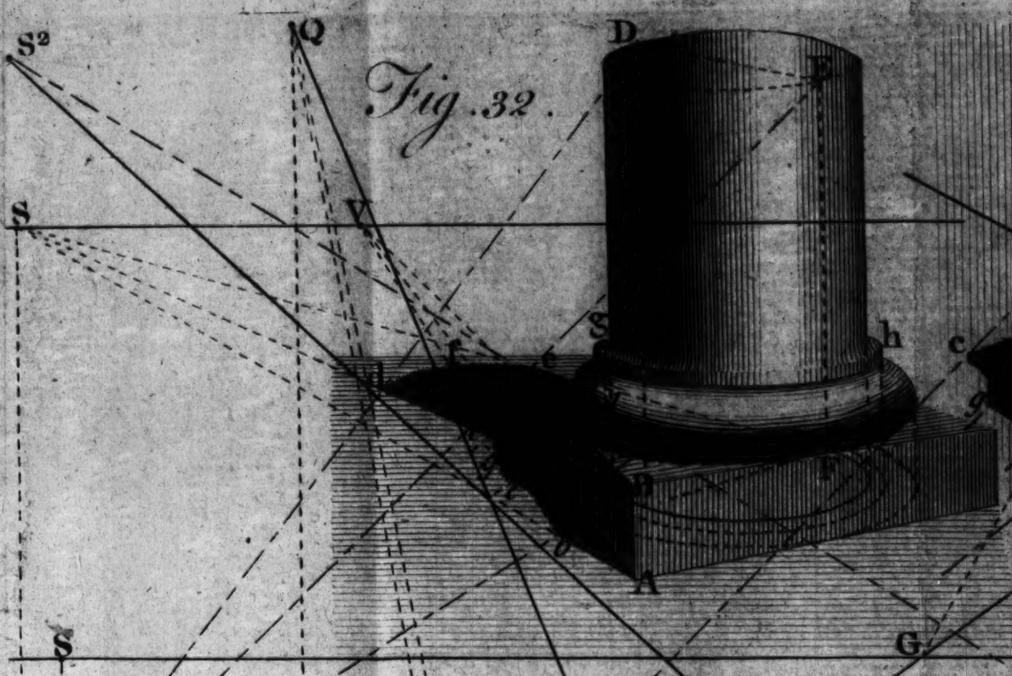


Fig. 33.



Fig. 35.

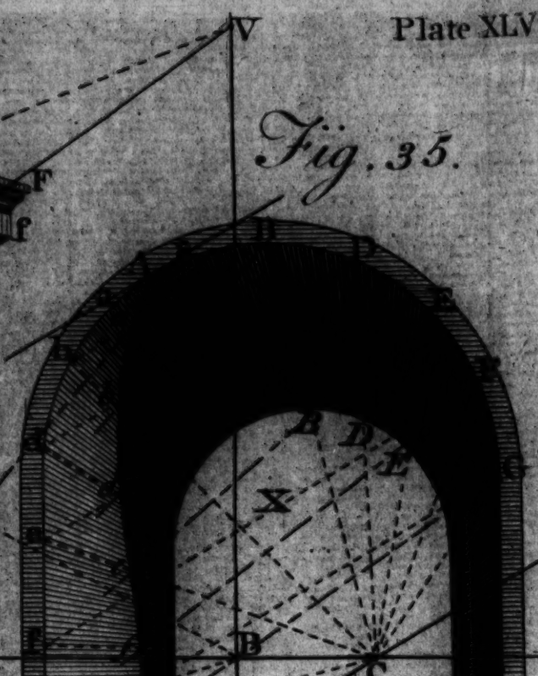


Fig. 34.

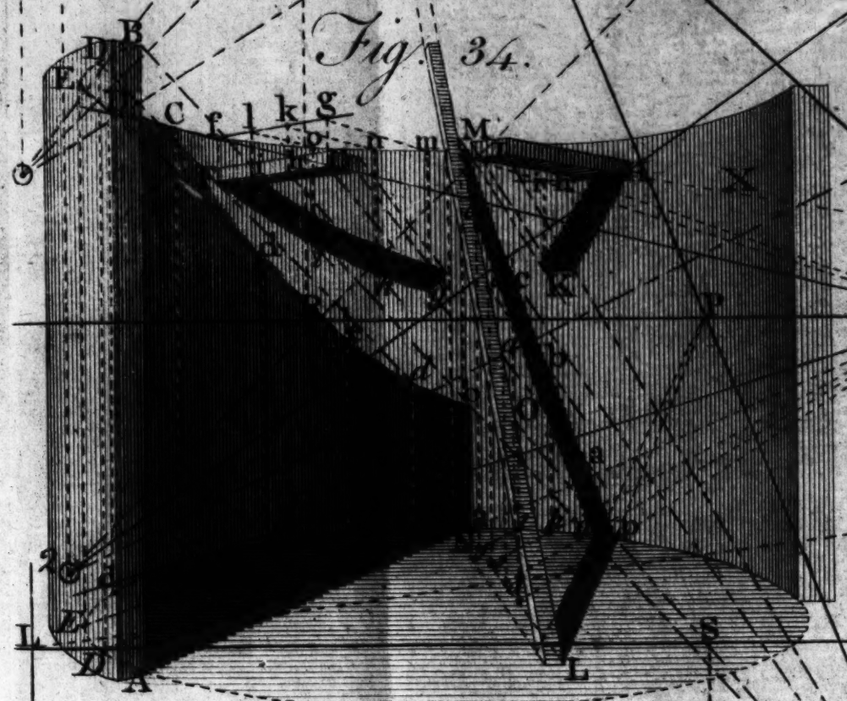


Fig. 36.

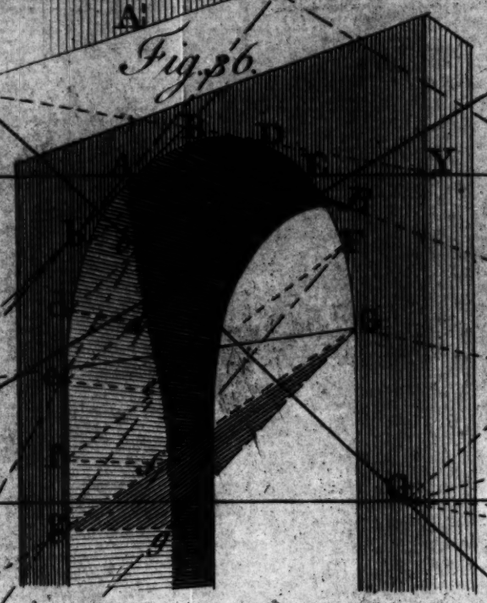


Fig. 37.

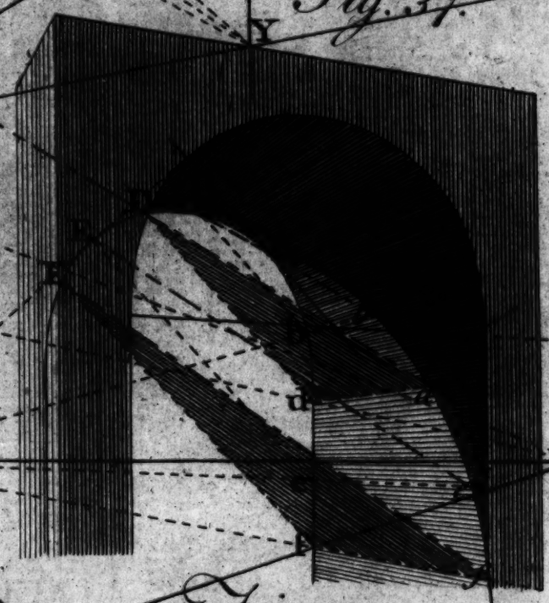


Fig. 38.

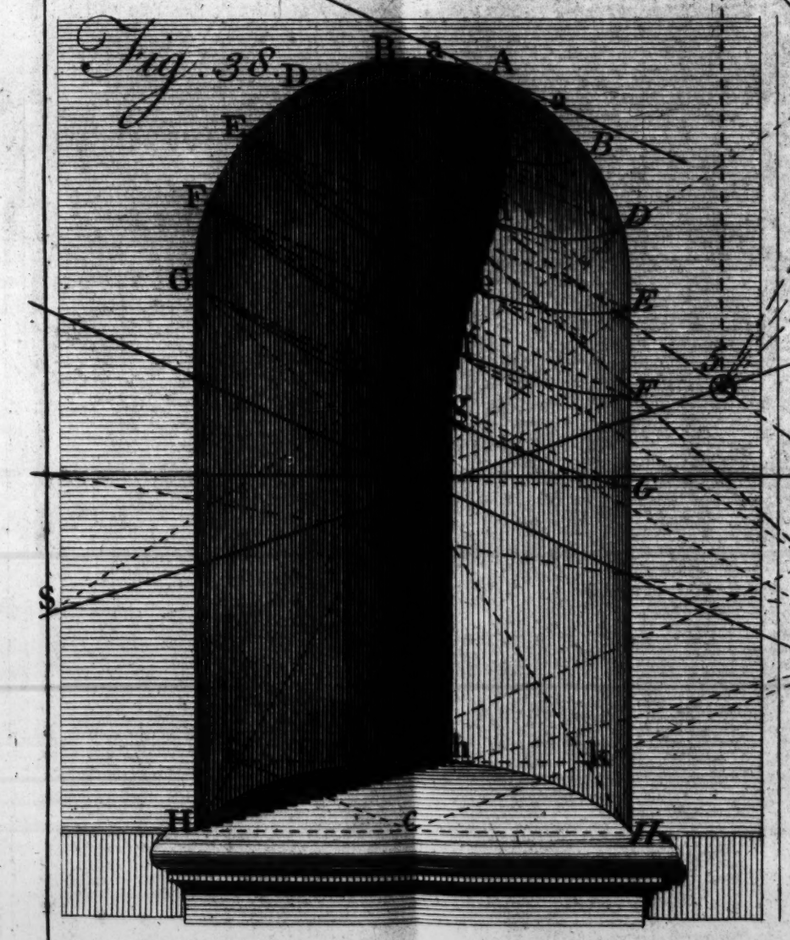


Fig. 39.



Fig. 40.



Publish'd by T. Malton June 20.th 1775.

Plate XLVI.

Fig. 41.

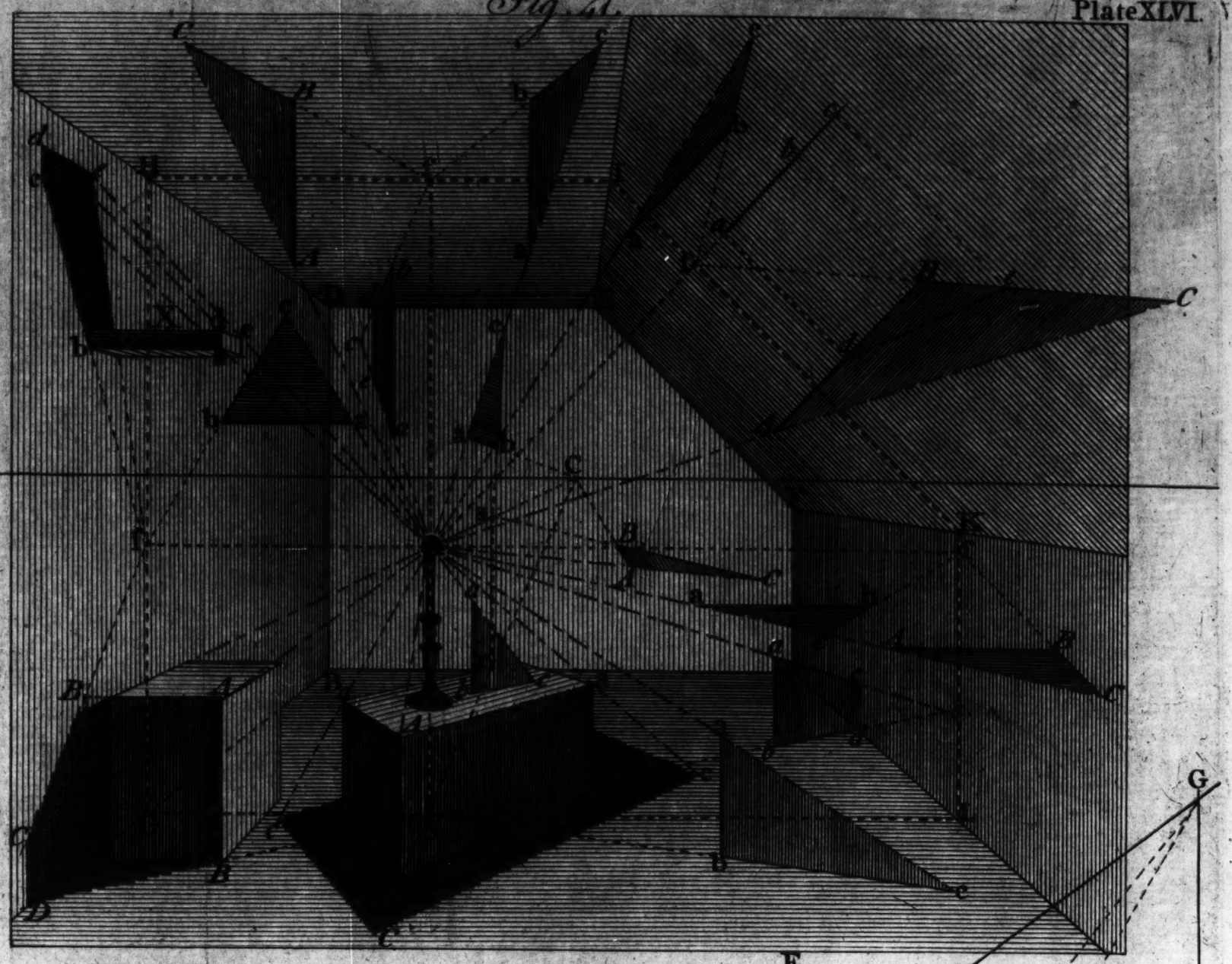


Fig. 43

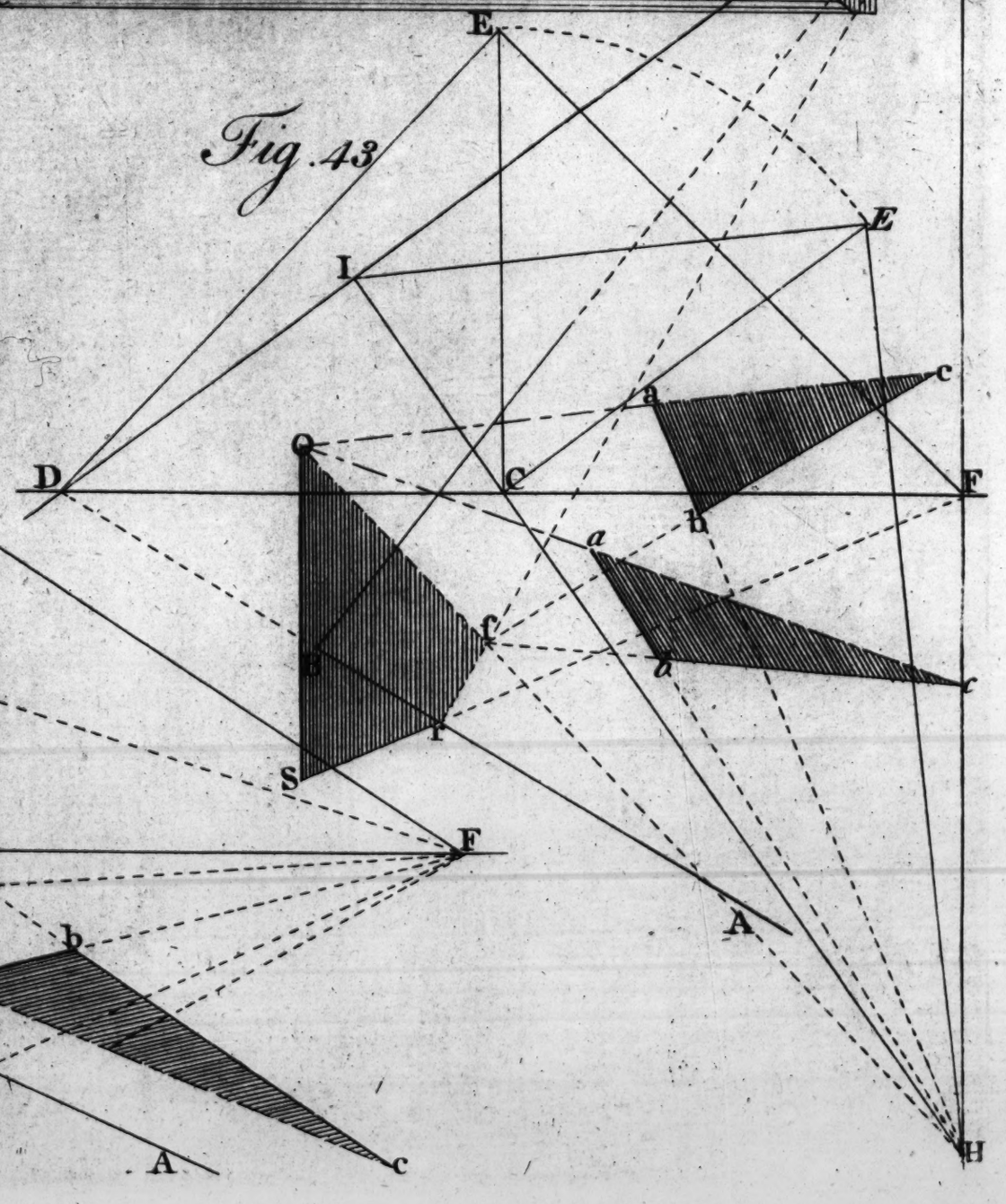


Fig. 42.

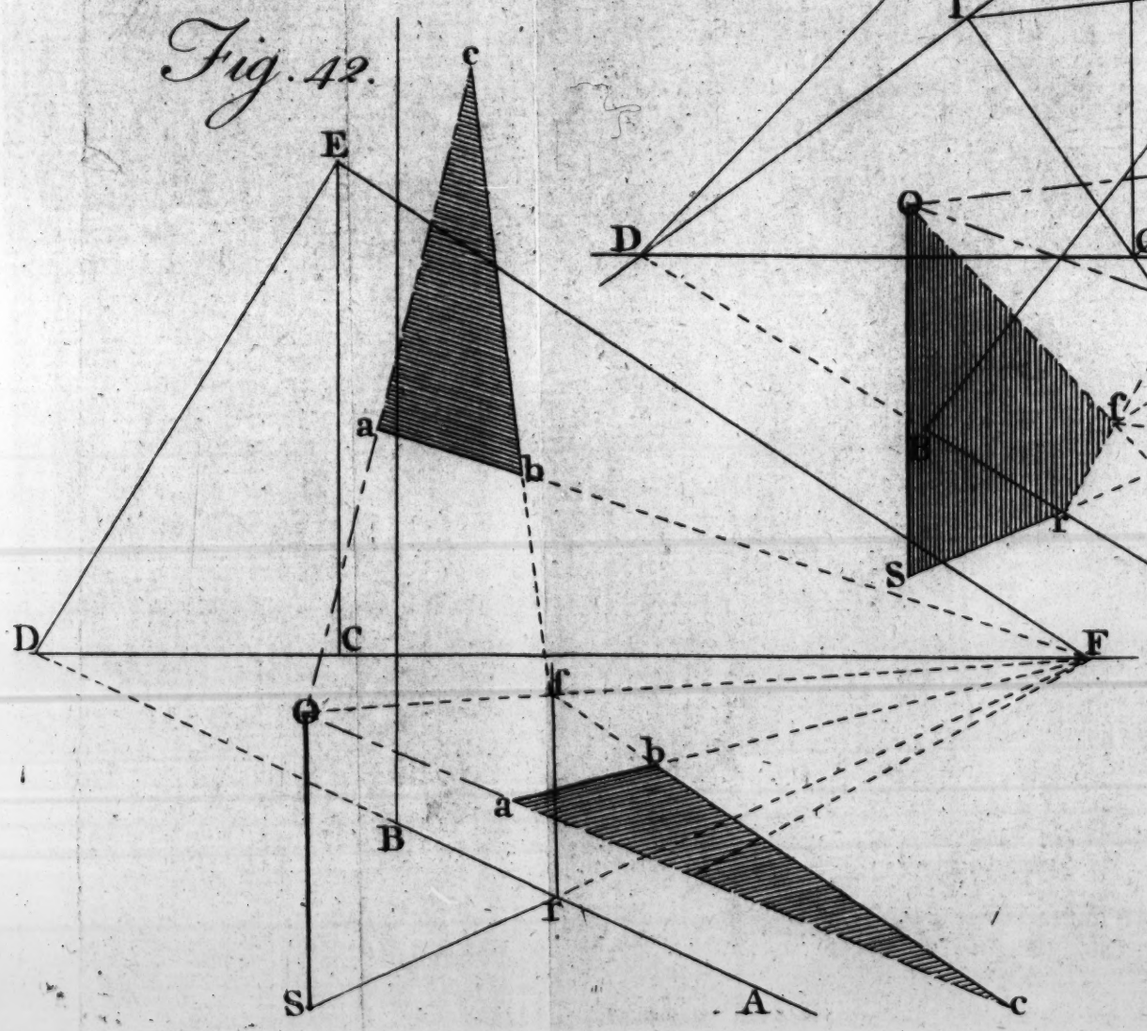


Plate XLVII

Fig. 45. N.6.

N.4

N.5

N.1

N.2

N.3

Fig. 44.

46. N.2.

Fig. 46.

N.6.

N.7

N.8

N.5

N.3

Published by T. Malton Aug.t 7.th 1774.

Plate XLVIII

Fig. 47.

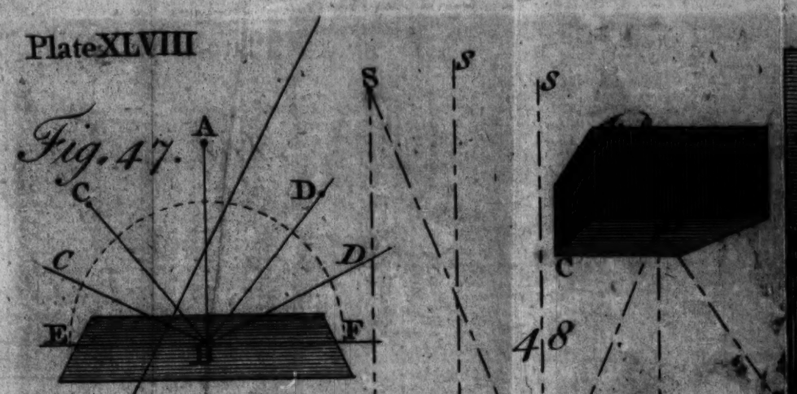


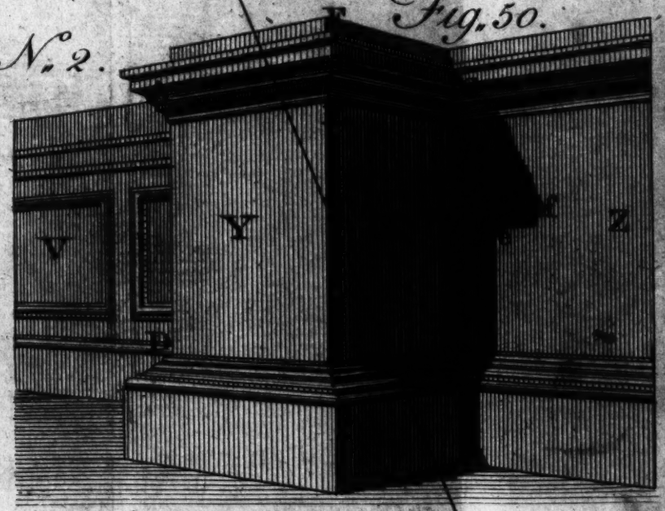
Fig. 48.

Fig. 49.



Fig. 50.

N.o 2.



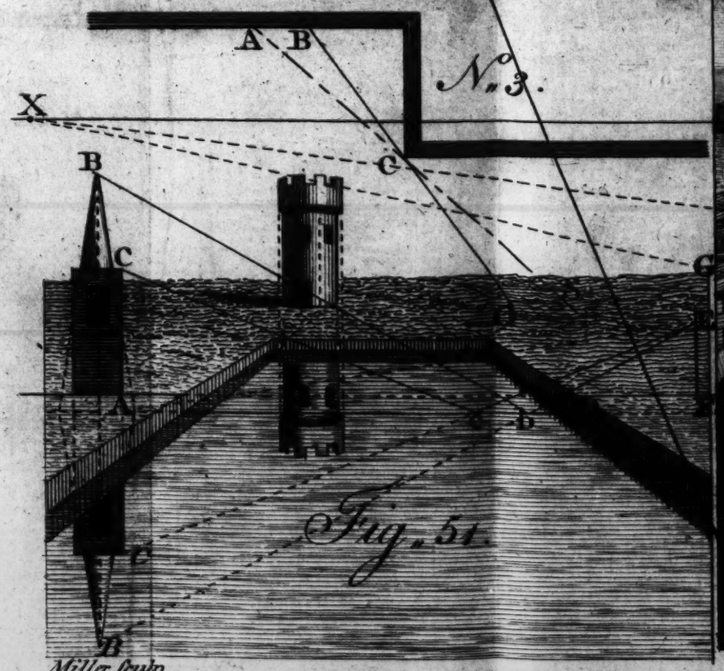


Fig. 51.

Miller sculp.

Fig. 53.



Fig. 52.



L Published by Thos Malton Sept.r 1.st 1775.